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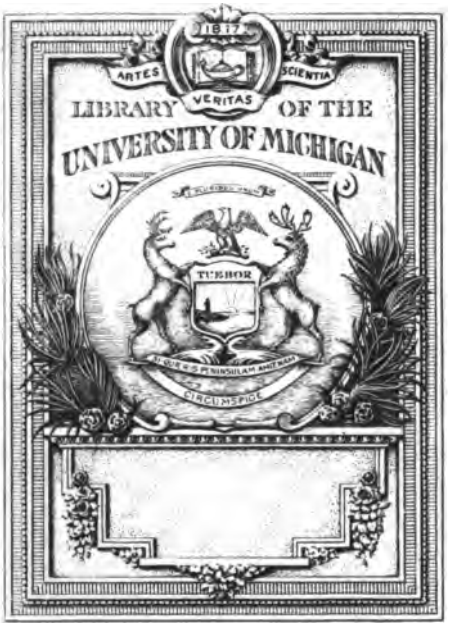

THE

UNITED STATES

ARITHMETIC.

DESIGNED FOR

ACADEMIES AND SCHOOLS.

By WILLIAM VOGDES, A. M.

PROFESSOR OF MATHEMATICS IN THE CENTRAL HIGH SCHOOL OF PHILADELPHIA.

PHILADELPHIA:
E. C. & J. BIDDLE, No. 6 SOUTH FIFTH STREET.
1847.

Adoption of the "United States Arithmetic" as a class book for the Public Schools of the City and County of Philadelphia.

CHAMBER OF THE CONTROLLERS OF PUBLIC SCHOOLS,
FIRST SCHOOL DISTRICT OF PENNSYLVANIA.

Philadelphia, April 9th, 1845.

At a meeting of the Controllers of Public Schools, held at their Chamber, on Tuesday, April 8th, 1845, the following resolution was adopted:—

Resolved, That the "UNITED STATES ARITHMETIC," by PROFESSOR VOGDES, be introduced, as a class book, into the Public Schools of the District.

From the minutes.

THOMAS B. FLORENCE,
Secretary.

Entered according to Act of Congress, in the year 1845,
By E. C. & J. BIDDLE,
In the Clerk's Office of the District Court of the United States,
of the Eastern District of Pennsylvania.

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PREFACE.

In preparing for press the present work, the author has endeavoured so to treat his subject as to draw forth the thinking powers of a child, and to insure the pupil's comprehension of it. For this purpose, every important rule has been followed by a demonstration or elucidation, in which are set forth, as clearly as possible, the reasons why the operations are to be performed according to the rule, and the principles on which the rule is founded.

The object of education being to prepare youth for the faithful performance of the duties of the respective situations in which they may be placed in after life, the author has not lost sight of the importance of making a school book *practical* in its character, but has introduced many matters with this end especially in view; and has so prepared the examples under the various rules, as to impart much useful information, and to give correct ideas of the modes of transacting business in mercantile and other industrial pursuits of men.

Many teachers have felt the want of a sufficient number of examples in most of the popular treatises on arithmetic. In this the number of examples exceeds that in any other book now in use, in proportion to its size.

The entire separation of mental and written exercises, and the placing of them in different volumes, as adopted in several popular works, the author deems objectionable, as it is calculated to make two distinct impressions of one and the same process, and therefore less likely to lead to a full understanding of the principles involved. In this work the mental and written exercises are placed together under one rule, and questions are appended by which the learner may examine and teach himself.

The principles of mensuration, embracing artificers' work, have been introduced, and it is believed treated of, at sufficient length to prove of lasting value to the future mechanic, who, as a schoolboy, may master these pages.

In the hope that it may not be found wanting in the points above set forth, this work is now submitted to the examination of the public.

☞ The first 104 pages of this work, embracing the rules preceding FRACTIONS, together with four pages of miscellaneous oral exercises, and twelve pages of miscellaneous written exercises, are published as THE FIRST PART OF THE UNITED STATES ARITHMETIC.

The vast number of pupils engaged in studying the elementary rules of Arithmetic, who never progress to the more advanced rules, and the greater length of time generally devoted to the former than the latter, by those who study the whole of the subject, satisfactorily prove that such a division will subserve economy; and therefore, it is presumed, will meet the approval of teachers and directors of schools.

Philadelphia, February, 1845.

A KEY to the United States Arithmetic, designed for the use of teachers, has been published.

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CHARACTERS USED IN THIS WORK.

$+$ This sign denotes addition; and is read plus, as $4+7$; that is, 4 added to 7, which is equal to 11.

$-$ This sign denotes subtraction; and is read minus, as $9-4$; that is, from 9 take 4, or 9 minus 4, which is equal to 5.

$\times$ The sign of multiplication; and is read multiplied by, as 5×3; that is, 5 multiplied by 3, which is equal to 15.

$\div$ The sign of division; and is read divided by, as $6\div2$; that is, 6 is to be divided by 2, which is equal to 3.

$=$ The sign of equality; and is read equal to, as $2+4=6$; that is, 2 added to 4, are equal to 6.

: is to : : so is : The sign of proportion; thus, 3 : 4 : : 6 : 8; that is, as 3 is to 4 so is 6 to 8.

4^2, 4^3, 4^4. Signs of involution; denoting that 4 is to be squared, cubed, or raised to the 4th power.

$\surd$ or $\sqrt{\ }$ Signs of the square root.

$\sqrt[3]{\ }$ Sign of the cube root.

$\therefore$ Therefore, or consequently.

ARITHMETIC.

§ 1. ARITHMETIC is the art of computing by numbers.

Q. What is the meaning of arithmetic?

A. It is the art of computing by numbers, and exhibits the doctrine of numbers, explains their properties, and teaches the mode of calculating by them.

Q. What is unity?

A. Unity, or a unit, is that by which every thing is regarded as one, being the beginning of number.

Q. What is meant by number?

A. Number is that which is composed of one or more units.

Q. What is an integer?

A. Any whole number is called an integer.

Q. What is meant by a composite number?

A. Any number which can be produced by multiplying two or more numbers together, each of which is greater than a unit, is called a composite number. Thus, 24 is a composite number, and its component parts are either 4 and 6, 3 and 8, or 2, 3 and 4.

Q. What is meant by a prime number?

A. A number which cannot be produced by the multiplication of two others, or which can be measured only by itself or a unit, is called a prime number. Thus, 1, 2, 3, 5, 7, 11, 13 and 17.

Q. What is an abstract number?

A. A number expressed without naming the species of the units; as when we say simply four or four times, five or five times.

Q. What is a concrete number?

A. A concrete number is always applied to some kind of thing, as when we say three dollars, four books, &c.

Q What is meant by quantity?

A. Every thing which is susceptible of increase or decrease; as extent, duration, weight, &c.

Q. What are the fundamental rules of arithmetic?

A. Notation, numeration, addition, subtraction, multiplication and division.

NOTATION AND NUMERATION.

§ **2.** Notation is the writing of numbers by figures; and numeration is the art of reading figures correctly.

The characters made use of in numeration, and the names of the numbers which they represent are as follows:

Cipher, or nought.	Unit, or one.	Two.	Three.	Four.	Five.	Six.	Seven.	Eight.	Nine.
0	1	2	3	4	5	6	7	8	9

These characters, with the exception of the cipher, are called digits, or significant figures.

Besides this value of the figures, they have another, which depends upon the place they stand in when joined together; as in the following

NUMERATION TABLE.

Hundreds of millions	Tens of millions	Millions	Hund. of thous.	Tens of thous.	Thousands	Hundreds	Tens	Units	
							2	1	One.
						3	2	1	Twenty-one.
					4	3	2	1	Three hundred and twenty-one.
				5	4	3	2	1	4 Thousand 321.
			6	5	4	3	2	1	54 Thousand 321.
		7	6	5	4	3	2	1	654 Thousand 321.
	8	7	6	5	4	3	2	1	7 Millions 654 thousand 321.
9	8	7	6	5	4	3	2	1	87 Millions 654 thousand 321.
									987 Millions 654 thousand 321.

The above Table is comprised in the following one.

Units	1	Units.
Tens	2	
Hundreds	3	
Thousands	4	Thousands.
Tens of thousands	5	
Hundreds of thousands	6	
Millions	7	Millions.
Tens of millions	8	
Hundreds of millions	9	

The figure in the first place, reckoning from right to left, denotes only its simple value; that in the second row, ten times its simple value; that in the third row, a hundred times its simple value, and so on, the value of any figure in each successive place, being always ten times its former value. Thus, in the number 1438, the 8 in the first place signifies only eight; 3 in the second place signifies three tens, or thirty; 4 in the third place four hundred; the 1 in the fourth place one thousand; and the whole number is read thus, one thousand four hundred and thirty-eight.

A nought, or cipher, stands for nothing of itself, but being placed on the right hand of other figures, it increases their value in the same tenfold proportion; thus 6 signifies only six; but 60 signifies six tens, or sixty; 600 is six hundred, &c.

For the more easy reading of large numbers they are divided into periods consisting of three figures each; the name of the first period being units, of the second thousands, of the third millions, of the fourth billions, of the fifth trillions, of the sixth quadrillions, &c. as in the following table:

6th period or period of quadrillions.	5th period or period of trillions.	4th period or period of billions.	3d period or period of millions.	2d period or period of thousands.	1st period or period of units.
123	456	789	698	765	432

The preceding table is thus read: one hundred and twenty-three quadrillions, four hundred and fifty-six trillions, seven hundred and eighty-nine billions, six hundred and ninety-eight millions, seven hundred and sixty-five thousand four hundred and thirty-two.

Q. Upon what does the value of figures depend?

A. The value of figures depends upon the place in which they stand.

Q. When is the word million used?

Q. When is the word billion used?

Q. How many places are there between units and millions?

Q. What difference in value does one place to the left make?

Q. What difference is made by two places to the left, and what by two places to the right?

THE ROMAN NOTATION.

I	1 one	XXIV	24 twenty-four
II	2 two	XXV	25 twenty-five
III	3 three	XXVI	26 twenty-six
IV	4 four	XXVII	27 twenty-seven
V	5 five	XXVIII	28 twenty-eight
VI	6 six	XXIX	29 twenty-nine
VII	7 seven	XXX	30 thirty
VIII	8 eight	XL	40 forty
IX	9 nine	L	50 fifty
X	10 ten	LX	60 sixty
XI	11 eleven	LXX	70 seventy
XII	12 twelve	LXXX	80 eighty
XIII	13 thirteen	XC	90 ninety
XIV	14 fourteen	C	100 one hundred
XV	15 fifteen	CC	200 two hundred
XVI	16 sixteen	CCC	300 three hundred
XVII	17 seventeen	CCCC	400 four hundred
XVIII	18 eighteen	D	500 five hundred
XIX	19 nineteen	DC	600 six hundred
XX	20 twenty	DCC	700 seven hundred
XXI	21 twenty-one	DCCC	800 eight hundred
XXII	22 twenty-two	DCCCC	900 nine hundred
XXIII	23 twenty-three	M	1000 one thousand

MDCCCXLV: one thousand eight hundred and forty-five.

EXAMPLES.

Write in words the following numbers.

1.	5	4.	1,634	7.	69,874
2.	35	5.	23,097	8.	431,245
3.	421	6.	58,702	9.	9,847,630

Write in figures the following numbers.

10. Sixty-one.
11. Two hundred and twenty-five.
12. One thousand one hundred and four.
13. Three hundred and twenty-eight thousand.
14. One million four hundred and six thousand.
15. Two millions four hundred and five thousand six hundred and forty-four.
16. Eighty-five millions two hundred and one.

17. Nine hundred and thirty-seven millions two hundred and fifty-three thousand two hundred and twenty-one.

18. One billion one million one thousand and one.

ADDITION.

§ 3. Addition is that operation by which we find the sum of two or more given numbers.

Q. What does addition mean?

A. It is the finding of one number that shall express as many as are contained in, or are equal to, any number of separate numbers.

Q. What are those numbers called?

A. They are called the given or particular numbers; and the number found, that which contains as many as are in all the particular ones, is called the sum of them.

Q. Does not the operation of adding, or addition, shorten the expression of numbers?

A. Yes. It enables one to read or write as one whole, that which is already written as a number of particulars.

Q. Does the sum always express the same value as all the particulars?

A. It does; for if it did not, the addition would be of no use.

Q. Are there any conditions required in order that the sum may do this?

A. There are, and they must always be carefully observed, otherwise there can be no meaning in that which is done.

Q. Cannot they be reduced to one?

A. They can; two numbers cannot be expressed by one number as their sum, unless a unit or 1, in one of them, be precisely equal to a unit or 1, in the other.

Q. What are the particular conditions by which this general one may be secured?

A. There are three: 1st. If the numbers stand for things, those things must be exactly of the same kind. 2d. They must both be of the same denomination. 3d. Two figures cannot be added together unless they hold the same place in the scale of numbers.

Q. Can you give me some instances?

A. Two dollars and ten yards, are not a sum or whole of any kind, and cannot be made or considered so by any

means. One dollar and four cents do make a sum, because they both mean money, but they cannot be expressed by the single number 5, though that number contains just as many times 1 as are in them both. But one dollar and four dollars, or four cents and one cent, can each be expressed by the sum 5, that being 5 dollars in the case of dollars, and 5 cents in the case of cents. One ten and four units, cannot be expressed by 5 of any thing, or by any other sum than 14, but one ten and four tens can be expressed by the sum 50, and one unit and four units by the sum 5.

Q. What sign is used for addition?

A. The sign +, which is called plus, or more; and means that the numbers between which it is placed are to be added together; as 3+2.

Q. What is meant by the sign =?

A. This sign is used to express equality.

Q. What is the operation of adding?

A. Finding a number which shall express the total value of several numbers.

MENTAL EXERCISES.

1. How many are 3 and 5? How many are 4 and 6? 7 and 2? 8 and 3? 9 and 5? 10 and 8? 7 and 5? 5 and 6? 5 and 11? 7, 3 and 8? 4 and 10? 4 and 7? 5 and 8? 6 and 8? 8 and 9? 9 and 6? 6 and 6? 6 and 10? 9 and 2? 7 and 9? 8 and 7? 6 and 12? 11 and 4? 4 and 7? 12 and 3? 4 and 14? 4, 2 and 3? 5, 1 and 7? 11, 7 and 2? 5, 8 and 4? 6, 7 and 4? 9, 6 and 7? 8, 3 and 2? 8, 5 and 6? 7, 4 and 9? 11, 7 and 8? 9+4+3? 6+1+2? 1+1+9? 8+9+7?

RULE.

1. Place the numbers under each other, so that the units may stand under units, tens under tens, &c., and draw a line under them.

2. Add up the figures in the row of units, and find how many tens are contained in their sum.

3. Write down what remains above the tens, or if nothing remains, a cipher, and carry as many ones to the next row as there were tens.

4. Add up the second row, together with the number carried in the same manner as the first; and proceed thus till the whole is finished.

The reason why we carry 1 for every 10, in adding integers, is, because numbers increase in a tenfold proportion from right to left, and therefore 10 in any column is equal to 1 in the next column towards the left hand.

If several numbers are to be added together, they will amount to the same sum, when placed regularly under each other, whichever line or row of figures stands uppermost.

PROOF.

1. Draw a line below the uppermost number, and suppose it cut off.

2. Add all the rest together, and write their sum under the number to be proved.

3. Add this last found number and the uppermost line together, and if the sum be the same as that found by the first addition, the work is correct.

This rule, as well as the method of proof, is founded on the known axiom, that the whole is equal to the sum of all its parts. All that requires explaining is the method of placing the numbers, and carrying for the tens, both which are evident from the nature of notation, for any other disposition of the numbers would entirely alter their value, and carrying one for every ten from an inferior line to a superior is evidently right, since a unit, in the latter case, is equal in value to ten in the former, or ten in the line of units make one in the line of tens; ten in the line of tens make one in the line of hundreds, &c. Besides the method here given, there is another very ingenious one of proving addition, by casting out the nines, thus:

1. Add the figures in the uppermost row together, and find how many times nine is contained in their sum.

2. Reject the nines, and write down the remainder directly even with the figures in the row.

3. Do the same with each of the other rows, and write all these excesses of nine together, in a line, and find their sum; then, if the excess of nines in this sum, found as before, be equal to the excess of nines in the total sum, the work is correct.

This method depends upon a property of the number 9, which, except 3, belongs to no other digit whatever; that any number divided by 9, will leave the same remainder as the sum of its figures, or digits, divided by 9, which may be thus demonstrated.

DEMONSTRATION.—Let there be any number, as 8467, this, separated into its several parts, becomes $8000+400+60+7$; but $8000=8\times1000=8\times(999+1)=8\times999+8$. In like manner $400=4\times99+4$, and $60=6\times9+6$. Therefore, $8467=8\times999+4\times99+6\times9+8+4+6+7$; and $8467\div9=(8\times999+4\times99+6\times9+8+4+6+7)\div9$. But $8\times999+4\times99+6\times9$ is evidently divisible by 9; therefore, if 8467 be divided by 9, it will leave the same remainder as $8+4+6+7$ divided by 9, and the same will be found true for any other number whatever.

This rule was first given by Dr. Wallis, in his Arithmetic published A. D. 1657, and is a very easy and simple method; though it is liable to this inconvenience, that a wrong operation may sometimes appear to be correct; for if we change the places of any two figures in the sum, it will still be the same; but then a true sum will always appear to be so by this proof; and to make a false one appear true, there must be at least two errors which are directly opposite to each other; and if there are two or more errors they must mutually balance, but the chance against this particular circumstance is so great, that we may as safely trust to this proof as to any other.

Q. How do you set down integers to be added?
Q. Why do you carry by 10 in adding integers?
Q. How do you prove addition?

EXAMPLES.

	(1.)		(2.)		(3.)		
	8214		32408		124673	5	Excess of nines.
	3101		68971		418742	8	
	2456		7342		389415	3	
	7940		678		910263	3	
	3042		45		645847	7	
	1421		78		215798	5	
Sum	26174	Sum	109522	Sum	2704738	4	Proof.
	17960		77114				
Proof	26174	Proof	109522				

4. $352+134+535+213+402$.
5. $244+434+244+145+435$.
6. $311+234+533+541+755$.
7. $423+342+645+613+604$.

8. 413+553+723+657+676.
9. 563+126+226+248+638.
10. 205+323+649+569+795.
11. 623+538+1121+1642+4972.
12. 278+147+1396+4576+7854.
13. 556+711+3141+5236+2618.
14. 489+8486+3174+9244+6217.
15. 81+6545+5784+5931+1745.
16. 12+3741+8165+2864+2749.
17. 47+89+444+1432+1375+2378.
18. 111+808+999+4296+4125+4556.
19. 2+101+888+2148+2062+9401.
20. 479+4286+6175+1582+7953+4700.
21. 84+143+7649+3164+4008+8334.
22. 71403+54270+24152+132+37046+75359.
23. 74126+14935+67398+98468+12976+892.
24. 1149+9411+1090+68798+84297+98049.
25. 3219+47312+4576+34399+984+37046.
26. What is the sum of 942, 376 and 352?
27. What is the sum of 1334, 4367 and 2421?
28. What is the sum of 842, 373 and 575?
29. What is the sum of 759, 889 and 999?
30. What is the sum of 876, 321 and 543?
31. What is the sum of 733, 345 and 677?
32. What is the sum of 540, 421, 372 and 1233?
33. What is the sum of 444, 588, 855 and 1297?
34. What is the sum of 168, 156, 124, 135 and 583?
35. What is the sum of 4621, 2146, 3754, 1405 and 3246?
36. What is the sum of 74752, 14521, 31662 and 54471?
37. What is the sum of 5246, 3625, 6474 and 5725?

38. A wholesale grocer, in four months sold sugar to the following amount:—in October, 4796, in November, 5794, in December, 7846, and in January, 8034 hhds. What was the whole number of hhds. sold?

39. A locomotive at one time drew a train of cars laden with 22321 pounds, at another, 41332, at a third, 12123, and at a fourth, 13220. How many pounds were drawn in all?

40. In one month there were paid into the treasury of the United States 222311 dollars, in another, 131232, in a third, 101221, and in a fourth, 234031. How much did the treasury receive altogether?

41. In 1839, there arrived in the U. States, 11188 Ameri-

can vessels, in 1840, 9706; in 1841, 11738, and in 1842, 10457. How many vessels arrived in the four years?

42. The increase of population in Pennsylvania from 1810 to 1820, was 239222; from 1820 to 1830, 298359; from 1830 to 1840, 376361: what was the whole increase in 30 years?

43. The eighteenth congressional district of Pennsylvania consists of Greene, Fayette and Somerset counties. The population of the first named county is 19140; of the second, 33574, and of the third, 19650. What is the population of the whole district?

44. The number of square miles in Somerset county is 1066; in Fayette, 824, and in Greene, 597. What is the whole number of square miles in the three counties?

45. The distance from Philadelphia to Lancaster by turnpike is 62 miles; from Lancaster to Harrisburg, 36 miles; from Harrisburg to Chambersburg is 50 miles; from Chambersburg to Bedford is 52 miles, and from Bedford to Pittsburg is 100 miles. What is the whole distance from Philadelphia to Pittsburg.

46. In the year 1838, there were 189000 dollars expended for school purposes in the city and county of Philadelphia; in 1839, 148000; in 1840, 175000, and in 1841, 216000. How much was expended for that purpose in the four years?

47. The capital of the Bank of North America is 1000000 dollars; that of the Philadelphia Bank is 1800000; that of the Farmers' and Mechanics' Bank is 1250000; of the Northern Liberties Bank, 500000, and of the Kensington Bank, 250000. What is the whole capital of the five banks?

48. From the death of king Arthur in 542, to the coronation of Egbert as king over all England in 800, elapsed 258 years; from Egbert to William the Conqueror, 266 years; from that king to William II., 21 years; from him to William III., 602 years, and from William III. to William IV., 141 years. How many years are reckoned from Arthur to William IV.

49. From Louis I. of France to Louis IX., called Saint Louis, are 412 years; from St. Louis to Louis XI., are 235 years; from Louis XI. to Louis XIV., are 182 years; from him to Louis Philip, or Louis XIX., are 187 years. How many years do you count from Louis I. to Louis Philip.

50. At the commencement of the 19th century, the gold and silver mines of New Spain produced ore to the value of

23000000 dollars; the value of the ore from the mines of Peru was 6240000; of Chili, 2060000; of Buenos Ayres, 4850000; of New Grenada, 8990000, and of Brazil, 4360000. What was the value of the products of all these American mines.

51. In one public library at Liverpool, there are 7550 volumes; in another, 2200; in a third, 17533; and in a fourth, 29987. How many volumes are contained in the four collections?

52. The number of men employed in mining in the United States is as follows: iron, 30497; lead, 1017; gold and other metals, 1774; coal, 6811. What is the total number of miners in all the different branches?

53. The average annual quantities of tea exported to various parts of the world from China, are nearly as follows: To Great Britain, 36000000 pounds; to America, 10000000; France, 350000; Holland, 2800000; Russia, 6500000; Cape of Good Hope, 200000; New South Wales, 500000; Bombay, Madras, and Calcutta, 2000000. What is the whole number of pounds of tea with which China supplies the world?

54. Colombia consists of the republics of Ecuador, New Grenada, and Venezuela. The population of these, in 1834, was as follows: Ecuador, 1687100; New Grenada, 900000; Venezuela, 600000. What is the whole population of Colombia?

55. The imports of Venezuela in 1834, were, from Great Britain, 897742 dollars; from the United States, 783061; from France, 61969; from Germany, 326972; from Spain, 82369; from New Grenada, 121; from Holland, 90957; from Denmark, 980161; from Mexico, 5030; and from other countries, 68416. What was the total value of the imports for that year?

56. The exports of the same republic for 1834, were, to Great Britain, 520542 dollars; to the United States, 1115490; to France, 205750; to Germany, 384527; to Spain, 205750; to New Grenada, 18764; to Holland, 155459; to Denmark, 740205; to Sardinia, 28658; to Hayti, 1109; and to other countries, 18316. What was the sum of the exports?

57. The British army in India is composed of 157758 native officers and soldiers; 111500 subsidiary native troops; and of a British army consisting of 205 staff officers, 1927 horse artillery, 4354 foot artillery, 77 engineers, 2585 cavalry,

13879 infantry, and 755 other officers. What, then, is the whole force employed in British India.

58. Ireland is divided into four provinces, the respective population of which is as follows: Leinster, 1909713; Munster, 2227152; Ulster, 2286622; Connaught, 1343914. What is the total population of Ireland?

59. The length of the river Amazon is 3300 miles. The lengths of its tributaries: Ucayali, 1350; Yutai, 750; Jaura, 750; Madeira, 1800; Topayos, 1000; Xingu, 1080; Napo, 800; Rio Negro, 1400. What is the sum of the lengths of the river and its tributaries?

60. During the year ending June 30, 1843, the mails were carried by coaches over 18414174 miles; by horse over 11146229; and by rail road over 5692402 miles. How many miles of mail transportation were performed in all?

SUBTRACTION

§ 4. Subtraction is that operation by which we find the difference between two given numbers.

Q. What does subtraction mean?

A. It is the finding how many of one number will be left after as many as there are in another number are taken away.

Q. In what relation does it stand to addition?

A. It is the reverse, and that which is done by addition may be again undone by subtraction.

Q. What is the object intended to be found or known by the process of subtraction?

A. Finding by how many one number differs from another, or how much would require to be added to the smaller of two numbers, so as to make the sum equal to the greater.

Q. Must there always be two given numbers before subtraction can be performed?

A. We must always have two things to compare, before there can be any difference.

Q. Are the given numbers for subtraction known by any particular names?

A. One is called the minuend, and the other the subtrahend.

Q. What do those terms signify?

A. Minuend means that which is to be diminished, and subtrahend means that which is to be taken away.

Q. What name is given to the result of the operation?

A. It is called difference, or remainder.

Q. Of what kind of numbers can the difference be taken?

A. They must be of the same kind, and not equal in value.

Q. What conditions are required before we can find the difference of two numbers?

A. First, they must both signify things of the same kind. Secondly, they must be of the same denomination; and their numbers must express their values, before the difference of the numbers will express the difference of the values.

Q. Is there any other principle which we must bear in mind in subtraction?

A. There is: if we add the same number to each of the two given numbers, or if we take away the same from each of them, we do not in either case, make the least alteration in the difference.

Q. What sign is used to denote the operation of subtracting?

A. The sign — which is called minus or less; and when placed between two numbers means their difference. As, 7—5.

MENTAL EXERCISES.

1. If 2 be taken from 5, how many will be left? 3 from 8? 6 from 9? 4 from 7? 2 from 11? 3 from 10? 4 from 12? 7 from 13? 9 from 14? 6 from 15? 9 from 16? 13 from 18? 14 from 25? 13 from 27? 9 from 21? 8 from 17? 6 from 19?

2. Take 6 from 21, and 9 from the remainder.

3. Take 8 from 19, and 7 from the remainder.

4. Take 12 from 31, and 15 from the remainder.

5. Take 16 from 40, and 24 from the remainder. 8—2—3; and 9—7—1.

RULE.

1. Place the less number under the greater, so that units may stand under units, tens under tens, &c., and draw a line under them.

2. Begin at the right hand, and take each figure in the

lower line from the figure above it, and write down the remainder.

3. But if the figure in the lower line be greater than that above it, add ten to the upper one, and then take the lower figure from it.

4. Write down the remainder, and carry one to the next lower figure; with which proceed as before; and so on, till the whole is finished.

PROOF.

Add the remainder to the less number, and if the sum be equal to the greater, the work is correct.

Or, subtract the remainder from the greater number, the difference will give the less number. It may also be proved by taking the excess of nines in the less number from the excess of nines in the greater; their difference will be equal to the excess of nines in the remainder. But if the excess of nines in the less line exceed that in the greater, add nine to the excess in the greater, and then proceed as before. The reason of this method will appear from what is shown in Addition.

DEMONSTRATION. 1. When all the figures of the least number are less than their correspondent figures in the greater, the difference of the figures in the several like places must altogether make the true difference sought; because, as the sum of the parts is equal to the whole, so must the sum of the differences of all the similar parts be equal to the difference of the whole.

2. When any figure of the greater number is less than its correspondent figure in the less, the ten, which is added by the rule, is the value of a unit in the next higher place by the nature of notation; and as the one which is added to the next place of the less number, diminishes the correspondent place of the greater accordingly; this is only taking from one place and adding as much to another, by which the total is never changed. So that, by this means the greater number is resolved into such parts as are each greater than, or equal to, the similar parts of the less; and, therefore, th difference of the corresponding figures, taken together, will make up the difference of the whole, as before.

The truth of the method of proof is evident; for the difference of two numbers added to the less, is manifestly equal to the greater.

Q. How do you set down integers to be subtracted?

Q. When the upper figure is less than the under one, how do you proceed?

Q. Since you add ten to the upper figure, why do you add only one to the next figure in the lower line?

Q. How do you prove subtraction?

EXAMPLES.

(1.)	(2.)	(3.)
From 85432 . . 4	From 983450 . . 2	From 7965014 . 5
Take 38721 . . 3	Take 712654 . . 7	Take 5832905 . . 5
Rem. 46711 . . 1	Rem. 270796 . . 4	Rem. 2132109 . 0
Proof 85432	Proof 983450	Proof 7965014

4. 786—287.
5. 936—604.
6. 387—114.
7. 849—542.
8. 998—789.
9. 1001—982.
10. 2414—1641.
11. 3828—2732.
12. 4872—3948.
13. 5381—4727.
14. 6871—5146.
15. 7068—6199.
16. 8476—7177.
17. 9489—8749.
18. 9989—8948.
19. 10101—7444.
20. 19898—11241.
21. 21741—9804.
22. 39489—22749.
23. 48249—37888.
24. 98737—69772.
25. 104982—94378.
26. 198979—48367.
27. 989991—746892.
28. 7854391—2784984.
29. 6321736—239216.
30. 9006654—999999.
31. 8810824—4056041.
32. 1173110—981119.
33. 27845678—12345678.
34. 57824316—41093279.
35. 89499560—78543984.
36. 47910108—46010237.
37. 31082460—30928341.
38. 91041002—7652549.
39. 80004112—8004104.

40. What is the difference between 50737 and 4298962?

41. What is the difference between 72649 and 6758386?

42. A merchant purchased goods amounting to 64188 dollars, and sold them for 67084 dollars. What did he gain?

43. In 1800, the population of Pennsylvania was 602545, in 1840, it was 1724033. Required the increase.

44. The State House in Philadelphia was erected in the year 1753; how many years have elapsed in 1845?

45. If you divide 37864 dollars between two men, giving one 19967 dollars, what will be the share of the other?

46. Bought 1085 bags of coffee for 9564 dollars, and sold them for 10453 dollars; what was gained by the transaction?

47. In the year 1841, France, with a population of 31 millions, raised 547550440 bushels of grain; during the same period, the United States with a population of 17 millions, raised 533988970 bushels; what is the difference in population, and cereal productions?

48. In 1828, the votes given at the presidential election in the United States, were 1150508, in 1840, the number was 2388900. Required the increase.

49. A grocer purchased 185 hhds. of sugar, weighing 186480 pounds, and sold 97 hhds., weighing 98796 pounds. How many hhds. had he left, and what was their weight?

50. Charles Carroll, the last surviving signer of the Declaration of Independence, died A. D., 1832, aged 95 years; in what year was he born?

51. The first complete treatise on Practical Arithmetic, in which the numerals now in use were employed, was published A. D. 1556. How many years since? (A. D. 1845.)

52. Sold goods to the amount of 29648 dollars, and gained 4749 dollars; what was the prime cost of the goods?

53. How much must you add to 1001 to make 100000?

54. The total population of the United States is 17068666, that of Great Britain, 26270151. Required the difference?

55. A Liverpool merchant shipped merchandize to the amount of 63484 dollars, but during a storm the captain threw overboard a part, valued at 13697 dollars. What was the value of the remainder?

56. Mount Sorato, the highest point of the Andes, rises 25400 feet above the level of the sea, and 9732 feet higher than Mount Blanc, the highest point of the Alps. What is the height of Mount Blanc?

57. A person bought goods to the amount of 17468 dollars, on a credit of three months, but preferring to pay the cash, an allowance of 269 dollars was made. What was the amount paid?

58. The number of inhabitants in the Middle States, in 1830, was 3664412; by the census of 1840, the number was 4604345. Required the difference in population?

59. The value of the imports at Philadelphia for the year 1842, was 7381788 dollars; the value of the domestic ex-

ports for the same time, was 3770729 dollars. What is the difference in value between the imports and exports?

EXERCISES IN ADDITION AND SUBTRACTION.

1 Subtract the sum of 56604, 4155, 4516, 3774, 2162 from 2514817?

2. Subtract 35637 from the sum of 250165, 92635, and 99097?

3. Subtract from 129601862, the sum of 40732359, 19-221328, 6291808, and 9364965.

4. From the sum of 6544654, 1407736, 9477343 and 32708, subtract the sum of 361558, 11615769, 1485360 and 681979?

5. From 258037 subtract the sum of 111, 999, 88, 666 and 108050?

6. From 1724033 subtract the sum of 908744 and 815-289.

7. Subtract from the sum of 188625 and 175574, the sum of 61617, 32500, 2705, 61, 727 and 24007.

8. What is the difference between the sum of 23583, 16872, 21891, and 22977; and the sum of 4226, 24531, 27873, 30699, and 23506?

9. The population of the Eastern States was, in 1840, as follows: Maine, 501793; New Hampshire, 284574; Vermont, 291948; Massachusetts, 737699; Rhode Island, 108-830; and Connecticut, 309978. Of the Middle States, the population of New York was 2428921; of New Jersey, 373306; of Pennsylvania, 1724033; and of Delaware, 78-085. How many more inhabitants were there in the Middle than in the Eastern States?

10. Give the difference between the total population of the Southern and the Western States in 1840, from the following list. Southern States: Maryland, 470019; Virginia, 1239797; North Carolina, 753419; South Carolina, 594398; Georgia, 691392; Alabama, 590756; Mississippi, 375651; Louisiana, 352411. Western States: Ohio, 1519467; Kentucky, 779828; Tennessee, 829210; Michigan, 212267; Indiana, 685866; Illinois, 476183; Missouri, 383702; Arkansas, 97574.

11. The value of the imports into New York state from October, 1841, to October, 1842, in American vessels, amounted to 51523055 dollars; in foreign vessels, to 6352549 dollars. Pennsylvania imported during the same period, in Ame-

rican vessels, 6757228 dollars; in foreign vessels, 628630 dollars. How much greater was the total value of the imports of New York than that of the imports of Pennsylvania?

12. Louisiana exports, in one year, foreign produce to the amount of 976727 dollars; domestic produce, 27427422 dollars. The total value of the exports, domestic and foreign, of New York, is 27576778 dollars. How much greater is the exporting trade of Louisiana than that of New York?

13. The imports of New York, from October, 1841, to October, 1842, were 57875604 dollars; of Pennsylvania, 7385858 dollars; of Louisiana, 8033590 dollars; and of Massachusetts, 17986433 dollars. What is the difference between the sum of the imports of these four states and the sum of those of the whole United States, which were for the same period 100162087 dollars?

14. The number of militia men and officers in the Eastern States, is stated to be, in Maine, 43352; New Hampshire, 32471; Massachusetts, 87215; Vermont, 27536; Rhode Island, 14995; and Connecticut, 46470. Of the Western States, the number in Tennessee, is 71252; Kentucky, 77276; Ohio, 180258; Indiana, 53913; Illinois, 83234; Missouri, 59689; Arkansas, 2028; and Michigan, 12786. How many more enrolled militia men are there in the Western than in the Eastern States?

15. The exports of the United States from October, 1841, to October, 1842, were as follows: Of products of the sea, 2823010 dollars; of the forest, 5518262; of agriculture, 73688113; of manufactures, 10940611. The total value of the imports for the same period, was 100162087 dollars. How much did the total value of the imports exceed that of the exports?

16. From the sum of 609458, 870406, 9004879, 23460401, and 7894763; subtract 21869909.

17. A person owed a merchant 1074 dollars, he paid him 695 dollars, and purchased goods to the amount of 598 dollars. How much did he then owe?

18. A merchant bought at one time 657 barrels of flour for 2625 dollars, at another time 956 barrels for 4025 dollars; he sold 845 barrels for 4235 dollars. How many barrels had he left, and what was their value?

19. The population of Philadelphia county in 1840, was 258037, of which the Northern Liberties contained 34474; Spring Garden, 27849; Kensington, 22314; Southwark,

27548; Moyamensing, 14573; and the townships, 37614. What was the remaining population, being that of the city?

20. A wholesale merchant purchased 6 hogsheads of sugar, weighing as follows: No. 1, 976 pounds; Nos. 2 and 3, each 1019 pounds; Nos. 4 and 5, each 1146 pounds; and No. 6, 1085 pounds; he sold to one person 2498 pounds, to another 1947 pounds, and to a third 763 pounds. How many pounds had he left?

21. In the city and districts of Philadelphia, for 1843, the assessed valuation of property, subject to county taxation, was as follows: the City, 58688499 dollars; Northern Liberties, 9224409 dollars; Spring Garden, 8862404 dollars; Kensington, 4023668 dollars; Southwark, 5578619 dollars; Moyamensing, 2330341 dollars; townships, 10613941 dollars. How much more was the city assessed than the districts?

22. In 1843, there were raised in the state of New York, 12479499 bushels of wheat; 24907553 bushels of oats; 3677222 bushels of rye; 2398354 bushels of buckwheat; and 15574590 bushels of corn. In Pennsylvania, 12215230 bushels of wheat; 19826938 bushels of oats; 9429637 bushels of rye; 2408508 bushels of buckwheat; and 15857-431 bushels of corn. How many more bushels of grain did Pennsylvania raise than New York?

23. The first landing of settlers in Pennsylvania, under Penn's charter, occurred in 1681, at Upland, now Chester; the boundary line between Pennsylvania and Maryland, after long and serious difficulties between the two colonies, was finally established by Mason and Dixon, in 1768. How many years elapsed between these two dates? And how many years between each of these dates and the year 1845?

24. How many years elapsed from the discovery of America, in 1492, to the Declaration of the Independence of the United States, in 1776? To the death of the author of that Declaration, Thomas Jefferson, in 1826? How many to 1845?

25. What is the difference between the population of New York, 312710, and that of Philadelphia, 258037? Between Philadelphia and New Orleans, 102103? Between New Orleans and Cincinnati, 46338? Between Cincinnati and Charleston, 29261? Between Charleston and Norfolk, 10920? Between Norfolk and Lexington, 6977?

MULTIPLICATION.

§ 5. Multiplication is that operation by which we increase one of two given numbers as often as there are units in the other; and is a compendious method of performing addition.

Q. What does multiplication mean?

A. It means the repetition of one number as many times as is expressed by another number, and showing to how much the repetition amounts.

Q. Must there always be two numbers given in every case of multiplication?

A. There must always be two numbers given, the one of which represents the number that is to be repeated, and the other the number of repetitions.

Q. What names are given to those numbers?

A. That which is to be multiplied is called the multiplicand; and the other, the multiplier. Both the multiplicand and multiplier are called terms, or factors.

Q. Is there any difference in these numbers?

A. The multiplicand is a number, or a number meaning things of any kind; but the multiplier means a number of times.

Q. What are the prime factors?

A. The prime factors of any number are those prime numbers by whose continued multiplication the number is formed. Thus, 2 and 3 are the prime factors of 12, for 2 multiplied by 2 gives 4, which multiplied by 3 is equal to 12.

Q. Is multiplying the same as adding equal numbers together?

A. It is; if we write down the multiplicand as often as there are units in the multiplier, and add the whole together, it is the same as multiplying.

Q. How can this long labour be avoided, and produce the same effect?

A. By the operation of multiplying, the result of which is called a product.

Q. What preparation is required, before you can perform the operation of multiplying?

A. That of committing the multiplication table carefully to memory.

Q. What is meant by the sign $\times$?

A. This sign, when written between numbers, means that they are to be multiplied together. Thus, 4×3.

THE MULTIPLICATION TABLE.

Twice		3 times		4 times		5 times		6 times		7 times	
1 make	2	1 make	3	1 make	4	1 make	5	1 make	6	1 make	7
2	4	2	6	2	8	2	10	2	12	2	14
3	6	3	9	3	12	3	15	3	18	3	21
4	8	4	12	4	16	4	20	4	24	4	28
5	10	5	15	5	20	5	25	5	30	5	35
6	12	6	18	6	24	6	30	6	36	6	42
7	14	7	21	7	28	7	35	7	42	7	49
8	16	8	24	8	32	8	40	8	48	8	56
9	18	9	27	9	36	9	45	9	54	9	63
10	20	10	30	10	40	10	50	10	60	10	70
11	22	11	33	11	44	11	55	11	66	11	77
12	24	12	36	12	48	12	60	12	72	12	84

8 times		9 times		10 times		11 times		12 times	
1 make	8	1 make	9	1 make	10	1 make	11	1 make	12
2	16	2	18	2	20	2	22	2	24
3	24	3	27	3	30	3	33	3	36
4	32	4	36	4	40	4	44	4	48
5	40	5	45	5	50	5	55	5	60
6	48	6	54	6	60	6	66	6	72
7	56	7	63	7	70	7	77	7	84
8	64	8	72	8	80	8	88	8	96
9	72	9	81	9	90	9	99	9	108
10	80	10	90	10	100	10	110	10	120
11	88	11	99	11	110	11	121	11	132
12	96	12	108	12	120	12	132	12	144

MENTAL EXERCISES.

Multiply 3 by 9. 4 by 7. 5 by 8. 6 by 7. 7 by 5. 8 by 6. 9 by 8. 10 by 6. 11 by 7. 12 by 6. 4 by 8. 5 by 9. 8 by 7. 9 by 9. 7 by 9. 8 by 8. 6 by 9. 12 by 8. 6 by 11. 6 by 5 by 3. 7 by 2 by 10. 5 by 4 by 6. 8 by 5 by 2. $2\times4\times8$. $7\times1\times4$. $9\times8\times7$.

RULE.

1. Place the multiplier under the multiplicand, so that the units may stand under units, tens under tens, &c., and draw a line under them.

2. Begin at the right hand, and multiply every figure in the multiplicand by each of the figures in the multiplier.

3. Reckon how many tens there are in the product of every two simple figures, and write down the remainder directly under the figure by which you are multiplying, and if nothing remains, a cipher.

4. Carry as many units as there were tens to the product of the next figure, and proceed, in like manner, till the whole is finished.

5. Add all the products together, and their sum will be the answer required.

If two numbers are to be multiplied together, they will give the same product, whichever number you make the multiplier.

If several numbers, as 5, 6, 7, &c., are to be multiplied together, it is the same thing whether 5 be multiplied by the product of 6 and 7, or it be multiplied first by 6 and then by 7, &c. And if several given numbers are to be multiplied by any number, and the sum of the products be taken, it will give the same result as the multiplication of the sum of those given numbers by that multiplier.

The product of any two numbers can have at most but as many places of figures as are in both the multiplier and multiplicand, and at least but one less.

METHOD OF PROOF.

Change the places of the multiplier and multiplicand, and if the product found from this operation is the same as before, the work is correct.

DEMONSTRATION. 1. When the multiplier is a single digit, it is evident that the product is properly determined by the rule; for by multiplying every figure by it, that is every part of the multiplicand, we multiply the whole; and writing down the products which are less than ten, or the excess of tens, in the places of the figures multiplied, and carrying the number of tens to the product of the next place, is only gathering together the similar parts of the respective products; and is, therefore, the same thing, in effect, as if we wrote down the multiplicand as often as the multiplier expresses, and added them together; for the sum of every column is the product of the figures in that column, and these products collected together, are evidently equal to the whole required product.

2. If the multiplier be a number made up of more than

one digit. After we have found the product of the multiplicand by the first figure of the multiplier, as above, we suppose the multiplier divided into parts, and find, after the same manner, the product of the multiplicand by the second figure of the multiplier; but, as the figure by which we are multiplying stands in the place of tens, the product must be ten times its simple value; and therefore, the first figure of this product must be placed in the place of tens; or which is the same thing, directly under the multiplying figure. And proceeding in this manner separately with all the figures of the multiplier, it is evident that we shall multiply all parts of the multiplicand by all the parts of the multiplier, or the whole of the multiplicand by the whole of the multiplier; therefore, these several products, being added together, will be equal to the whole required product.

The following examples are subjoined to make the reason of the rule appear as plain as possible.

(1.)		(2.)		
6435		1547535		
7		5476		
35=	5×7	9285210=	6	times the multiplicand.
210=	30×7	10832745 =	70	" "
2800=	400×7	6190140 =	400	" "
42000=	6000×7	7737675 =	5000	" "
45045=	6435×7	8474301660=	5476	" "

Besides the method of proof given above, there is another very convenient and easy one, by the help of that peculiar property of the number 9, before mentioned, which is performed thus:

RULE. 1. Cast the nines out of the two factors as in addition, and write down the remainders.

2. Multiply the two remainders together, and if the excess of nines in their product be equal to the excess of nines in the total product, the work is correct.

EXAMPLE.

3416—5=excess of 9s in the multiplicand.
786—3= " " in the multiplier.

20496
27328
23912

2684976—6= " " in the product=excess of 9s in 5×3.

Multiplication may also very naturally be proved by Division, for the product being divided by either of the factors, will evidently give the other; but it would have been contrary to order to have given this rule in the text, because the pupil is supposed as yet to be unacquainted with Division.

EXAMPLES.

(1.)	(2.)	
483691257	4278	746
2	746	4278
967382514	25668	5968
	17112	5222
	29946	1492
	3191388 Product.	2984
		3191388 Proof.

Proof by casting out the nines.

The excess of 9s in the multiplicand =3.
" " in the multiplier =8.
" " in the product =6, which equal the excess of 9s in 3×8.

3. 789×3.
4. 4237×5.
5. 7241×6.
6. 9372×7.
7. 5384×8.
8. 8378×10.
9. 2345×11.
10. 6789×12.
11. 54321×4.
12. 98765×6.
13. 12348×7.
14. 462868×9.
15. 198275×11.
16. 7424394×2.
17. 9836276×3.
18. 4752314×9.
19. 72531409×4.
20. 99764514×7.
21. 864257981×10.
22. 189752468×11.
23. 971117740×12.
24. 3715296871×2.
25. 1786925173×3.
26. 1234567890×4.
27. 9081234567×5.
28. 2545878554×8.
29. 3775566894×1.
30. 742×13.
31. 890×17.
32. 419×19.
33. 847×23.
34. 489×26.
35. 372×29.
36. 567×31.
37. 764×34.
38. 837×14.
39. 738×15.
40. 123×16.
41. 4834×18.
42. 1444×20.
43. 6101×21.
44. 7854×24.
45. 5487×27.
46. 3141×25.

47. 61315×28.
48. 887744×30.
49. 742101×32.
50. 123546×45.
51. 278145×49.
52. 198576×89.
53. 634215×67.
54. 428613×74.
55. 148432×54.
56. 371529×98.
57. 518040×57.
58. 6172441×66.
59. 1442716×88.
60. 144×111.
61. 175×121.
62. 189×231.
63. 842×389.
64. 941×438.
65. 149×201.
66. 846×782.
67. 574×647.
68. 647×561.
69. 464×872.
70. 982×187.
71. 1411×1234.
72. 2345×2141.
73. 5678×3456.
74. 6587×4678.
75. 3786×5248.
76. 4012×6716
77. 34112×14234.
78. 5721467×428742.
79. 99748201×8175672.
80. 84310072×987234.

CONTRACTIONS.

§ 6. Case 1. When there are ciphers in the numbers to be multiplied.

RULE.

1. If the ciphers are at the right hand of the numbers, multiply the other figures only, and place as many ciphers at the right hand of the product, as are at the right hand of both the factors.

2. When the ciphers are in any part of the multiplier, neglect them as before, observing to place the first figure of every product precisely under the figures you are multiplying by.

EXAMPLES.

(1.)

Multiply 837000
by 36000

5022
2511

Product 30132000000

(2.)

Multiply 7054059
by 60050

35270295
42324354

Product 423596242950

3. 1234×400.
4. 54781×5710.
5. 82361×69420.
6. 987258×74600.
7. 314837×829700.
8. 386927×9876000.

9. 4627381×1167000.	14. 80024×8104.
10. 7854392×4800000.	15. 98701×71004.
11. 5869×305.	16. 300056×21006.
12. 5086×508.	17. 605705×60050.
13. 7402×1008.	18. 2340053×120003.

§ 7. Case 2. When the multiplier is the product of two or more numbers, or a composite number, greater than 12.

RULE.

Multiply by each of those parts separately, instead of the whole number at once.

The reason of this method is obvious; for any number multiplied by the component parts of another, must give the same product as if it were multiplied by that number: thus 9 times a given number multiplied by 8, gives 72 times the same number, as clearly as 9 times 8 make 72.

EXAMPLES.

1. Multiply 213789465 by 36, or by 6 times 6.

213789465
6
1282736790
6
7696420740 Product.

2. 582×25.	8. 23456×49.	14. 9765432×90.
3. 648×28.	9. 78921×56.	15. 7128112×132.
4. 774×32.	10. 84347×64.	16. 6183574×144.
5. 823×35.	11. 46543×72.	17. 1325678×121.
6. 925×42.	12. 1234567×81.	18. 8712653×990.
7. 1214×45.	13. 4657237×96.	19. 874683×1296.

§ 8. Case 3. To multiply by any number of nines in one line, as 9, 99, 999, &c.

RULE.

Annex as many ciphers to the multiplicand as there are nines in the multiplier, and from this number subtract the multiplicand, the difference will be the product required.

EXAMPLES.

1. Multiply 81041 by 99.

8104100	
81041	
8023059	

2. Multiply 81641 by 998.

998+2	=1000
81641×1000	=81641000
81641×2	= 163282
84641×998	=81477718

3. 237×99.
4. 1761×999.
5. 69207×997.
6. 118052×9991.

§ 9. CASE 4. To find the product of one or more figures from the product of a figure already found.

RULE.

Multiply the product found by that number which, when multiplied by the given number, produces the other part of the multiplier.

EXAMPLES.

(1.)	(2.)	(3.)
5524	7420	8746
84	369	763
22096	66780	61222
44192	267120	550998
464016	2737980	6673198

In the 1st example the product of 4 is multiplied by 2, in the 2d, the product of 9 is multiplied by 4; and in the 3d, the product of 7, (or rather 700,) is multiplied by 9.

4. Multiply 36284 by 54279.
5. " 2841 " 96324.
6. " 42061 " 6183.
7. Multiply 74820 by 27030.
8. " 56142 " 36484.
9. " 17864 " 3612.

Q. How do you multiply integers when the multiplier consists of several figures?

Q. How do you multiply, when ciphers are annexed to any or both of the factors?

Q. How do you multiply, when the multiplier is a composite number?

Q. How do you prove multiplication?

MISCELLANEOUS EXERCISES.

1.	Multiply	41152263 by 2.	8.	Multiply	771604932 by 16.
2.	"	881834208 " 14.	9.	"	7346184 " 168.
3.	"	128600822 " 12.	10.	"	64300411 " 192.
4.	"	205761315 " 6.	11.	"	36743092 " 336.
5.	"	293944736 " 7.	12.	"	18371546 " 672.
6.	"	73486184 " 12.	13.	"	9185773 " 1344.
7.	"	9185773 " 14.	14.	"	35234 " 52424.

15. Moscow contains 305631 inhabitants. If there were sixteen times as many, what number would there be?

16. The university of Moscow had in 1835, 120 professors, and 430 pupils. If there were 430 pupils for each professor, how many would there be?

17. If one acre of well tilled land produces 218 bushels of potatoes, how many bushels will 573 acres produce?

18. For every ox sold in a certain market there are sold 98 sheep. If 74184 oxen were disposed of, how many sheep would be sold?

19. If 383042 pounds of lead are sold in one year, how many pounds will be sold in 9 years?

20. If a man can walk 9875 miles in a year, how far could he walk in 16 years?

21. If one house cost 1263 dollars, how much will a row of 13 houses cost?

22. If one calf be worth 11 dollars, how many dollars would 14303 calves cost?

23. The population of Brussels in 1837, was about 134-265. If Spain contains 33 times as many inhabitants as the city of Brussels, what is the number of its population?

24. From Charleston to Madeira is 3600 miles. Over how many miles does a ship sail in making 37 voyages between the two places?

25. From Norfolk to Portugal is 3700 miles. From New York to Portugal is 3400 miles. How many more miles will a ship sail in making 19 voyages between Norfolk and Portugal, than another vessel which makes the same number of voyages between that country and New York?

26. How much must a manufacturer receive for 695876 yards of cloth, at 7 dollars per yard?

27. A grocer sold 4569 cwt. of sugar, at the rate of 9 dollars per cwt., how many dollars did he receive?

28. What is the value of 13487 barrels of flour at 6 dollars per barrel?

29. In the District of Columbia there are 100 square miles, and on each mile 437 inhabitants; what is the population?

30. What will be the cost of building a ship of 576 tons, at 56 dollars per ton?

31. There are 365 days in a year; how many in 4395 years?

32. Bought a farm of 267 acres, at the rate of 75 dollars per acre; what did it cost?

33. A planter sold to a merchant, on his note at 6 months, 3297 bales of cotton, at 75 dollars per bale; what was the amount of the note?

34. What is the content of a rectangular piece of ground whose length is 465, and breadth 95 perches?

35. There are 1760 yards in a mile; how many yards in 458 miles?

36. To square a number, is to multiply the number by itself; what is the square of 6045?

37. How many cubic inches in 47095 wine gallons, each gallon containing 231 inches?

38. At New Gibraltar, of a still night, the sentinel's cry of "All's well," can be heard in 55 seconds after it has been uttered at Old Gibraltar; required the distance between the two places, sound moving at the rate of 1142 feet in a second?

39. What will 143 head of cattle come to, at 19 dollars per head?

40. There are 640 acres in one square mile, how many acres in 69075 square miles?

41. What is the cost of constructing 609 miles of rail road, at 15085 dollars a mile?

42. How many square feet of boards in a floor 72 feet long and 45 feet wide?

43. If 36 men can perform a piece of work in 159 days, how long will it take one man?

44. How many times will a wheel turn in 1475 miles, supposing it to turn 320 times in one mile?

45. At 225 dollars per ton, what will 498 tons of hemp cost?

46. The earth casts a shadow whose mean length is equal to 108 times the diameter; required the length of the shadow, the diameter being 7935 miles?

47. To cube a number is to multiply the square of the number by the number itself: what is the cube of 625?

48. A steamer moves at the rate of 16 miles an hour; what distance will she pass over in 24 hours?

49. How many feet of earth can be removed in 35 days, by a cart carrying 15 loads a day and 36 feet at a load?

50. A merchant purchased 34 bales of cloth, each bale contained 23 pieces, and each piece 27 yards. How many pieces and yards, and what is the value of the whole at 5 dollars per yard.

51. How many dollars are equal to 23 bags, each containing 643 doubloons, the doubloon being valued at 16 dollars.

52. The pressure of the atmosphere upon a square inch is 15 pounds, what is the pressure upon a square foot which contains 144 inches?

53. How many ounces of provisions must a ship's company of 525 men have to last 93 days, allowing to each man 16 ounces per day?

54. To dig a canal 137280 yards long, 48 yards wide, and 3 yards deep, how many solid yards of earth must be removed?

55. It has been estimated that the river Ganges, during the flood season, discharges daily into the sea a mass of mud equal to 74 times the weight of the great Egyptian pyramid. Required the quantity discharged in 26 days, the weight of the pyramid being 2164956550400 pounds.

DIVISION.

§ **10.** Division is that operation by which we find how often one number is contained in another, and is a short method of performing subtraction.

Q. What does division mean?

A. It means the separating into equal parts: thus, two numbers are given, one of which is to be divided into parts; the other shows how many parts there are to be; and the object of the operation is to find the value of one of those parts.

Q. Do these numbers get particular names?

A. The number to be divided is called the dividend; the number you divide by is called the divisor, and shows how many parts there are; and the number of times the divisor is

contained in the dividend is called the quotient, and expresses one of these parts.

Q. Does division never mean any thing else than finding the proposed part of a number?

A. It also means finding what part one number is of another.

Q. Is division the reverse of any other operation?

A. It is the reverse of multiplication.

Q. Why?

A. Because the product in multiplication is made up of as many parts, each equal to the multiplicand, as the number of the multiplier; and when the quotient in division is found, the dividend is made up of as many parts, each equal to the divisor or the quotient, as the number of the other of these.

Q. Is the product of the divisor and quotient always equal to the dividend?

A. Not always; for it may not be an exact number of times the divisor.

Q. What is meant by the sign ÷?

A. This is the sign of division, and means that the former of the numbers between which it is placed is to be divided by the latter. Thus $8 \div 4 = 2$.

Q. Is not division a very tedious operation?

A. With a large quotient it would be; but by means of the scale of numbers, and other operations, we are enabled to shorten it.

MENTAL EXERCISES.

Divide 9 by 3. 12 by 6. 14 by 2. 18 by 6. 15 by 5. 21 by 3. 24 by 8. Divide 17 by 4 and give the remainder.

$28 \div 4$. $26 \div 5$. $38 \div 9$. $42 \div 7$. $54 \div 6$. $72 \div 8$. $63 \div 9$. $49 \div 7$. $45 \div 9$. $56 \div 7$. $68 \div 6$. $50 \div 8$. $79 \div 8$. $62 \div 8$.

CASE 1.

§ 11. When the divisor does not exceed 12.

RULE.

Find how often the divisor is contained in the first, or first and second figures of the dividend, and write the quotient figure under it; prefix (in your own mind) what remains to the next figure in the dividend, and divide as before.

CASE 2.

§ 12. When the divisor consists of several figures.

RULE.

1. Draw a line on the right and left of the dividend, and write the divisor on the left.

2. Find how often the divisor is contained in as many of the left hand figures of the dividend as are just necessary, and place that number on the right.

3. Multiply the divisor by that number, and place the product under the above mentioned figures of the dividend.

4. Subtract the said product from that part of the dividend under which it stands, and bring down the next figure of the dividend to the right of the remainder.

5. Divide the remainder so increased, as before; and if at any time it be found less than the divisor, put a cipher in the quotient, bring down the next figure of the dividend, and continue the operation until the whole is completed.

METHOD OF PROOF.

Multiply the quotient by the divisor, and if this product, together with the remainder, be equal to the dividend the work is correct.

An even number cannot divide or measure an odd number, nor a greater a less.

Half the sum of any two numbers, increased by half their difference, will give the greater number; and half their sum diminished by half their difference, will give the less number.

The quotient, arising from the division of the sum of two, or more numbers, is equal to the sum of the quotients arising from the division of each number separately, by the same divisor.

If any two numbers be separately divided by 9 or 3, and the two remainders multiplied together, and that product divided by 9 or 3, the last remainder will be the same as if you divide the product of the two first numbers by 9 or 3.

Any number divided by 9 or 3, will leave the same remainder as the sum of its digits divided by 9 or 3. Hence, if any number be divisible by 9 or 3, the sum of its digits is likewise divisible by 9 or 3, and *vice versa.*

A greater number is said to be a multiple of a less, when it contains the less an exact number of times, and in this case the less is called an aliquot part, or simply a part of the greater.

By the rule, we resolve the dividend into parts, and find by trial the number of times the divisor is contained in each

of these parts; the only thing then, which remains to be proved is, that the several figures of the quotient taken as one number, according to the order in which they are placed, form the true quotient of the whole dividend by the divisor, which may be thus demonstrated.

DEMONSTRATION. The complete value of the first part of the dividend is, by the nature of notation, 10, 100, or 1000, &c., times the value of which it is taken in the operation, according as there are 1, 2, 3, &c. figures standing after it; and consequently the true value of the quotient figure belonging to that part of the dividend is also 10, 100, or 1000, &c. times its simple value. But the true value of the quotient figure belonging to that part of the dividend, as found by the rule, is also 10, 100, or 1000, &c. times its simple value; for there are as many figures set after it as the number of remaining figures in the dividend; and therefore, this first quotient figure, taken in its complete value, from the place it stands in, is the true quotient of the divisor in the complete value of the first part of the dividend. For the same reason, all the rest of the figures of the quotient taken according to their places, are each the true quotient of the divisor in the complete value of the several parts of the dividend belonging to each; because as the first figure on the right hand of each succeeding part of the dividend has a less number of figures by one standing after it, so in like manner have their quotients; and, consequently, taking all the quotient figures in order, as they are placed by the rule, they make one number, which is equal to the sum of the true quotient of all the several parts of the dividend; and this, therefore, is the true quotient of the whole dividend by the divisor. This may be illustrated by an example, in which set down the several parts of the dividend and quotient, according to their true values; for this purpose let 11850 be divided by 48, and the work will stand thus:

	Divisor 48)11850 dividend.	
1st part of the dividend,	11000	
48×200	9600	. . . 200 the 1st quotient.
1st remainder, . . .	1400	
Add . .	850	
2d part of the dividend,	2250	
48×40	1920	. . . 40 the 2d quotient.
2d remainder, . . .	330	(*Carried over.*)

(*Brought over,*)	330		
Add . .	0		
3d part of the dividend,	330		
48×6	288	 6	the 3d quotient.
Last remainder, . .	42	. . . 246	sum of all the quotients, or answer.

When there is no remainder, the quotient is the perfect answer to the question; but where there is a remainder, it may be observed that it goes so much towards another time as it approaches to the divisor; thus, if the remainder be a third part of the divisor, it will go one-third of a time more, and so on.

It is sometimes difficult to find how often the divisor is contained in the numbers of the several steps of the operation; we may try how often the first figure of the divisor is contained in the first figure, or first and second figures of the dividend; that number diminished by one or two is generally the quotient figure.

The reason of the method of proof is plain, for since the quotient is the number of times the dividend contains the divisor, the product of the quotient and divisor must evidently be equal to the dividend.

There are several other methods made use of to prove division, the best and most useful of which are the following:

1. Subtract the remainder from the dividend, and divide this number by the quotient, and the quotient found by this division will be equal to the former divisor, when the work is correct.

2. Add the remainder, and all the products of the several quotient figures by the divisor together, according to the order in which they stand in the work, and the sum will be equal to the dividend when the work is right.

3. Subtract the remainder from the dividend, and what remains will be equal to the product of the divisor and quotient.

4. Cast the 9s out of the divisor and quotient, and multiply the remainders together; what remains, after casting the 9s out of the product and adding the remainder, if any, will be equal to what remains after casting the 9s out of the dividend.

5. Divide the divisor and quotient by any number, multiply the remainder after this division, and add the product to

the remainder in the question; divide this sum again by the same number, and the remainder so found will be equal to the remainder after dividing the dividend by the same number. The several methods of proof may be illustrated by an example, thus

```
86)14387(167 Quot.
   86
  578
  516
  627
  602
   25
```

```
14387
   25
167)14362(86 Proof by
 1336      division.
 1002
 1002
```

```
  167
   86
 1002
1336
   25
14387 Proof by multi-
        plication.
```

PROOF BY CASTING OUT THE 9's.

5=excess of 9's in 167.
5= do. do. in 86.
5= do. do. in 14387.
7= do. do. in 25.

Therefore 5×5=25 (product of the excesses in quotient and divisor) excess of 9s=7+the excess of 9s in the remainder=7=5 which =5 the excess of 9s in the dividend.

PROOF BY USING ANY NUMBER.

1=the remainder after dividing the divisor by 5.
2= „ „ „ quotient by 5.

1×2=the product of the two remainders, and 2+25 (the remainder in the question)=27, which sum being divided by 5 leaves a remainder of 2; which is=2 the remainder after dividing the dividend by 5.

EXAMPLES.

(1.)

```
5)1914569130
  382913826
```

Proof.

```
382913826
        5
1914569130
```

(2.)

```
6)784698424
  130781404
```

Proof.

```
130781404
        6
784698424
```

(3.)

```
465)178426359(383712 Quot.
    1395
    3892
    3720
    1726
    1395
    3313
    3255
     585
     465
     1209
      930
      279 Rem.
178426359 Proof by addition.
```

```
383712 Quot.
   465 Divisor.
1918560
2302272
1534848
    279 Rem.
178426359 Proof by mult.
```

PROOF BY CASTING OUT THE 9s.

6=excess of 9s in the divisor.
6=excess of 9s in the quotient.
0=excess of 9s in the remainder.

Therefore, $6\times6=36$ (product of the excess in quotient and divisor,) excess of 9s=0. The excess of 9s in the remainder=0; and $0+0=0$=excess of 9s, which equal the excess of 9s in the dividend, consequently the work is right.

PROOF BY USING ANY NUMBER.

1=the remainder after dividing the divisor by 4.
0=the remainder after dividing the quotient by 4.

$1\times0=0$=the product of the two remainders; and $0+279$ (the remainder in the question,)=279, which sum being divided by 4, leaves a remainder of 3, which is equal to 3, the remainder in the dividend after dividing by the same number.

EXAMPLES IN CASE 1.

4. 237626÷2.	12. 472894÷ 6.	20. 1234560÷10.
5. 458436÷2.	13. 742494÷ 6.	21. 404340÷ 5.
6. 474328÷3.	14. 381423÷ 7.	22. 876736÷ 7.
7. 927546÷3.	15. 862576÷ 8.	23. 987654÷ 8.
8. 582628÷4.	16. 108744÷ 8.	24. 548977÷11.
9. 248648÷4.	17. 346239÷ 9.	25. 720012÷12.
10. 258670÷5.	18. 948780÷ 9.	26. 8143188÷ 4.
11. 694835÷5.	19. 8745670÷10.	27. 90879867÷ 3.

EXAMPLES IN CASE 2.

28.	169÷ 13.	40.	3917253÷ 228
29.	256÷ 16.	41.	38406278÷ 533.
30.	289÷ 17.	42.	19203139÷ 372.
31.	36119÷ 19.	43.	30928144÷ 728.
32.	48422÷ 22.	44.	15464072÷ 662.
33.	8731047÷ 74.	45.	604284680÷ 824.
34.	2526074÷ 98.	46.	569304137÷ 3177.
35.	4848879÷111.	47.	12345678÷ 7304.
36.	123477÷ 65.	48.	113864146÷ 6354.
37.	5052148÷139.	49.	11426140092÷ 1356.
38.	96817650÷425.	50.	4637075990÷28803.
39.	12768423÷986.	51.	123457075÷ 365.

CONTRACTIONS.

CASE 1.

§ 13. When there are ciphers annexed to the divisor.

RULE.

Cut off the ciphers from the divisor, and point off as many figures from the right of the dividend; then divide the remaining figures by each other, and the quotient will be the answer.

If any thing remains after this division, annex the figures pointed off to the remainder, and it will be the true remainder.

The reason of this rule is obvious; for the cutting off the same number of figures from each, is the same as dividing each of them by 10, 100, 1000, &c., and it is evident that as often as the whole divisor is contained in the whole dividend, so often must any part of the divisor be contained in a like part of the dividend. This method is only to avoid a needless repetition of ciphers, which would happen in the common way, as may be seen by working an example at large.

EXAMPLES.

1. Divide 897348432 by 30.

3,0)89734843,2

29911614+12

2. Divide 6234517890 by 8100.

81,00)62345178,90(769693 Quotient.
567
564
486
785
729
561
486
757
729
288
243
4590 Remainder.

3. 4721634÷20.	12. 811023456÷590.
4. 4712934÷40.	13. 107843948÷990.
5. 13986112÷80.	14. 1867472976÷438600.
6. 50446874÷90.	15. 7380964812÷894200.
7. 69724879÷110.	16. 2962543628÷374000.
8. 28731607÷190.	17. 31806217901÷1424000.
9. 98347748÷200.	18. 456377129461÷6782000.
10. 30623980÷300.	19. 980725099214÷7143100.
11. 79194372÷410.	20. 617384594200÷9842000.

CASE 2.

§ **14.** When the divisor is the exact product of two or more numbers.

RULE.

Divide by each of these numbers separately, instead of the whole divisor at once.

The actual remainder in questions worked by this rule, is found thus—

1. Multiply the quotient by the divisor and subtract the product from the dividend, and the result will be the true remainder. The truth of this is evident; for if the product of the quotient and divisor added to the remainder be equal to the dividend, their product taken from the dividend must leave the remainder.

2. Or multiply the last remainder by the preceding divisor, or last but one, and to the product add the preceding remainder; multiply this sum by the next preceding divisor, and to the product add the next preceding remainder; and so on till you have gone through all the divisors and remainders to the first, thus

Divide 73879 by 140.

7)73879
5)10554+1
4)2110+4
527+2

2 last remainder.
Mult. 5 preceding divisor
10
Add 4 the 2d remainder.
14
Mult. 7 the 1st divisor.
98
Add 1 the 1st remainder.
99 the true remainder.

EXAMPLES.

1. Divide 3571084 by 24.

24=6×4, 6)3571084
4)595180+4
Quotient 148795+0

The remainder=0×6+4=4

2. Divide 51433948 by 162.

162=9×6×3. 9)51433948
6)5714883+1
3)952480+3
Quotient 317493+1

Remainder 82=(1×6+3)×9+1.

3. 34608550÷28.
4. 71285951÷42.
5. 28566031÷63.
6. 14283016÷84.
7. 789352÷99.
8. 76941168÷108.
9. 123456708÷81.
10. 875423104÷121.
11. 942702845÷144.
12. 233741864÷132.
13. 3114174908÷126=(7×6×3.)
14. 647643881÷252=(4×9×7.)
15. 813779070÷315=(7×5×9.)
16. 470028489÷336=(8×7×6.)

17. 5878543141÷432=(6×8×9.)
18. 1048702482÷576=(8×8×9.)
19. 83374790214÷693=(11×9×7.)
20. 90801114768÷756=(12×7×9.)
21. 45480567344÷891=(9×9×11.)
22. 1635494406÷1188=(9×12×11.)
23. 28398916874÷960=(12×8×10.)
24. 5130956227÷4032=(8×7×6×4×3.)
25. 830116572808÷15120=(6×7×8×9×5.)

CASE 3.

§ **15.** When the divisor can be made hundreds, thousands, &c., by multiplying it by any number.

RULE.

Multiply the dividend also by that number, and then proceed as before.

This rule is deduced from the principle that any number multiplied and divided by the same number, remains the same as before.

EXAMPLES.

1. Divide 785414 by 25.

25) 785414
4 4
1,00)31416,56
31416.$\frac{56}{100}$

2.	Divide	51647	by 125.	6.	Divide	117668494	by 875.
3.	"	23731	" 250.	7.	"	497782676	" 75.
4.	"	936132	" 175.	8.	"	614879048	" 1875.
5.	"	3584410	" 625.	9.	"	7831416	" 225.

CASE 4.

§ **16.** To divide by any number of 9s.

RULE.

Place under the dividend a part of the same, leaving as many figures to the right hand as there are nines in the divisor. Place it again with the like change of place, as often as the length of the dividend admits; add these together, and cut off as many figures from the right hand of the

sum as there are nines in the divisor; the figures on the left hand constitute the quotient, and those cut off the remainder.

EXAMPLES.

(1.)

99)314167
3141 67
31 41
3173 40 08

3173 quot.
40 rem.

(2.)

999)953046
953 046
953046
953, &c.
953999 999999, &c. = 1.
1
quot. 954000 0 rem.

The figures extended without the line show the reason of carrying the unit when necessary.

3. Divide 33740 by 99.
4. " 553086 " 999.
5. " 43701631 " 9999.

Q. When the divisor is above 12, how do you divide?
Q. How do you divide, when the divisor is a composite number?
Q. How do you prove division?

CASE 5.

§ **17.** To resolve any composite number into its prime factors.

RULE.

Divide the number by any prime number which will divide it without a remainder; then divide the quotient in the same way, and so continue until a quotient is obtained which is a prime number. Then the successive divisors together with the last quotient will form the prime factors required.

The reason of this rule may be found in the fact, that all numbers which are not prime, are composed of prime factors. For, all numbers which are not prime are composite, and may be therefore separated into two or more factors. If these factors are not prime, they can again be separated into other factors, and thus the decomposition can be continued until all the factors are prime.

EXAMPLES.

1. Resolve 14500 into its prime factors.

2	14500
2	7250
5	3625
5	725
5	145
	29

Therefore, $2\times2\times5\times5\times5\times29=14500$, are the prime factors required.

2. Resolve 450 into its prime factors.
3. Resolve 786 into its prime factors.
4. Resolve 897 into its prime factors.
5. Resolve 1998 into its prime factors.
6. Resolve 2799 into its prime factors.
7. Resolve 3976 into its prime factors.
8. Resolve 14950 into its prime factors.
9. Resolve 32368 into its prime factors.
10. Resolve 786542 into its prime factors.
11. Resolve 98765432 into its prime factors.

Q. What are prime factors?

Q. How do you resolve a number into prime factors?

MISCELLANEOUS EXERCISES.

1. Divide 29629608 by 2.
2. Divide 4938268 by 4.
3. Divide 269867904 by 12.
4. Divide 503700480 by 15.
5. Divide 844439040 by 18.
6. Divide 4552933760 by 32.
7. Divide 7125880320 by 37.
8. Divide 103703040 by 28.
9. Divide 4320960 by 35.
10. Divide 99550080 by 84.
11. How many times is 72 contained in 7167605760?
12. How many times is 144 contained in 967057920?
13. How many times is 741 contained in 91481480649?
14. How many times is 703 contained in 86790122667?
15. Divide 165679010838 by 1342.
16. Divide 2922839479575 by 23675.
17. Divide 104366295346563 by 845367.
18. Divide 168434689825269 by 1364321

19. Divide 596566908151488 by 4832192.

20. Divide 7927200051319269 by 64210321.

21. Multiply 13717421 by 72, and divide their product by 8.

22. Multiply 13717421 by 1827, and divide their product by 203.

23. An army of 19000 men having plundered a city of 266000 dollars. How much must each man have?

24. There was a certain number of men concerned in the payment of 1272 dollars, and each one paid three dollars. What was the number of men?

25. If 2233 barrels of flour were divided among eleven persons, how much would each one receive?

26. The guardians of the poor have 2214 cords of wood to divide among 738 persons. How many cords will each one receive?

27. France has 32000000 inhabitants. The number of square miles in her territory is 154000. How many persons has she to each mile?

28. Eight children agreed to divide equally the estate left by their father. The sum divided was 67456 dollars. What was the share of each?

29. In planting a large orchard a farmer made ninety rows. How many trees should he put in each row to make up 4320, the whole number of trees planted?

30. Sixty-six German emigrants bought 42240 acres of land in the state of Ohio, which they resolve to divide equally. How many acres will each receive?

31. In starting upon a journey of 2376 miles, a traveller resolves to ride 44 miles a day. How many days, at that rate, will it take him to finish his journey?

32. From New York to Greece is 4800 miles. How many miles must a ship sail in a day to make the passage in 24 days?

33. In the year 1755, a vessel sailed from France to Canada, 2800 miles, in four weeks. How many miles did she sail in a week?

34. How many days would it require to sail from Scotland to Labrador, a distance of 2100 miles, at the rate of 150 miles each day?

35. From Norway to Greenland is 1800 miles. From Chili to New Zealand is 5800 miles. If a ship start from Norway and another from Chili at the same time, one for

Greenland, the other for New Zealand, and each sail at the rate of 100 miles a day, how much longer will one vessel be in reaching New Zealand than the other in making Greenland?

36. In the state of Virginia there are 1239797 inhabitants, and 64000 square miles. How many persons are there to each square mile?

37. Ninety-nine labourers pick in one year 12474 pounds of cotton. How many pounds is that for each?

38. In a regiment consisting of 1000 men, there were 50 officers, how many men to an officer?

39. The value of the machinery manufactured annually in the United States, is about 10985845 dollars. The number of men employed is 13001. What amount of machinery is produced by each man?

40. The number of smelting houses for the manufacture of lead is 120. The number of pounds of lead produced is about 31239480. How many pounds would that give for each smelting house?

41. The number of men employed in the manufacture of hardware and cutlery is about 5500. The value of the manufacture may be estimated at 6451500 dollars. How much in that case does each man produce?

42. The number of wholesale commercial and commission houses in the city and county of Philadelphia is 149; that of the stores for retailing dry goods, groceries, &c. is about 2086. How many of the latter are there for each of the former?

43. The number of forges, furnaces, &c. for the manufacture of wrought and cast iron in the United States, is about 1600. The capital invested in the whole of them is nearly 20433600 dollars. What would be the capital of each were it equally divided?

44. The total number of houses annually erected in the United States, is showed by the census of 1840, to be 54113. The erection of these costs about 41911609 dollars. How much is that for each?

45. By the same census, the number of powder mills in the United States is 137. The number of pounds of powder manufactured annually, is 8977336. What quantity would that give for each, allowing every mill to produce an equal number of pounds?

COMPOUND ADDITION.

§ **18.** Compound Addition is the addition of several numbers of different denominations.

RULE.

1. Place all the given numbers of the same denomination directly under each other.

2. Add up the figures in the lowest denomination, and divide the sum by as many of that denomination as will make one of the next greater.

3. Place the remainder under its column, and carry the quotient to the next; which add up in the same manner as before.

4. Proceed thus through all the denominations to the highest, whose sum, together with the several remainders, will be the answer required.

The method of proof is the same as in addition of integers.

Q. What is compound addition?

Q. How are the numbers placed to be added?

Q. Where must you begin to add?

Q. If the amount of the column added is more than will make one of the next higher denomination, what do you do?

Q. If the amount of the column is less than one of the next higher denomination, how do you proceed?

A. Set down the whole amount of the column.

Q. How do you prove compound addition?

FEDERAL MONEY.

§ **19.** Federal money is the currency of the United States.

Q. What are the denominations of federal money?

A. Eagles, dollars, dimes, cents, and mills.

Repeat the table.

TABLE.

10 mills, *m.* make . . . 1 cent, *c.*
10 cents . . . . . 1 dime, *d.*
10 dimes, or 100 cents, 1 dollar, $
10 dollars . . . . . 1 eagle, *E.*

The coins of the United States are of three kinds; gold, silver, and copper.

The gold coins are:			The silver coins are:		
The eagle,	=	$10	The dollar,	=	100 *cts*
Half eagle,	=	$5	Half dollar,	=	50 *cts*
Quarter eagle,	=	$2½	Quarter dollar,	=	25 *cts.*
			Dime,	=	10 *cts.*
			Half dime,	=	5 *cts.*

The copper coins are—cent, half cent.

The denominations dime and eagle, are not used in keeping accounts.

EXAMPLES.

(1.)

$	*cts.*
36	45
43	62
18	1
16	4
19	69
133	81

(2.)

$	*cts.*	*m.*
28	11	8
6	00	4
19	48	6
14	16	9
124	1	8
191	79	5

(3.)

E.	$	*d.*	*cts.*	*m.*
72	3	1	5	3
41	6	9	7	4
18	3	7	7	9
98	9	9	8	1
15	4	4	6	8
246	8	3	5	5

	$	*c.*	*m.*	$	*c.*	*m.*	$	*c.*	*m.*	$	*c.*	*m.*	
4. Add	111	11	4+	48	08	2+	16	41	9+	41	06	7	together.
5. "	487	16	2+	121	17	8+	48	00	1+	99	44	4	"
6. "	1728	06	7+	225	19	7+	29	49	9+	112	49	6	"
7. "	987	22	8+	117	00	0+	89	16	4+	97	01	1	"
8. "	1210	16	1+	872	14	1+	70	69	1+	100	00	2	"
9. "	1987	41	9+	931	12	7+	99	98	9+	188	79	8	"

ENGLISH MONEY.

§ **20.** Q. What are the denominations of English money?

A. Pounds, shillings, pence, and farthings.

Repeat the table.

TABLE.

4 farthing, *qrs.* make	. . . .	1 penny, *d.*
12 pence	. . . . .	1 shilling, *s.*
20 shillings	. . . . .	1 pound, £
21 shilling	. . . . .	1 guinea.
5 shillings	. . . . .	1 crown.

Farthings are commonly written as fractions of a penny. Thus, 1 farthing is written ¼*d.*, 2 farthings. ½*d.*, 3 farthings, ¾*d.*

EXAMPLES.

	(1.) £	s.	d.	(2.) £	s.	d.	(3.) £	s.	d.	qr
	741	19	11	43	0	$10\frac{3}{4}$	24	1	9	1
	421	16	7	48	7	$0\frac{1}{2}$	10	0	4	2
	342	4	6	11	6	$1\frac{1}{4}$	7	8	10	3
	30	0	1	25	2	8	17	19	2	1
Sum	1536	1	1							
	794	1	2							
Proof	1536	1	1							

4. Add 41£ 1*s.* $11\frac{3}{4}$*d.*+83£ 19*s.* 10*d.*+10£ 11*s.* $4\frac{1}{2}$*d.*+104£ 17*s.* $6\frac{1}{4}$*d.* together.

5. Add 21£ 4*s.* 10*d.*+7£ 19*s.* 1*d.*+59£ 0*s.* 8*d.*+81£ 1*s.* 9*d.*

6. Add 89£ 5*s.* 5*d.*+32£ 0*s.* 9*d.*+71£ 14*s.* 2*d.*+6£ 15*s.* 0*d.*

7. Add 110£ 18*s.* 6*d.*+99£ 17*s.* 8*d.*+30£ 13*s.* 4*d.*+19£ 1*s.* 7*d.*

8. Add 5£ 2*s.* $4\frac{1}{2}$*d.*+48£ 10*s.* $6\frac{1}{4}$*d.*+84£ 6*s.* $1\frac{3}{4}$*d.*+10£ 2*s.* $5\frac{1}{2}$*d.*

9. Add 72£ 3*s.* 7*d.*+27£ 3*s.* 7*d.*+37£ 4*s.* $8\frac{1}{2}$*d.*+12£ 13*s.* $0\frac{1}{4}$*d.*+92£ 3*s.* $8\frac{1}{4}$*d.*

10. Add 35£ 15*s.* 2*d.*+91£ 17*s.* 6*d.*+16£ 3*s.* $7\frac{3}{4}$*d.*+29£ 8*s.* $3\frac{1}{2}$*d.*+53£ 9*s.* $1\frac{1}{4}$*d.*

11. Add 27£ 17*s.* 4*d.*+£62 19*s.* $11\frac{1}{2}$*d.*+12£ 14*s.* $6\frac{3}{4}$*d.*+9£ 14*s.* $7\frac{3}{4}$*d.*+7£ 15*s.* 8*d.*+1£ 7*s.* $4\frac{1}{2}$*d.*

12. Add 147 13*s.* $7\frac{1}{4}$*d.*+116£ 9*s.* $3\frac{1}{2}$*d.*+173£ 12*s.* $2\frac{1}{4}$*d.*+489£ 0*s.* $1\frac{3}{4}$*d.*+731£ 1*s.* $2\frac{1}{2}$*d.*+258£ 7*s.* $4\frac{1}{4}$*d.*

13. Add 297£ 2*s.* 7*d.*+448£ 4*s.* $7\frac{1}{2}$*d.*+671£ 2*s.* $0\frac{3}{4}$*d.*+98£ 16*s.* $11\frac{3}{4}$*d.*+125£ 9*s.* 0*d.*+19£ 19*s.* $11\frac{3}{4}$*d.*+0£ 8*s.* 1*d.*

14. Add 888£ 0*s.* 0*d.*+297£ 17*s.* $2\frac{1}{2}$*d.*+258£ 19*s.* $0\frac{1}{4}$*d.*+137£ 1*s.* 1*d.*+713£ 12*s.* 3*d.*+610£ 14*s.* $8\frac{1}{4}$*d.*+741£ 4*s.* 7*d.*

15. Add 55£ 5*s.* 4*d.*+460£ 7*s.* 6*d.*+13£ 3*s.* $7\frac{1}{2}$*d.*+729£ 7*s.* 7*d.*+20£ 10*s.* 6*d.*+515£ 16*s.* 10*d.*+91£ 6*s.* $4\frac{1}{2}$*d.*+998£. 19*s.* 11*d.*

TROY WEIGHT.

§ **21.** Q. What are the denominations of troy weight?
A. Pounds, ounces, pennyweights, and grains.
Repeat the table.

TABLE.

24 grains, *gr.* make . . 1 pennyweight, *dwt.*
20 pennyweights . . . 1 ounce, *oz.*
12 ounces . . . . . 1 pound, *lb.*

Gold, silver, jewels, precious stones, and liquors, are weighed by this weight.

EXAMPLES.

(1.)			(2.)				(3.)			
lbs.	*oz.*	*dwts.*	*lbs.*	*oz.*	*dwts.*	*gr.*	*lbs.*	*oz.*	*dwts.*	*gr.*
5	7	13	1	6	16	10	0	3	11	15
7	3	8	9	8	10	18	0	4	5	3
9	2	12	4	2	3	7	7	9	14	12
8	4	11	8	9	14	9	4	2	19	2
11	11	19	0	3	15	12	11	6	7	10
10	6	5	2	5	5	16	6	9	16	14
53	0	8	27	0	6	0				

4. Add 4*lbs.* 10*oz.* 14*dwts* 18*gr.*+4*lbs.* 0*oz.* 14*dwts.* 0*gr.* +2*lbs.* 7*oz.* 4*dwts.* 4*gr.*+0*lbs.* 1*oz.* 8*dwts.* 1*gr.*

5. Add 12*lbs.* 3*oz.* 11*dwts.* 4*gr.*+14*lbs.* 7*oz.* 0*gr.*+0*lb.* 11*oz.* 19*dwts.*+5*lbs.* 15*dwts.* 23*gr.*+15*lbs.* 4*oz.* 1*dwt.* 4*gr.*+1*lb.* 11*oz.* 19*dwts.* 7*gr.*

6. Add 19*lbs.* 1*oz.* 10*dwts.* 7*gr.*+29*lbs.* 4*oz.* 4*dwts.* 22*gr.* +18*lbs.* 11*oz.* 19*dwts.* 23*gr.*+31*lbs.* 7*oz.* 1*dwt.* 1*gr.*+36*lbs.* 0*oz.* 0*dwt.* 16*gr.*+14*lbs.* 5*oz.* 6*gr.*+9*oz.* 9*dwts.* 9*gr.*

APOTHECARIES WEIGHT.

§ **22.** Q. What are the denominations of apothecaries weight?

A. Pounds, ounces, drachms, scruples, and grains.

Repeat the table.

TABLE.

20 grains, *gr.* make . . . 1 scruple, ℈
3 scruples . . . . . 1 drachm, ℥
8 drachms . . . . . 1 ounce, ℥
12 ounces . . . . . 1 pound, ℔

Apothecaries compound their medicines by this weight, but buy and sell by avoirdupois weight.

EXAMPLES.

(1.)

℥	ʒ	℈
1	7	1
0	2	0
1	1	1
6	6	2
0	0	2
0	6	1
11	0	1

(2.)

℔	℥	ʒ	℈
9	7	3	0
3	4	5	1
6	0	4	2
8	5	3	1
9	0	0	2
1	1	1	1
37	7	2	1

(3.)

℔	℥	ʒ	℈	*gr*
1	2	4	1	11
4	8	7	2	19
7	11	1	0	14
9	0	3	1	4
10	11	7	2	19
6	10	5	1	12

4. Add 2℔ 8℥ 1ʒ 1℈ 1*gr.*+7℥ 7ʒ 2℈ 18*gr.*+9℔ 1℥ 5ʒ 11*gr.*+17℔ 11℥ 2ʒ 13*gr.*

5. Add 11℔ 4℥ 6ʒ 2℈ 17*gr.*+22℔ 5℥ 1℈ 14*gr.*+7℔ 4ʒ 6*gr.*+16℔ 1℥ 1ʒ 2℈ 19*gr.*+11℔ 5ʒ 1℈ 4*gr.*+1℔ 3℥ 3ʒ 2℈ 5*gr.*

6. Add 17℔ 3ʒ 7ʒ 2℈ 11*gr.*+4℔ 11℥ 6ʒ 1℈ 19*gr.*+6℔ 7℥ 6ʒ 1℈ 17*gr.*+14℔ 9℥ 4ʒ 2℈+1℔ 5℥ 3ʒ 0℈ 1*gr.*+7℔ 2℥ 2ʒ 1℈ 12*gr.*

AVOIRDUPOIS WEIGHT.

§ **23.** Q. What are the denominations of avoirdupois weight?

A. Tons, hundreds, quarters, pounds, ounces, and drachms Repeat the table.

TABLE.

16 drachms (*dr.*) make 1 ounce, *oz.*
16 ounces . . . . . 1 pound, *lb.*
28 pounds . . . . . 1 quarter, *qr.*
4 quarters, or 112*lbs.* 1 hundred weight, *cwt.*
20 hundred weight . . 1 ton, *T.*

By this weight, all coarse and drossy goods, groceries, and all metals, except gold and silver, are weighed.

A hundred weight is often used to signify 100lbs.; and the hundred weight of 112lbs. is called a hundred weight gross.

EXAMPLES.

(1.)

T.	*cwt.*	*qr.*	*lbs.*	*oz.*	*dr.*
6	13	1	18	13	5
7	6	0	12	0	6
5	8	3	14	6	0
0	1	2	0	7	15
4	15	1	10	4	8
24	5	1	0	0	2

(2.)

T.	*cwt.*	*qr.*	*lbs.*	*oz.*	*dr.*
9	19	0	12	11	15
4	2	3	27	1	13
8	1	1	0	10	11
7	0	0	18	5	0
11	14	2	9	4	7

3. Add 2 *T.* 10*cwt.* 3*qr.*+5 *T.* 19*cwt.* 1*qr.* 11*lb.*+7 *T.* 8*cwt.* 3*qr.* 17*lb.*+6 *T.* 12*cwt.* 2*qr.* 12*lb.*+0 *T.* 1*cwt.* 1*qr.* 15*lb.*

4. Add 21 *T.* 16*cwt.* 1*qr.* 4*lb.* 13*oz.* 2*dr.*+17 *T.* 7*cwt.* 0*qr.* 7*lb.* 6*oz.* 14*dr.*+7 *T.* 17*cwt.* 3*qr.* 17*lb.* 14*oz.* 15*dr.*+1 *T.* 19*cwt.* 3*qr.* 27*lb.* 15*oz.* 15*dr.*

5. Add 6 *T.* 13*cwt.* 1*qr.* 15*lb.* 13*oz.* 5*dr.*+9 *T.* 11*cwt.* 0*qr.* 13*lb.* 8*oz.* 7*dr.*+18 *T.* 1*cwt.* 2*qr.* 19*lb.* 0*oz.* 9*dr.*+11 *T.* 17*cwt.* 3*qr.* 21*lb.* 5*oz.* 12*dr.*+1 *T.* 1*cwt.* 1*qr.* 1*lb.* 1*oz.* 1*dr.*

6. Add 9 *T.* 19*cwt.* 2*qr.* 10*lb.* 14*oz.* 14*dr.*+11 *T.* 10*cwt.* 3*qr.* 11*lb.* 10*oz.* 2*dr.*+17 *T.* 17*cwt.* 0*qr.* 19*lb.* 15*oz.* 15*dr.*+28 *T.* 14*cwt.* 1*qr.* 16*lb.* 5*oz.* 9*dr.*+31 *T.* 8*cwt.* 3*qr.* 25*lb.* 6*oz.* 3*dr.*+4*cwt.* 10*lb.* 2*dr.*

LONG MEASURE.

§ **24.** Q. What are the denominations of long measure?

A. Leagues, miles, furlongs, rods or poles, yards, feet, inches, and barleycorns.

Repeat the table.

TABLE.

3 barleycorns (*b. c.*) make	1 inch, *in.*
12 inches	1 foot, *ft.*
3 feet	1 yard, *yd.*
5½ yards, or 16½ feet	1 rod or pole, *p.*
40 poles, or 220 yards	1 furlong, *fur.*
8 furlongs, or 1760 yards	1 mile, *M.*
3 miles	1 league, *L.*
60 geographic, or 69½ statute miles,	1 degree, *Deg.*
360 degrees the circumference of the earth.	
4 inches	1 hand.
6 feet	1 fathom.
$7\frac{92}{100}$ inches	1 link.
4 rods, or 66 feet, or 100 links . .	1 chain.
80 chains.	1 mile.

A hand is used to measure the height of horses.

A fathom is chiefly used to measure the depth of water.

This measure is used to ascertain length, or distances, without regard to breadth.

EXAMPLES.

(1.)

L.	*m.*	*fur.*	*p.*	*yd.*	*ft.*	*in.*
16	2	6	14	0	2	4
12	1	5	9	5	1	9
19	0	4	27	4	2	11
27	2	7	10	0	0	10
76	1	7	22	0	1	10

(2.)

L.	*m.*	*fur.*	*p.*	*yd.*	*ft.*	*in.*	*bar.*
13	1	4	10	1	0	1	0
29	0	7	29	0	1	2	2
64	2	5	11	3	2	9	1
98	1	6	33	5	1	11	2

3. Add 37*m.* 6*fur.* 35*p.* 0*yds.* 2*ft.* 11*in.* 1*bar.*+28*m.* 1*fur.* 12*p.* 3*yd.* 1*ft.* 8*in.* 2*bar.*+17*m.* 4*fur.* 4*p.* 3*yd.* 2*ft.* 9*in.* 1*bar.* +29*m.* 2*fur.* 19*p.* 5*yd.* 0*ft.* 10*in.* 0*bar.*

4. Add 20*L.* 2*m.* 7*fur.* 39*p.*+18*L.* 2*m.* 5*fur.* 31*p.*+16*L.* 1*m.* 6*fur.* 17*p.*+25*L.* 1*m.* 4*fur.* 28*p.*+6*L.* 0*m.* 2*fur.* 11*p.*

5. Add 256*m.* 7*fur.* 29*p.* 5*yd.*+233*m.* 3*fur.* 37*p.* 4*yd.*+ 711*m.* 6*fur.* 37*p.* 3*yd.*+91*m.* 6*fur.* 17*p.* 3*yd.*+0*m.* 0*fur.* 39*p.* 2*yd.*+0*m.* 4*fur.* 3*p.* 5*yd.*

LAND OR SQUARE MEASURE.

§ **25.** Q. What are the denominations of square measure?

A. Square miles, square acres, square roods, square poles, square yards, square feet, and square inches.

Repeat the table.

TABLE.

144 square inches, (*S. I.*) make 1 square foot, *S. F.*
9 square feet . . . . . 1 square yard, *S. Y.*
30¼ square yards 1 square rod, pole or perch, *S. P.*
272¼ square feet . . . . 1 square rod.
40 square poles . . . . 1 square rood, *S. R.*
4 square roods or 4840 sq. yds. 1 square acre, *S. A.*
640 square acres . . . . 1 square mile, *S. M.*
100 feet . . . . . 1 square of flooring.
16 square rods . . . . . 1 square chain.
10 square chains, ... 1 acre.

This measure is applied to surface, or whatever has length and breadth.

EXAMPLES.

(1.)

A.	*R.*	*P.*	*ft.*	*in.*
167	2	4	260	116
81	1	2	270	41
14	0	0	198	0
36	3	39	211	140
1	1	13	155	4
301	0	22	7	13

(2.)

A.	*R.*	*P.*	*ft.*
19	3	7	271
61	1	39	199
18	0	13	187
31	2	4	271
14	11	21	166

3. Add 7*A.* 0*R.* 16*P.* 267*ft.* 21*in.*+15*A.* 3*R.* 31*P.* 270*ft* 141*in.*+19*A.* 3*R.* 8*P.* 261*ft.* 108*in.*+9*A.* 2*R.* 17*P.* 241*ft* 28*in.*+1*A.* 2*R.* 0*P.* 50*ft.* 4*in.*

4. 87*A.* 2*R.* 19*P.* 181*ft.* 131*in.*+21*A.* 3*R.* 39*P.* 240*ft.* 61*in.*+36*A.* 1*R.* 1*P.* 174*ft.* 99*in.*+46*A.* 1*R.* 0*P.* 261*ft.* 84*in.*+4*A.* 0*R.* 0*P.* 233*ft.* 30*in.*

5. 7*A.* 3*R.* 39*P.*+144*A.* 2*R.* 14*P.*+99*A.* 0*R.* 11*P.*+ 81*A.* 6*R.* 31*P.*+71*A.* 1*R.* 21*P.*+97*A.* 3*R.* 36*P.*

6. 641*A.* 3*R.* 38*P.*+748*A.* 0*R.* 11*P.*+888*A.* 1*R.* 18*P.* +248*A.* 1*R.* 19*P.*+374*A.* 2*R.* 27*P.*+438*A.* 3*R.* 39*P.*

SOLID OR CUBIC MEASURE.

§ **26.** Q. What are the denominations of solid measure?
A. Cords, tons, solid yards, solid feet, solid inches.
Repeat the table.

TABLE.

1728 solid inches (*S. I.*) make	1 solid foot, *Sd. F.*
27 solid feet	1 solid yard, *Sd. Y.*
40 feet of round timber	1 ton, *T.*
50 feet of hewn timber	1 ton, *T.*
40 solid feet	1 ton of shipping.
16 feet of wood	1 cord foot.
128 feet (8 feet long, 4 feet wide and 4 feet high) or 8 cord feet	1 cord, *Cd.*
24¾ solid feet	1 solid perch of stone.

This measure is applied to whatever has length, breadth, and thickness.

EXAMPLES.

(1.)			(2.)			(3.)		
T.	*ft.*	*in.*	*Cds.*	*ft.*	*in.*	*Cds.*	*ft.*	*in.*
41	25	1411	25	118	671	16	127	1721
6	14	678	8	91	401	18	110	869
110	36	807	17	127	1589	9	105	1633
121	17	117	28	1	84	19	116	1168
99	39	444	9	112	71	37	4	1041
380	13	1	90	66	1088			

4. Add 41*yds.* 22*ft.* 1111*in.*+67*yds.* 26*ft.* 1727*in.*+23*yds.* 18*ft.* 841*in.*+87*yds.* 9*ft.* 1689*in.*+78*yds.* 21*ft.* 441*in.*

5. Add 68 *T.* 39*ft.* 718*in.*+79 *T.* 21*ft.* 1628*in.*+126 *T.* 18*ft.* 601*in.*+17 *T.* 27*ft.* 468*in.*+118 *T.* 14*ft.* 1727*in.*

6. Add 19*cds.* 127*ft.* 111*in.*+17*cds.* 110*ft.* 1711*in*+21*cds.* 13*ft.* 748*in.*+61*cds.* 108*ft.* 1699*in.*+5*cds.* 112*ft.* 889*in.*

CLOTH MEASURE.

§ **27.** Q. What are the denominations of cloth measure?

A. Yards, quarters, nails, inches, and ells.

Repeat the table.

TABLE.

2¼ inches (*in.*) make 1 nail, *n.*
4 nails . . . . . 1 quarter of a yard, *qr.*
4 quarters . . . . . 1 yard, *yd.*
2½ quarters . . . . . 1 ell Hamburgh, *E. H.*
3 quarters . . . . . 1 ell Flemish, *E. Fl.*
5 quarters . . . . . 1 ell English, *E. E.*
6 quarters . . . . . 1 ell French, *E. Fr.*

This measure is used for cloths, tapes, &c.

EXAMPLES.

(1.)			(2.)			(3.)			(4.)		
yds.	*qrs.*	*n.*	*E.E.*	*qrs.*	*n.*	*E.Fl.*	*qrs.*	*n.*	*yds.*	*qrs.*	*n.*
21	3	2	8	4	3	5	2	1	14	0	3
32	2	2	7	1	2	3	1	3	7	3	2
18	0	1	9	3	1	11	0	2	13	0	1
16	3	2	6	2	0	9	2	1	2	2	2
8	1	1	0	1	2	7	1	2	9	3	3
97	3	0	32	3	0						

5. Add 4*yds.* 3*qrs.* 3*n.*+17*yds.* 1*qr.* 0*n.*+21*yds.* 1*qr.* 3*n.*+7*yds.* 2*qrs.* 2*n.*+12*yds.* 2*qrs.* 1*n.*+5*yds.* 0*qrs.* 3*n.*

6. 7*E. E.* 4*qrs.* 1*n.*+17*E. E.* 3*qrs.* 2*n.*+35*E. E.* 2*qrs.* 3*n.*+41*E. E.* 1*qr.* 1*n.*+14*E. E.* 2*qrs.* 2*n.*+5*E. E.* 2*qrs.* 1*n.*

7. 7*E. Fl.* 2*qrs.* 1*n.*+6*E. Fl.* 2*qrs.* 2*n.*+9*E. Fl.* 1*qr.* 1*n.*+2*E. Fl.* 1*qr.* 3*n.*+7*E. Fl.* 1*qr.* 1*n.*+12*E. Fl.* 0*qrs.* 2*n.*

8. 2*E. Fr.* 5*qrs.* 2*n.*+1*E. Fr.* 4*qrs.* 0*n.*+21*E. Fr.* 3*qrs.* 3*n.*+13*E. Fr.* 2*qrs.* 3*n.*+8*E. Fr.* 1*qr.* 3*n.*+17*E. Fr.* 0*qrs.* 1*n.*

9. 17*yds.* 3*qrs.* 1*n.*+31*yds.* 2*qrs.* 2*n.*+45*yds.* 1*qr.* 1*n.*+71*yds.* 0*qrs.* 3*n.*+6*yds.* 0*qrs.* 2*n.*+0*yds.* 3*qrs.* 3*n.*+3*yds.* 0*qrs.* 0*n.*+5*yds.* 2*qrs.* 1*n.*

WINE MEASURE.

§ 28. Q. What are the denominations of wine measure?

A. Tuns, pipes, puncheons, hogsheads, tierces, barrels, gallons, quarts, pints, and gills.

Repeat the table.

TABLE.

4	gills, (*gi.*) make	1 pint, *pt.*
2	pints	1 quart, *qt.*
4	quarts	1 gallon, *gal.*
31½	gallons	1 barrel, *bar.*
42	gallons	1 tierce, *tie.*
63	gallons	1 hogshead, *hhd.*
84	gallons, or 2 tierces, . .	1 puncheon, *pun.*
2	hogsheads	1 pipe, *P.*
2	pipes, or 4 hogsheads, or 252 gallons,	1 tun, *T.*

The wine gallon contains 231 cubic inches.

This measure is applied to all spirituous liquors, vinegar, oil, &c.

EXAMPLES.

(1.)

T.	*hhd.*	*gal.*	*qt.*
18	0	59	3
7	3	21	1
17	1	48	0
28	2	24	3
5	1	17	1
77	1	45	0

(2.)

T.	*hhd.*	*gal.*	*qt.*	*pt.*
21	3	14	3	1
10	2	62	1	0
11	0	31	2	1
19	3	11	0	1
38	1	47	1	0

(3.)

P.	*hhd.*	*gal.*	*qt.*	*pt.*
6	1	8	1	0
8	0	18	2	1
16	1	16	0	1
12	0	31	3	0
7	1	60	1	1

4. Add 23*hhd.* 17*gal.* 3*qt.* 1*pt.* 3*gi.*+41*hhd.* 0*gal.* 2*qt.* 1*pt.* 1*gi.*+13*hhd.* 61*gal.* 2*qt.* 0*pt.* 2*gi.*+19*hhd.* 49*gal.* 0*qt.* 0*pt.* 3*gi.*+7*hhd.* 38*gal.* 1*pt.* 3*gi.*

5. Add 23 *T.* 3*hhd.* 62*gal.* 1*qt.*+15 *T.* 1*hhd.* 7*gal.* 3*qt.* +39 *T.* 0*hhd.* 4*gal.* 0*qt.*+11 *T.* 3*hhd.* 18*gal.* 2*qt.*

6. Add 1*P.* 1*hhd.* 27*gal.* 3*qt.*+6*P.* 0*hhd.* 61*gal.* 1*qt.*+ 3*P.* 0*hhd.* 49*gal.* 2*qt.*+2*P.* 1*hhd.* 23*gal.* 0*qt.*+5*P.* 1*hhd.* 18*gal.* 3*qt.*

7. Add 17 *T.* 2*P.* 1*tie.* 41*gal.*+29 *T.* 1*P.* 1*tie.* 19*gal.*+ 34 *T.* 1*P.* 0*tie.* 9*gal.*+8 *T.* 2*P.* 0*tie.* 6*gal.*+25 *T.* 0*P.* 1*tie.* 13*gal.*

ALE AND BEER MEASURE.

§ **29.** Q. What are the denominations of ale measure?

A. Tuns, butts, hogsheads, barrels, gallons, quarts, and pints.

Repeat the table.

TABLE.

2 pints, (*pts.*) make . . .	1 quart, *qt.*
4 quarts	1 gallon, *gal.*
36 gallons	1 barrel, *bbl.*
54 gallons, or 1½ barrels, .	1 hogshead, *hhd.*
2 hogsheads	1 butt, *bt.*
2 butts	1 tun, *T.*

The ale gallon contains 282 cubic inches.

EXAMPLES.

(1.)

T.	*bt.*	*hhd.*	*gal.*	*qt.*	*pt.*
23	0	1	51	2	1
31	1	0	27	0	1
56	1	0	28	3	0
12	0	1	11	1	1
64	1	1	53	2	0
189	0	0	10	1	1

(2.)

Hhd.	*gal.*	*qt.*	*pt.*
8	51	2	1
16	3	3	1
19	52	0	0
33	17	2	1
79	53	0	1

3. Add 47 *T.* 1*bt.* 1*hhd.* 27*gal.* 2*qt.*+28 *T.* 0*bt.* 1*hhd.* 53*gal.* 0*qt.*+12 *T.* 1*bt.* 1*hhd.* 0*gal.* 3*qt.*+33 *T.* 1*bt.* 1*hhd.* 49*gal.* 0*qt.*

4. Add 91*hhd.* 50*gal.* 2*qt.* 1*pt.*+78*hhd.* 48*gal.* 3*qt.* 0*pt.* +19*hhd.* 37*gal.* 3*qt.* 1*pt.*+9*hhd.* 52*gal.* 0*qt.* 1*pt.*+49*hhd.* 39*gal.* 1*qt.* 0*pt.*

5. Add 45*hhd.* 25*gal.* 2*qt.* 0*pt.*+36*hhd.* 18*gal.* 1*qt.* 1*pt* +96*hhd.* 13*gal.* 3*qt.* 1*pt.*+87*hhd.* 13*gal.* 0*qt.* 1*pt.*+11*hhd.* 47*gal.* 1*qt.* 0*pt.*

6. Add 56 *T.* 1*bt.* 1*hhd.* 51*gal.* 3*qt.*+46 *T.* 0*bt.* 1*hhd.* 28*gal.* 1*qt.*+47 *T.* 1*bt.* 0*hhd.* 39*gal.* 2*qt.*+10 *T.* 1*bt.* 0*hhd.* 15*gal.* 3*qt.*+25 *T.* 0*bt.* 1*hhd.* 50*gal.* 2*qt.*

DRY MEASURE.

§ **30.** Q. What are the denominations of dry measure?

A. Bushels, pecks, quarts, and pints.

Repeat the table.

TABLE.

2 pints, (*pt.*)	make . .	1 quart, *qt.*
8 quarts	. . . . .	1 peck, *p.*
4 pecks	. . . . .	1 bushel, *bu.*
36 bushels	. . . . .	1 chaldron, *chal.*

A gallon by dry measure contains $268\frac{4}{5}$ cubic inches, and a bushel $2150\frac{42}{100}$.

This measure is used for grain, salt, coal, fruit, &c.

EXAMPLES.

(1.)				(2.)				(3.)			
Bu.	*pk.*	*qt.*	*pt.*	*Ch.*	*bu.*	*pk.*	*qt.*	*Bu.*	*pk.*	*qt.*	*pt.*
4	2	5	1	14	32	3	2	41	0	3	1
7	3	7	0	7	17	1	1	32	3	7	0
8	2	3	1	16	33	2	7	71	2	5	1
5	1	2	1	21	19	0	6	87	3	6	0
9	3	7	0	11	8	3	5	14	1	1	1
36	2	1	1								

4. Add 17*bu.* 3*pk.* 6*qt.* 1*pt.*+15*bu.* 2*pk.* 5*qt.* 1*pt.*+21*bu.* 2*pk.* 4*qt.* 0*pt.*+37*bu.* 3*pk.* 7*qt.* 1*pt.*

5. Add 32*bu.* 1*pk.* 2*qt.* 1*pt.*+81*bu.* 1*pk.* 4*qt.* 0*pt.*+91*bu.* 3*pk.* 1*qt.* 0*pt.*+18*bu.* 3*pk.* 2*qt.* 1*pt.*+9*bu.* 1*pk.* 1*qt.* 1*pt.*

6. Add 21*chal.* 38*bu.* 3*pk.* 2*qt.* 1*pt.*+17*chal.* 10*bu.* 2*pk.* 0*qt.* 0*pt.*+84*chal.* 17*bu.* 0*pk.* 1*qt* 0*pt.*+99*chal.* 31*bu.* 1*pk.* 3*qt.* 1*pt.*+8*chal.* 3*pk.* 5*qt.*+25*bu.* 2*pk.* 1*pt.*+3*pk.* 0*qt.* 1*pt.*

TIME.

§ **31.** What are the denominations of time?

A. Years, months, weeks, days, hours, minutes, and seconds.

Repeat the table.

TABLE.

60 seconds, (*sec.*) make . . 1 minute, *min.*
60 minutes . . . . . 1 hour, *h.*
24 hours . . . . . 1 day, *d.*
7 days . . . . . 1 week, *wk.*
4 weeks . . . . . 1 lunar month, *l. mo.*
13 l. mos., 1 day, 5 hours, 48 min., 48 sec., *or* 365 days, 5 hrs. 48 min. 48 sec. } 1 solar year, *yr.*
12 calendar months, *or* 365 ds. 1 civil or calendar year, *yr.*
100 years . . . . . 1 century, *C.*

Jan.	the	1st	month	has	31	days.	July	the	7th	month	has	31	days.
Feb.	"	2d	"	"	28	"	Aug.	"	8th	"	"	31	"
Mar.	"	3d	"	"	31	"	Sep.	"	9th	"	"	30	"
April	"	4th	"	"	30	"	Oct.	"	10th	"	"	31	"
May	"	5th	"	"	31	"	Nov.	"	11th	"	"	30	"
June	"	6th	"	"	30	"	Dec.	"	12th	"	"	31	"

One day is added to the month of February (making it 29 days) in every fourth calendar year, called *leap year*, that the civil may nearly correspond with the solar year.

A TABLE, *showing the number of days from any day in one month, to the same day in any other month.*

From January	Feb.	Mar.	Apr.	May	Jun.	July	Aug.	Sep.	Oct.	Nov.	Dec.
To Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

EXAMPLES.

1. How many days from 2d of April to 2d of September?

Look for April at the left hand, and September at the top, in the angle is 153.

2. How many days from 1st of May to 1st of January?

Look for May at the left hand and January at the top, in the angle is 245.

EXAMPLES.

(1.)

yr.	*l.mo.*	*wk.*	*ds.*
7	4	2	4
8	3	3	5
5	11	0	6
6	0	1	3
2	10	3	2
30	4	3	6

(2.)

yr.	*l.mo.*	*wk.*	*ds.*	*hr.*	*min.*
2	3	1	3	11	31
7	11	3	6	14	50
5	9	2	5	19	11
6	7	1	4	21	23
3	2	0	6	15	4
25	8	2	6	9	59

(3.)

yr.	*l.mo.*	*wk.*	*ds.*	*hr.*	*min.*	*s.*
17	2	0	5	11	3	41
8	1	1	6	19	31	29
16	3	3	3	23	17	17
0	11	3	4	9	59	32
14	0	0	0	4	2	12

4. Add 6*yrs.* 3*mo.* 2*wks.*+8*yrs.* 4*mo.* 3*wks.*+9*yrs.* 11*mo.* 1*wk.*+7*yrs.* 7*mo.* 2*wks.*+4*yrs.* 0*mo.* 2*wks.*

5. 5*yrs.* 11*mo.* 2*wks.* 6*ds.* 40*min.*+5*yrs.* 8*mo.* 3*wks.* 0*ds.* 18*h.* 42*min.*+11*yrs.* 3*mo.* 1*wk.* 5*ds.* 12*h.* 31*min.*+21*yrs.* 9*mo.* 2*wks.* 3*ds.* 23*h.* 11*min.*

6. 15*yrs.* 9*mo.* 3*wks.* 6*ds.* 2*h.* 21*min.* 15*sec.*+27*yrs.* 11*mo.* 0*wk.* 3*ds.* 11*h.* 52*min.* 1*sec.*+2*yrs.* 10*mo.* 2*wks.* 6*ds.* 23*h.* 48*min.* 59*sec.*+1*yr.* 11*mo.* 3*wks.* 6*ds.* 23*h.* 59*min.* 59*sec.*

CIRCULAR MOTION.

§ **32.** Q. What are the denominations of motion?

A. Signs, degrees, minutes, and seconds.

Repeat the table.

TABLE.

60 seconds ($''$) make . 1 minute, *m.* $'$
60 minutes . . . . 1 degree, *d.* °
30 degrees . . . . 1 sign, *s.*
90 degrees . . . . 1 quadrant.
12 signs, or 360 degrees, 1 circle of the Zodiac.

This table is used by navigators, astronomers, &c., and relates to the heavenly bodies.

EXAMPLES.

(1.)			(2.)				(3.)			
°	′	″	s.	°	′	″	s.	°	′	″
4	8	17	0	11	4	15	7	29	18	19
9	31	58	10	9	54	45	11	21	50	40
11	54	36	11	0	45	37	9	18	31	57
8	1	11	8	4	18	1	6	4	20	27
6	59	54	9	21	52	28	5	15	19	6
40	35	56								

4. Add $4° 6′ 15″ + 9° 16′ 50″ + 18° 51′ 39″ + 27° 0′ 17″ + 21° 0′ 56″ + 0° 1′ 59″ + 2° 49′ 35″$.

5. Add $2s. 3° 4′ 5″ + 5s. 8° 9′ 6″ + 4s. 7° 8′ 31″ + 11s. 27° 30′ 24″ + 9s. 18° 17′ 38″$.

6. Add $1s. 27° 12′ 31″ + 8s. 3° 33′ 31″ + 9s. 22° 40′ 55″ + 10s. 10° 45′ 44″ + 0s. 20° 55′ 15″ + 6s. 15° 24′ 24″$.

MISCELLANEOUS TABLE.

§ **33.**

24 sheets of paper make	1 quire.
20 quires	1 ream.
2 reams	1 bundle.
10 reams	1 bale.
2 leaves or 4 pages . .	1 sheet folio.
4 leaves or 8 pages . .	1 sheet quarto.
8 leaves or 16 pages .	1 sheet octavo.
12 leaves or 24 pages .	1 sheet duodecimo.
18 leaves or 36 pages .	1 sheet eighteens.
24 leaves or 48 pages .	1 sheet twenty-fours
12 skins of parchment .	1 dozen.
5 dozen or 60 skins .	1 roll.
12 particulars or single things	1 dozen.
12 dozen	1 gross.
12 gross or 144 dozen .	1 great gross.
20 single things . . .	1 score.
5 score	1 hundred.
Firkin of butter . . .	56 lbs.
Fother of lead	19½ cwt.
Stone of iron	14 lbs.
Stone of butcher's meat .	8lbs.
Barrel of raisins . . .	112 lbs.
Barrel of flour	196 lbs.
Barrel of soap	256 lbs.

Barrel of pork or beef	200 lbs.
Last of gunpowder	42 barrels.
Hide of land	100 acres
Load of bricks	500.
Load of tiles	1000.
Cubit	18 inches.
Sacred cubit	22 inches nearly.
Span	9 inches.
Gallon of train oil	7½ lbs.
Gallon of molasses	11 lbs.
Quintal of fish	100 lbs.
Sack	3 bushels.
Fagot of steel	120 lbs.
Tod	28 lbs.
Barrel of cider or beer	32 gallons.
Barrel of fish	30 gallons.
Russian mile	1100 yards.
Italian	1467 "
Irish and Scotch	2200 "
Polish	4400 "
Spanish	5028 "
German	5866 "
Swedish or Danish	7233 "
Hungarian	8800 "

MISCELLANEOUS EXERCISES.

1. If B have owing to him on note £1641 8*s.* 4½*d.*, and interest due thereon £142 15*s.* 7½*d.*, how much is the amount?

2. Just received 4 parcels of cloth; in the first 100*yd.* 3*qr.* 2*n*; in the second 170*yd.* 1*qr.* 1*n.*; in the third 199*yd.*; and in the fourth 205*yd.* 3*qr.*; how many yards in all?

3. A goldsmith bought 5 ingots of silver, the first weighing 11*lb.* 4*oz.* 16*dwt.* 11*gr.*; the second 9*lb.* 10*oz.* 14*dwt.*; the third 8*lb.* 5*oz.* 15*dwt.* 14*gr.*; the fourth 2*lb.* 6*oz.* 0*dwt* 8*gr.*; and the fifth 6*lb.* 4*oz.* 10*dwt.* 20*gr.*; what is the weight of the whole?

4. Suppose a merchant bought 4 hogsheads of sugar weighing as follows: No. 1, 9*cwt.* 2*qr.* 14*lb.*; No. 2, 8*cwt.* 3*qr.* 12*lb.*; No. 3, 7*cwt.* 2*qr.* 21*lb.*; and No. 4, 6*cwt.* 0*qr.* 2*lb.*; how much did the whole weigh?

5. If a druggist mix several simples together, first, 3*oz.* 4*dra.* 1*scr.*; second, 4*oz.* 3*dra.* 2*scr.* 18*gr.*; third, 4*dra.*

16*gr.*; fourth, 6*oz.* 5*dra.* 2*scr.* 2*gr.*: how much do they all weigh?

6. There are 5 pieces of linen, No. 1, 27*yd.* 3*qr.* 3*n.*; No. 2, 41*yd.* 2*qr.* 3*n.*; No. 3, 30*yd.* 2*qr.* 1*n.*; No. 4, 33*yd.* 1*qr.* 2*n.*; and No. 5, 28*yd.* 0*qr.* 1*n.*: what quantity do they contain?

7. If one field contain 17*ar.* 3*r.* 27*p.*; another 27*ar.* 3*r.* 36*p.*; and a third 31*ar.* 1*r.* 18*p.*: how much in all?

8. What day of the year was the 25th of the 8th month, 1844?

9. A man had to travel a certain distance, and he first walked 100*m.* 6*fur.*; then rode 216*m.* 5*fur.* 8*p.*; then walked 117*m.* 19*p.*; then rode again 200*m.*: what was the extent of his journey?

10. A gentleman's stock of wines was 4*pi.* 1*hhd.* of port; 2*pun.* of sherry; 3*pi.* of claret; of Madeira 72 *gal.*; of mountain 1*hhd.* 44*gal.* 1*pt.*: what quantity of liquor did his cellar contain?

11. Admit John to be 27*y.* 5*m.* and 2*w.* old, Jane 25*y.*; William 20*y.* 7*m.* 3*w.* 4*d.*; Francis 17*y.* 4*d.*; Frances 14*y.* 11*m.* 1*w.*; and Edward 12*y.* 1*m.* 6*d.*: what is the sum of their ages?

12. Add 14*bu.* 2*pk.* 5*qt.*; 23*bu.* 3*pk.*; 8*bu.* 7*qt.*; 19*bu.* 1*pk.* to a granary that contains 69*bu.* 4*qt.*; and tell the amount.

13. Required the sum of 3*s.* 24° 45′; 1*s.* 2′; 2*s.* 20° 13′; and 3*s.* 15° 30′ 40″.

14. Bought 4 lots of wood, the first containing 42*cd.*; second 8*cd.* 76*ft.*; third 16*cd.* 97*ft.*; and the fourth 31*cd.* 118*ft.*: what was the quantity purchased?

15. A merchant divided a ship's cargo into four parts, the first of which he sold for $2017. 22*ct.*; the second for $1749. 49*ct.*; the third for $3797. 21*ct.*; and the last for $1993. 87*ct.*: what did he receive for the whole cargo?

16. A manufacturer shipped 3 cases of straw bonnets, for the first of which he received $130. 74*ct.*; for the second $129. 13*ct.*; for the third $169. 81*ct.* For the cases containing the goods, the purchaser paid $4. 85*ct.* What sum did he receive for the whole shipment?

17. A shoe dealer purchased at auction four trunks filled with shoes, valued as follows: first, $500; second, $114. 28*ct.*; third, $175. 15*ct.*; fourth, $356. 94*ct.* What was the cost of the whole?

18. A merchant sold three bales of cottons; for the first

he received $582. 16*ct.*; for the second, $852. 11*ct.*; for the third, $1987. What did he receive for the three bales?

19. A grocer bought a barrel of oil containing 23*gal.* 3*qt.*; a tierce which held 41*gal.* 2*qt.* 1*pt.*; and a hogshead containing the usual quantity. How many gallons were contained in the three vessels?

20. A ship sailing round the world, sails in the first month over 1*s.* 13° 45′ of longitude; in the second, over 1*s.* 0° 53′ 21″; in the third, over only 19° 22′: how far was the vessel from the port from which she sailed at the end of the third month?

21. A silversmith sells five dozen of silver spoons, weighing 10*lb.* 9*oz.* 12*dwt.*; two ladles, weighing 1*lb.* 3*oz.* 10*dwt.*; one dozen of plates, weighing 2*lb.* 11*oz.* 19*dwt.*; and four dozen of plates of another pattern, the weight of which amounted to 6*lb.* 4*oz.* 17*dwt.* What was the weight of the whole?

22. A gentleman has five sons, of which the fifth, or youngest, is 1 year, 8 months, and 17 days old. The fourth is 2 years, 3 months, and 8 days older. Between the fourth and third, there is an interval of 3 years, 8 months, and 25 days. The third son is 3 years younger than the second, who wants 2 years, 9 months, and 14 days of being as old as the first. What is the age of the first born?

23. An American merchant had remitted to him from his agent in London, at five different times, the following sums: 1st, £792 4*s.* 7¼*d.*; 2d, £484 7*s.* 4½*d.*; 3d, £176 13*s.* 2¾*d.*; 4th, £568 17*s.* 1½*d.*; 5th, £251 12*s.* 6¼*d.* How much did he receive in all?

24. A sea captain carried for a merchant 12*cwt.* 2*qr.* 12*lb.* of hops; 5*cwt.* 2*qr.* of wool; 1*cwt.* 2*qr.* 14*lb.* of rice; 10*cwt.* 1*qr.* 12*lb.* of sugar; and 2*qr.* 21*lb.* of salt. For what amount of weight must the captain receive freight?

25. If a postrider travel on one day 33 miles, 7 furlongs, 8 poles; on the next, 48 miles, 3 furlongs, 37 poles; on the third, 47 miles, 7 furlongs, 17 poles; on the fourth, 53 miles, 6 furlongs, 14 poles; and on the fifth, 49 miles, 3 furlongs, 29 poles. How many miles will he travel in the whole five days?

26. A corder has in one pile 22 cords, 35 feet; in another, 74 cords, 2 feet; in a third, 48 cords, 7 feet; in a fourth, 19 cords, 9 feet; and in a fifth, 37 cords, 5 feet. How much has he in all?

COMPOUND SUBTRACTION.

§ **34.** Compound subtraction teaches the method of finding the difference between numbers of several denominations.

RULE.

1. Place the less number under the greater, so that like denominations may stand under each other.

2. Begin at the right hand, and take each number in the lower line from that above it, and place the remainder under it.

3. But if the number in the lower line be greater than that above it, increase the upper number by as many as make one of the next higher denomination, then subtract the lower number from the upper one, and write down what remains.

4. Carry the unit borrowed to the next number in the lower line, which subtract from the number above it, as before, and proceed in the same manner till the whole is finished, and the several remainders taken together will be the whole difference required.

The method of proof is the same as in subtraction of integers.

Q. What is compound subtraction?

Q. How are the numbers placed to be subtracted?

Q. Where do you begin to subtract?

Q. If the number in the lower line be greater than the one above it, how do you proceed?

Q. How do you prove compound subtraction?

§ 35. FEDERAL MONEY.

EXAMPLES.

(1.)			(2.)				(3.)				
$	*ct.*	*m.*	$	*d.*	*ct.*	*m.*	*E.*	$	*d.*	*ct.*	*m.*
154	19	3	243	3	8	6	7	4	5	8	0
79	10	4	184	4	9	7	3	6	8	7	1
75	08	9	58	8	8	9	3	7	7	0	9

4. From $178. 14*ct.* 8*m.* take $89. 15*ct.* 6*m.*
5. From $872. 0*ct.* 4*m.* take $642. 18*ct.* 7*m.*
6. From $184. 1*ct.* 3*m.* take $99. 2*ct.* 1*m.*
7. From $1870. 99*ct.* 9*m.* take $1642. 1*ct.* 0*m.*
8. From $999. 99*ct.* 9*m.* take $149. 89*ct.* 7*m.*

9. From $778. 62*ct.* 5*m.* take $516. 42*ct.* 1*m.*
10. From $1000. 0*ct.* 0*m.* take $784. 82*ct.* 6*m.*
11. From $682. 21*ct.* 8*m.* take $444. 44*ct.* 4*m.*
12. From $2148. 0*ct.* 7*m.* take $1249. 16*ct.* 8*m.*
13. From six eagles take four dollars and eight cents.
14. From ten half eagles take six dollars and six cents.

§ 36. ENGLISH MONEY.

EXAMPLES.

	(1.) £	*s.*	*d.*	(2.) £	*s.*	*d.*	(3.) £	*s.*	*d.*
From	11	8	$6\frac{1}{2}$	471	19	$11\frac{3}{4}$	781	19	$11\frac{1}{2}$
Take	7	5	$8\frac{3}{4}$	348	15	$10\frac{1}{2}$	299	0	$11\frac{3}{4}$
Remains	4	2	$9\frac{3}{4}$	123	4	$1\frac{1}{4}$			
Proof	11	8	$6\frac{1}{2}$	471	19	$11\frac{3}{4}$			

4. £28 8*s.* 4*d.*—£17 18*s.* 9*d.*
5. £126 19*s.* $8\frac{1}{2}$*d.*—£87 0*s.* 11*d.*
6. £243 11*s.* 4*d.*—£199 12*s.* $6\frac{1}{4}$*d.*
7. £714 18*s.* $3\frac{1}{4}$*d.*—£524 19*s.* $10\frac{1}{2}$*d.*
8. £483 16*s.* $5\frac{1}{2}$*d.*—£279 9*s.* $11\frac{1}{4}$*d.*
9. £345 13*s.* 9*d.*—£227 14*s.* $9\frac{3}{4}$*d.*
10. £249 3*s.* 7*d.*—£137 17*s.* $5\frac{1}{2}$*d.*
11. £74 4*s.* 5*d.*—£18 6*s.* $4\frac{1}{4}$*d.*
12. £8 0*s.* $0\frac{1}{4}$*d.*—£6 4*s.* $0\frac{1}{2}$*d.*
13. £10 10*s.* 10*d.*—£5 19*s.* $3\frac{1}{2}$*d.*
14. £7 7*s.* 7*d.*—£4 9*s.* 11*d.*
15. £879 1*s.* $4\frac{1}{2}$*d.*—£680 2*s.* 2*d.*
16. £1114 3*s.* 1*d.*—£978 3*s.* 7*d.*
17. £921 2*s.* $2\frac{1}{2}$*d.*—£710 7*s.* $1\frac{1}{4}$*d.*
18. £0 19*s.* $11\frac{3}{4}$*d.*—£0 4*s.* 4*d.*
19. £1 5*s.* 4*d.*—£0 15*s.* 5*d.*
20. £67 6*s.* 0*d.*—£19 19*s.* $1\frac{1}{2}$*d.*

§ 37. TROY WEIGHT.

EXAMPLES.

(1.) *lb.*	*oz.*	*dwt.*	(2.) *lb.*	*oz.*	*dwt.*	*gr.*	(3.) *lb.*	*oz.*	*dwt.*	*gr.*
31	8	9	87	7	14	19	111	0	16	21
11	9	12	18	11	16	21	86	11	9	16
19	10	17	68	7	17	22				

4. 12*lb.* 7*dwt.* 8*gr.*—4*lb.* 8*dwt.* 5*gr.*
5. 18*lb.* 2*oz.* 0*dwt.*—11*lb.* 8*oz.* 4*dwt.*
6. 0*lb.* 10*oz.* 6*dwt.* 9*gr.*—0*lb.* 4*oz.* 18*dwt.* 11*gr.*
7. 5*lb.* 7*oz.* 14*dwt.* 16*gr.*—2*lb.* 9*oz.* 18*dwt.* 16*gr.*
8. 11*lb.* 9*oz.* 15*dwt.* 3*gr.*—5*lb.* 10*oz.* 6*dwt.* 19*gr.*
9. 18*lb.* 0*oz.* 1*dwt.* 2*gr.*—12*lb.* 17*dwt.* 9*gr.*
10. 7*oz.* 11*dwt.* 8*gr.*—4*oz.* 19*dwt.* 23*gr.*
11. 12*oz.* 7*dwt.* 16*gr.*—5*oz.* 10*dwt.* 17*gr.*
12. 15*lb.* 10*oz.* 8*dwt.*—10*lb.* 11*dwt.* 4*gr.*
13. 8*oz.* 13*dwt.* 9*gr*—7*oz.* 16*dwt.* 0*gr.*
14. 19*lb.* 4*oz.* 0*dwt.* 1*gr.*—14*lb.* 9*oz.* 7*dwt.* 11*gr.*
15. 27*lb.* 11*oz.* 19*dwt.* 4*gr.*—19*lb.* 10*oz.* 0*dwt.* 22*gr.*
16. 41*lb.* 5*oz.* 0*dwt.* 6*gr.*—29*lb.* 6*oz.* 18*dwt.* 15*gr.*
17. 99*lb.* 0*oz.* 0*dwt.* 0*gr.*—78*lb.* 11*oz.* 11*dwt.* 21*gr.*
18. 67*lb.* 11*oz.* 19*dwt.* 20*gr.*—51*lb.* 11*oz.* 19*dwt.* 23*gr.*
19. 73*lb.* 4*oz.* 1*dwt.* 1*gr.*—67*lb.* 10*oz.* 13*dwt.* 4*gr.*
20. 121*lb.* 11*oz.* 13*dwt.* 7*gr.*—109*lb.* 4*oz.* 14*dwt.* 8*gr.*

§ 38. APOTHECARIES' WEIGHT.

EXAMPLES.

(1.)

℔	℥	ʒ
13	6	7
4	2	3
9	4	4

(2.)

℔	℥	ʒ	℈	*gr.*
98	3	4	1	17
19	7	6	2	19
78	7	5	1	18

(3.)

℔	℥	ʒ	℈
79	11	7	2
38	9	4	1
41	2	3	1

4. 16℔ 9℥ 3ʒ 0℈ 14*gr.*—11℔ 10℥ 7ʒ 1℈ 9*gr.*
5. 14℔ 7℥ 7ʒ 2℈ 17*gr.*—9℔ 8℥ 6ʒ 0℈ 19*gr.*
6. 19℔ 8℥ 6ʒ 1℈ 19*gr.*—17℔ 10℥ 7ʒ 1℈ 17*gr.*
7. 23℔ 11℥ 4ʒ 0℈ 12*gr.*—15℔ 5℥ 4ʒ 1℈ 2*gr.*
8. 9℔ 2℥ 5ʒ 2℈ 3*gr.*—6℔ 2℥ 6ʒ 2℈ 4*gr.*
9. 17℔. 1℥ 6ʒ 0℈ 18*gr.*—12℔ 0℥ 7ʒ 0℈ 16*gr.*
10. 37℔ 0℥ 1ʒ 1℈—29℔ 11℥ 7ʒ 2℈.
11. 49℔ 11℥ 7ʒ 0℈—40℔ 11℥ 7ʒ 2℈.
12. 8℔ 0℥ 6ʒ 4*gr.*—2℔ 5℥ 4ʒ 1℈ 15*gr.*
13. 18℔ 4ʒ 2℈—13℔ 2℥ 11*gr.*
14. 13℔ 1℈ 5*gr.*—11℥ 3ʒ 6*gr.*
15. 5℔ 7℥ 1ʒ 2℈ 12*gr.*—1℔ 3ʒ 1℈ 14*gr.*
16. 10℔ 4℥ 5ʒ 1℈ 18*gr.*—7℔ 5℥ 6ʒ 2℈ 12*gr.*
17. 15℔ 10℥ 3ʒ 0℈ 17*gr.*—9℔ 0℥ 5ʒ 1℈ 18*gr.*
18. 12℔ 11℥ 2℈ 1*gr.*—7℔ 6℥ 2ʒ 2℈ 2*gr.*
19. 11℔ 0℥ 11*gr.*—2℔ 4ʒ 2℈.
20. 64℔ 2℥ 7ʒ 2℈ 10*gr.*—5℔ 3℥ 6ʒ 1℈ 16*gr.*

§ 39. AVOIRDUPOIS WEIGHT.

EXAMPLES.

(1.)

T.	*cwt.*	*qr.*	*lb.*
18	1	3	14
4	2	2	27
13	19	0	15

(2.)

T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
141	18	2	10	7	11
79	19	3	12	9	13
61	18	2	25	13	14

3. 14*cwt.* 0*qr.* 4*lb.*—4*cwt.* 3*qr.* 16*lb.*
4. 46*cwt.* 3*qr.* 17*lb.*—29*cwt.* 2*qr.* 19*lb.*
5. 33*cwt.* 1*qr.* 23*lb*—17*cwt.* 3*qr.* 12*lb.*
6. 51*cwt.* 0*qr.* 14*lb*—42*cwt.* 2*qr.* 15*lb.*
7. 6*cwt.* 2*qr.* 17*lb.* 13*oz.*—4*cwt.* 1*qr.* 11*lb.* 10*oz.*
8. 12*cwt.* 3*qr.* 21*lb.* 6*oz.*—8*cwt.* 2*qr.* 18*lb.* 12*oz.*
9. 14*cwt.* 1*qr.* 6*lb.* 2*oz.*—9*cwt.* 0*qr.* 4*lb.* 7*oz.*
10. 17*cwt.* 2*qr.* 0*lb.* 15*oz.*—14*cwt.* 3*qr.* 27*lb.* 14*oz.*
11. 19*cwt.* 0*qr.* 17*lb.* 0*oz.* 4*dr.*—15*cwt.* 1*qr.* 1*lb.* 11*oz.* 9*dr.*
12. 11*cwt.* 1*qr.* 6*lb.* 11*oz.* 6*dr.*—5*cwt.* 2*qr.* 16*lb.* 15*oz.* 15*dr.*
13. 16*cwt* 13*lb.* 2*dr.*—6*cwt.* 1*qr.* 9*dr.*
14. 23*cwt.* 15*oz.* 14*dr.*—20*cwt.* 18*lb.* 15*dr.*
15. 13*cwt.* 15*oz.* 12*dr.*—11*cwt.* 1*qr.* 7*dr.*
16. 2 *T.* 13*cwt.* 1*qr.* 14*lb.*—1 *T.* 16*cwt.* 3*qr.* 11*lb.*
17. 114 *T.* 19*cwt.* 0*qr.* 19*lb.*—84 *T.* 15*cwt.* 1*qr.* 24*lb.*
18. 211 *T.* 13*cwt.* 1*qr.* 6*lb.* 13*oz.* 4*dr.*—199 *T.* 10*cwt.* 0*qr.* 25*lb.* 14*oz.* 13*dr.*
19. 345 *T.* 16*cwt.* 3*qr.* 22*lb.* 4*oz.* 11*dr.*—284 *T.* 1*cwt.* 2*qr.* 17*lb.* 14*oz.* 12*dr.*
20. 87 *T.* 2*qr.* 12*oz.* 8*dr.*—42 *T.* 15*cwt.* 18*lb.* 14*oz.*

§ 40. LONG MEASURE.

EXAMPLES.

(1.)

deg.	*m.*	*fur.*	*p.*	*yd.*	*ft.*	*in.*	*bar.*
48	29	5	28	3	1	8	1
27	31	6	38	2	0	2	2
20	57	6	30	1	1	5	2

(2.)

L.	*m.*	*fur.*	*p.*	*yd.*	*ft.*	*in.*	*bar.*
61	2	3	17	2	2	11	1
39	1	6	29	1	2	11	2
22	0	4	28	0	2	11	2

3. 45*deg.* 31*m.* 4*fur.* 21*p.* 12*ft.*—27*deg.* 29*m.* 7*fur.* 16*p.* 9*ft.*
4. 121*m.* 3*fur.* 11*p.* 4*ft.*—97*m.* 6*fur.* 18*p.* 2*ft.*
5. 111*deg.* 49*m.* 5*fur.*—79*deg.* 53*m.* 7*fur.*
6. 49*m.* 3*fur.* 19*p.* 5*yd.*—36*m.* 6*fur.* 25*p.* 4*yd.*
7. 7*fur.* 5*p.* 4*yd.* 2*ft.* 9*in.* 2*bar.*—5*fur.* 4*p.* 3*yd.* 2*ft.* 11*in.* 1*bar.*
8. 87*deg.* 59*m.* 2*fur.* 39*p.* 4*yd.*—69*deg.* 53*m.* 6*fur.* 18*p.* 3*yd.*
9. 116*m.* 6*fur.* 38*p.* 5*yd.*—81*m.* 7*fur.* 39*p.* 4*yd.*
10. 7*deg.* 41*m.* 2*fur.* 9*p.* 3*yd.* 2*ft.* 10*in.*—3*deg.* 30*m.* 3*fur.* 17*p.* 2*yd.* 11*in.*
11. 37*m.* 7*fur.* 14*p.* 3*yd.*—29*m.* 6*fur.* 19*p.* 2*yd.*
12. 146*m.* 0*fur.* 31*p.* 2*yd.*—111*m.* 7*fur.* 32*p.* 2*yd.*
13. 237*m.* 2*fur.* 8*p.* 4*yd.*—204*m.* 6*fur.* 12*p.* 3*yd.*
14. 101*m.* 5*fur.* 17*p.* 2*yd.*—88*m.* 7*fur.* 21*p.* 1*yd.*
15. 421*m.* 2*fur.* 35*p.* 1*yd.*—375*m.* 5*fur.* 36*p.* 1*yd.*
16. 119*m.* 3*fur.* 20*p.* 2*yd.* 2*ft.*—71*m.* 6*fur.* 22*p.* 1*yd.* 2*ft.*
17. 112*m.* 5*fur.* 23*p.* 3*yd.* 1*ft.*—101*m.* 6*fur.* 24*p.* 2*yd.* 1*ft.*
18. 222*m.* 7*fur.* 33*p.* 4*yd.* 2*ft.*—111*m.* 0*fur.* 34*p.* 3*yd.* 2*ft.*
19. 333*m.* 0*fur.* 36*p.* 5*yd.* 1*ft.*—231*m.* 1*fur.* 37*p.* 4*yd.* 0*ft.*
20. 41*deg.* 53*m.* 4*fur.* 30*p.* 3*yd.* 2*ft.* 9*in.* 2*bar.*—34*deg.* 46*m.* 5*fur.* 32*p.* 2*yd.* 2*ft.* 10*in.* 1*bar.*

§ 41. LAND OR SQUARE MEASURE.

EXAMPLES.

(1.)

A.	*R.*	*P.*	*yd.*	*ft.*	*in.*
17	2	17	28	8	114
11	3	18	24	7	101
5	2	39	4	1	13

(2.)

A.	*R.*	*P.*	*yd.*	*ft.*	*in.*
34	1	36	30	7	131
29	2	38	27	8	116
4	2	38	2	8	15

3. 67*A.* 3*R.* 17*p.*—41*A.* 0*R.* 17*p.*
4. 118*A.* 2*R.* 39*p.*—97*A.* 3*R.* 11*p.*
5. 231*A.* 1*R.* 12*p.*—198*A.* 2*R.* 21*p.*
6. 576*A.* 3*R.* 13*p.*—410*A.* 0*R.* 14*p.*
7. 84*A.* 0*R.* 29*p.*—48*A.* 1*R.* 26*p.*
8. 71*A.* 3*R.* 31*p.*—19*A.* 2*R.* 37*p.*
9. 29*A.* 0*R.* 35*p.*—22*A.* 1*R.* 38*p.*
10. 119*A.* 3*R.* 19*p.*—77*A.* 2*R.* 23*p.*
11. 46*A.* 0*R.* 2*p.* 6*yd.*—28*A.* 3*R.* 15*p.* 5*yd.*
12. 11*A.* 2*R.* 18*p.* 8*yd.*—4*A.* 0*R.* 1*p.* 6*yd.*
13. 69*A.* 3*R.* 31*p.* 7*yd.*—41*A.* 1*R.* 34*p.* 7*yd.*

14. 152*A.* 1*R.* 22*p.* 8*yd.*—114*A.* 2*R.* 17*p.* 4*yd.*
15. 236*A.* 3*R.* 2*p.* 4*yd.* 4*ft.*—144*A.* 2*R.* 18*p.* 3*yd.* 5*ft.*
16. 568*A.* 1*R.* 27*p.* 29*yd.* 2*ft.*—479*A.* 3*R.* 37*p.* 21*yd.* 7*ft.*
17. 901*A.* 0*R.* 30*p.* 21*yd.* 3*ft.*—799*A.* 2*R.* 14*p.* 18*yd.* 8*ft.*
18. 26*A.* 1*R.* 2*p.* 30*yd.* 2*ft.* 111*in.*—16*A.* 2*R.* 11*p.* 22*yd.* 3*ft.* 112*in.*
19. 75*A.* 1*R.* 17*p.* 204*ft.* 78*in.*—33*A.* 2*R.* 39*p.* 117*ft.* 84*in.*
20. 11*A.* 3*R.* 21*p.* 116*ft.* 143*in.*—2*A.* 1*R.* 13*p.* 99*ft.* 112*in.*

§ 42. SOLID OR CUBIC MEASURE.

EXAMPLES.

(1.)			(2.)			(3.)		
T.	*ft.*	*in.*	*T.*	*ft.*	*in.*	*T.*	*ft.*	*in.*
11	39	1200	87	17	1545	114	22	666
6	25	842	42	18	1698	49	32	1500
5	14	358	44	38	1575			

4. 36 *T.* 14*ft.* 14*in.*—19 *T.* 16*ft.* 1111*in.*
5. 29 *T.* 4*ft.* 128*in.*—16 *T.* 5*ft.* 1041*in.*
6. 45 *T.* 39*ft.* 1312*in.*—24 *T.* 28*ft.* 1289*in.*
7. 56 *T.* 18*ft.* 631*in.*—39 *T.* 19*ft.* 404*in.*
8. 61 *T.* 27*ft.* 1346*in.*—43 *T.* 15*ft.* 1486*in.*
9. 72 *T.* 33*ft.* 1578*in.*—57 *T.* 36*ft.* 1608*in.*
10. 84 *T.* 17*ft.* 721*in.*—59 *T.* 16*ft.* 729*in.*
11. 101 *T.* 43*ft.* 1197 *in.*—66 *T.* 45*ft.* 824*in.*
12. 243 *T.* 5*ft.* 16*in.*—151 *T.* 45*ft.* 15*in.*
13. 175 *T.* 47*ft.* 1701*in.*—127 *T.* 49*ft.* 1672*in.*
14. 312 *T.* 31*ft.* 432*in.*—222 *T.* 12*ft.* 324*in.*
15. 1112 *T.* 39*ft.* 1727*in.*—1009 *T.* 21*ft.* 1524*in.*
16. 94 *T.* 44*ft.* 410*in.*—87 *T.* 48*ft.* 301*in.*
17. 10 *Cd.* 41*ft.* 1004*in.*—6 *Cd.* 122*ft.* 974*in.*
18. 12 *Cd.* 126*ft.* 739*in.*—5 *Cd.* 121*ft.* 899*in.*
19. 100 *Cd.* 98*ft.* 576*in.*—62 *Cd.* 127*ft.* 625*in.*
20. 384 *Cd.* 49*ft.* 1078*in.*—291 *Cd.* 118*ft.* 1602*in.*

§ 43. CLOTH MEASURE.

EXAMPLES.

(1.)			(2.)				(3.)			
yd.	*qr.*	*n.*	*yd.*	*qr.*	*n.*	*in.*	*yd.*	*qr.*	*n.*	*in.*
17	2	3	114	3	1	1	94	0	2	2
11	1	2	38	0	3	2	39	3	3	1
6	1	1	76	2	1	$1\frac{1}{4}$				

4. 141*yd.* 0*qr.* 1*n.*—81*yd.* 3*qr.* 3*n.*
5. 69*yd.* 3*qr.* 3*n.*—44*yd.* 3*qr.* 2*n.*
6. 114*yd.* 2*qr.* 1*n.*—55*yd.* 3*qr.* 2*n.*
7. 98*yd.* 1*qr.* 2*n.*—64*yd.* 2*qr.* 3*n.*
8. 271*yd.* 0*qr.* 3*n.*—119*yd.* 1*qr.* 1*n.*
9. 399*yd.* 1*qr.* 2*n.*—249*yd.* 2*qr.* 3*n.*
10. 123*yd.* 2*qr.* 1*n.*—104*yd.* 3*qr.* 0*n.*
11. 178*yd.* 0*qr.* 0*n.*—99*yd.* 1*qr.* 1*n.*
12. 460*yd.* 1*qr.* 3*n.*—372*yd.* 2*qr.* 2*n.*
13. 28*E. E.* 1*qr.* 1*n.*—16*E. E.* 3*qr.* 2*n.*
14. 65*E. E.* 0*qr.* 3*n.*—37*E. E.* 4*qr.* 2*n.*
15. 44*E. Fr.* 1*qr.* 2*n.*—27*E. Fr.* 5*qr.* 3*n.*
16. 55*E. Fl.* 2*qr.* 1*n.*—33*E. Fl.* 2*qr.* 2*n.*
17. 48*E. H.* 2*qr.* 2*n.*—17*E. H.* 1*qr.* 3*n.*
18. 114*yd.* 1*qr.* 3*n.* 2*in.*—89*yd.* 2*qr.* 1*n.* 2*in.*
19. 1160*yd.* 2*qr.* 2*n.* 1*in.*—988*yd.* 2*qr.* 2*n.* 1*in.*
20. 1736*yd.* 2*qr.* 1*n.* 2*in.*—1647*yd.* 3*qr.* 3*n.* 1*in.*

§ 44. WINE MEASURE.

EXAMPLES.

(1.)

T.	*hhd.*	*gal.*	*qt.*	*pt.*
84	3	17	1	1
15	1	27	2	1
69	1	52	3	0

(2.)

hhd.	*gal.*	*qt.*	*pt.*	*g.*
8	41	2	1	3
5	56	3	0	2
2	47	3	1	1

3. 16*T.* 2*hhd.* 17*gal.* 3*qt.*—10*T.* 1*hhd.* 34*gal.* 2*qt.*
4. 27*T.* 3*hhd.* 38*gal.* 2*qt.*—19*T.* 2*hhd.* 16*gal.* 3*qt.*
5. 51*hhd.* 18*gal.* 3*qt.* 1*pt.*—39*hhd.* 19*gal.* 2*qt.* 0*pt.*
6. 111*hhd.* 61*gal.* 2*qt.* 0*pt.*—97*hhd.* 62*gal.* 3*qt.* 1*pt*
7. 88*hhd.* 11*gal.* 1*qt.*—49*hhd.* 14*gal.* 3*qt.*
8. 99*hhd.* 45*gal.* 2*qt.*—89*hhd.* 54*gal.* 3*qt.*
9. 122*hhd.* 28*gal.* 3*qt.*—79*hhd.* 31*gal.* 2*qt.*
10. 16*hhd.* 60*gal.* 1*qt.* 1*pt.*—12*hhd.* 61*gal.* 2*qt.* 1*pt.*
11. 231*hhd.* 58*gal.* 2*qt.*—147*hhd.* 59*gal.* 3*qt.*
12. 35*hhd.* 16*gal.* 2*qt.* 1*pt.*—17*hhd.* 17*gal.* 3*qt.* 0*pt.*
13. 44*hhd.* 7*gal.* 1*qt.* 0*pt.* 2*gi.*—24*hhd.* 8*gal.* 2*qt.* 1*pt.* 3*gi*
14. 21*T.* 2*hhd.* 16*gal.*—18*T.* 3*hhd.* 49*gal.*
15. 101*T.* 2*hhd.* 33*gal.* 1*qt.*—81*T.* 3*hhd.* 44*gal.* 2*qt.*
16. 19*T.* 2*pun.* 1*tie.* 41*gal.*—12*T.* 1*pun.* 1*tie.* 22*gal.*
17. 83*T.* 2*pun.* 1*tie.* 41*gal.* 2*qt.*—56*T.* 1*pun.* 1*tie.* 36*gal* 1*qt.*

18. 32 *T.* 1*pun.* 1*hhd.* 62*gal.* 2*qt.*—27 *T.* 0*pun.* 1*hhd.* 49*gal.* 3*qt.*
19. 91 *T.* 0*P.* 0*hhd.* 14*gal.* 1*qt.*—68 *T.* 1*P.* 1*hhd.* 16*gal.* 2*qt.*
20. 90*hhd.* 61*gal.* 2*qt.* 1*pt.* 2*gi.*—53*hhd.* 62*gal* 3*qt.* 0*pt.* 1*gi.*

§ 45. ALE AND BEER MEASURE.

EXAMPLES.

(1.)

T.	*hhd.*	*gal.*	*qt.*	*pt.*
78	1	18	1	0
13	0	51	1	1
65	0	20	3	1

(2.)

hhd.	*gal.*	*qt.*	*pt.*
201	0	0	0
97	52	2	1
103	1	1	1

3. 16*hhd.* 31*gal.* 2*qt.* 0*pt.*—14*hhd.* 20*gal.* 1*qt.* 1*pt.*
4. 110*hhd.* 6*gal.* 0*qt.* 0*pt.*—51*hhd.* 37*gal.* 2*qt.* 1*pt.*
5. 236*hhd.* 17*gal.* 1*qt.* 1*pt.*—201*hhd.* 18*gal.* 3*qt.* 1*pt.*
6. 49*hhd.* 53*gal.* 0*qt.* 1*pt.*—31*hhd.* 27*gal.* 1*qt.* 0*pt.*
7. 129*hhd.* 29*gal.* 2*qt.*—102*hhd.* 36*gal.* 3*qt.*
8. 340*hhd.* 42*gal.* 0*qt.*—214*hhd.* 51*gal.* 2*qt.*
9. 199*hhd.* 6*gal.* 2*qt.*—117*hhd.* 7*gal.* 3*qt.*
10. 38*hhd.* 14*gal.* 0*qt.* 1*pt.*—0*hhd.* 51*gal.* 1*qt.* 1*pt.*
11. 300*hhd.* 0*gal.* 0*qt.* 0*pt.*—129*hhd.* 49*gal.* 2*qt.* 1*pt.*
12. 78 *T.* 1*bt.* 1*hhd.* 13*gal.* 0*qt.*—60 *T.* 1*bt.* 1*hhd.* 48*gal*
13. 400*hhd.* 53*gal.* 2*qt.*—177*hhd.* 50*gal.* 3*qt.*
14. 51 *T.* 1*bt.* 0*hhd.* 39*gal.*—37 *T.* 0*bt.* 0*hhd.* 40*gal.*
15. 12 *T.* 1*bt.* 1*hhd.* 16*gal.*—8 *T.* 0*bt.* 0*hhd.* 50*gal.*
16. 17 *T.* 0*bt.* 1*hhd.* 8*gal.* 2*qt.*—7 *T.* 1*bt.* 1*hhd.* 1*gal.* 1*qt.*
17 501*hhd.* 19*gal.* 3*qt.*—392*hhd.* 20*gal.* 2*qt.*
18. 36 *T.* 1*bt.* 1*hhd.* 35*gal.*—25 *T.* 1*bt.* 1*hhd.* 41*gal.*
19. 111*hhd.* 0*gal.* 1*qt.* 1*pt.*—87*hhd.* 0*gal.* 2*qt.* 1*pt.*
20. 98 *T.* 1*bt.* 1*hhd.* 15*gal.*—70 *T.* 1*bt.* 1*hhd.* 49*gal.*

§ 46. DRY MEASURE.

EXAMPLES.

(1.)

chal.	*bu.*	*p.*	*qt.*	*pt.*
41	2	2	1	1
7	1	3	2	0
34	0	2	7	1

(2.)

bu.	*p.*	*qt.*	*pt.*
71	1	1	1
26	2	7	1
44	2	2	0

3. 56*bu.* 3*p.* 5*qt.* 1*pt.*—29*bu.* 0*p.* 7*qt.* 0*pt*
4. 88*bu.* 2*p.* 6*qt.* 0*pt.*—58*bu.* 3*p.* 7*qt.* 1*pt.*
5. 13*bu.* 1*p.* 5*qt.*—7*bu.* 2*p.* 6*qt.*
6. 98*bu.* 2*p.* 4*qt.*—49*bu.* 3*p.* 5*qt.*
7. 116*bu.* 0*p.* 2*qt.*—75*bu.* 2*p.* 3*qt.*
8. 201*bu.* 1*p.* 3*qt.*—102*bu.* 2*p.* 4*qt.*
9. 145*bu.* 2*p.* 2*qt.*—115*bu.* 3*p.* 5*qt.*
10. 333*bu.* 0*p.* 1*qt.*—111*bu.* 2*p.* 2*qt.*
11. 1002*bu.* 3*p.* 4*qt.*—709*bu.* 3*p.* 5*qt.*
12. 1237*bu.* 0*p.* 1*qt.*—864*bu.* 2*p.* 2*qt.*
13. 1727*bu.* 2*p.* 0*qt.*—1000*bu.* 3*p.* 1*qt.*
14. 1687*bu.* 3*p.* 7*qt.* 0*pt.*—998*bu.* 3*p.* 7*qt.* 1*pt.*
15. 760*bu.* 0*p.* 0*qt.* 0*pt.*—631*bu.* 2*p.* 1*qt.* 1*pt.*
16. 2104*bu.* 3*p.* 6*qt.*—1305*bu.* 2*p.* 7*qt.*
17. 9*chal.* 30*bu.* 3*p.* 6*qt.*—4*chal.* 32*bu.* 2*p.* 7*qt.*
18. 14*chal.* 23*bu.* 2*p.* 1*qt.*—6*chal.* 23*bu.* 2*p.* 2*qt.*
19. 59*chal.* 1*bu.* 3*p.* 5*qt.* 0*pt.*—43*chal.* 18*bu.* 3*p.* 6*qt.* 1*pt.*
20. 88*chal.* 34*bu.* 2*p.* 7*qt.*—64*chal.* 35*bu.* 0*p.* 4*qt.*

§ 47. TIME.

EXAMPLES.

(1.)

yr.	*l.mo.*	*w.*	*d.*	*h.*	*min.*	*sec.*
71	11	0	6	18	49	51
42	9	3	2	22	51	54
29	1	1	3	19	57	57

(2.)

yr.	*l.mo.*	*w.*	*d.*	*h.*	*min.*	*sec.*
46	8	2	4	18	23	38
38	9	3	5	22	31	56

3. 10*yr.* 4*mo.* 3*w.* 3*d.*—6*yr.* 5*mo.* 2*w.* 4*d.*
4. 12*yr.* 1*mo.* 1*w.* 3*d.*—7*yr.* 2*mo.* 3*w.* 6*d.*
5. 14*yr.* 0*mo.* 2*w.* 4*d.*—8*yr.* 1*mo.* 1*w.* 5*d.*
6. 19*yr.* 6*mo.* 1*w.* 6*d.*—13*yr.* 8*mo.* 2*w.* 1*d.*
7. 11*mo.* 2*w.* 4*d.* 8*h.*—7*mo.* 3*w.* 3*d.* 12*h.*
8. 10*mo.* 0*w.* 0*d.* 15*h.*—5*mo.* 0*w.* 0*d.* 22*h.*
9. 4*mo.* 1*w.* 3*d.* 1*h.* 6*min.*—2*mo.* 2*w.* 4*d.* 2*h.* 3*min.*
10. 9*mo.* 3*w.* 2*d.* 18*h.* 16*min.*—4*mo.* 2*w.* 6*d.* 10*h.* 42*min.*
11. 7*mo.* 0*w.* 6*d.* 20*h.* 34*min.* 10*sec.*—3*mo.* 2*w.* 4*d.* 21*h.* 36*min.* 4*sec*
12. 4*yr.* 11*mo.* 2*w.* 4*d.* 7*h.* 21*min.* 6*sec.*—1*yr.* 11*mo.* 3*w.* 5*d.* 8*h.* 7*min.* 8*sec.*
13. 6*d.* 14*h.* 15*min.* 41*sec.*—2*d.* 18*h.* 16*min.* 32*sec.*
14. 3*w.* 2*d.* 23*h.* 51*min.* 52*sec.*—2*w.* 3*d.* 4*h.* 32*min.* 56*sec.*
15. 4*w.* 4*d.* 19*h.* 0*min.* 3*sec.*—1*w.* 5*d.* 20*h.* 1*min.* 4*sec.*

16. 101*yr.* 10*mo.* 18*h.*—87*yr.* 11*mo.* 22*h.*
17. 17*yr.* 0*mo.* 2*h.*—10*yr.* 1*mo.* 12*h.*
18. 21*yr.* 2*mo.* 2*w.* 4*d.* 6*h.* 4*min.*—9*yr.* 5*mo.* 3*w.* 5*d.* 7*h.* 5*min.*
19. 11*mo.* 3*w.* 6*d.* 15*h.* 13*min.* 17*sec.*—7*mo.* 3*w.* 6*d.* 18*h.* 11*min.* 50*sec.*
20. 5*d.* 0*h.* 16*min.* 1*sec.*—3*d.* 21*h.* 16*min.* 2*sec.*

§ 48. CIRCULAR MOTION.

EXAMPLES.

(1.)

s.	°	′	″
7	10	19	49
5	11	21	51
1	28	57	58

(2.)

s.	°	′	″
9	0	25	11
3	19	28	12
5	10	56	59

3. 23° 17′ 42″—18° 0′ 51″.
4. 27° 51′ 15″—14° 2′ 17″.
5. 16° 18′ 1″—11° 21′ 4″.
6. 18° 31′ 42″—9° 15′ 49″.
7. 15° 19′ 53″—5° 57′ 54″.
8. 25° 0′ 17″—13° 2′ 14″.
9. 9° 10′ 49″—6° 40′ 37″.
10. 12° 29′ 47″—5° 52′ 56″.
11. 29° 52′ 21″—18° 58′ 32″.
12. 3*s.* 2° 4′—2*s.* 8° 21′.
13. 8*s.* 11° 21′ 8″—4*s.* 20° 33′ 11″.
14. 11*s.* 21° 14′ 51″—8*s.* 25° 18′ 32″.
15. 22° 27′ 29″—18° 28′ 50″.
16. 19° 0′ 0″—4° 50′ 13″.
17. 15° 2′ 1″—11° 2′ 12″.
18. 29° 59′ 58″—12° 0′ 59″.
19. 10*s.* 0° 31′ 17″—6*s.* 29° 38′ 14.″
20. 9*s.* 17° 0′ 33″—2*s.* 21° 4′ 57″.

MISCELLANEOUS EXERCISES.

1. A man owed $14874.44, of which he paid $7983.87. how much does he still owe?
2. A owes B, $7543, and B takes goods of A to the amount of $4282.67: how much has A still to pay?
3. A man owns one house which is worth $11000, and another worth $17823.48: what is the difference of the values of the two houses?

4. A merchant bought a quantity of goods for $763.66, on which he paid $334.78: what must he still pay?

5. A ship sails from Amsterdam for Philadelphia with a cargo, valued at $38700; but storms and leaks damage the cargo so that it is only worth $27800: what is the amount of the loss?

6. A ship sails from Philadelphia with a cargo worth $25165.86, on arriving at her destined port the cargo is sold for $40000: what is the amount gained?

7. From 200*l.* take 99*l.* 19*s.* 11*d.* 3*f.*

8. Take 146*l.* 11*s.* 9*d.* 1*f.* from 300*l.*

9. Bought 48 yards of muslin, out of which I sold 25*yds.* 3*qr.* 2*n.*: what quantity have I left?

10. The great bell at Oxford weighs 7*tons* 11*cwt.* 3*qrs.* 4*lb.*, and that at St. Paul's 5*tons* 2*cwt.* 1*qr.* 22*lb.*: how much does the former exceed the latter in weight?

11. A youth has served 3 years 3 months 2 weeks and 5 days of his 5 years' apprenticeship: how much longer has he yet to serve?

12. Between the towns D. and C. there are two roads; the one measures 32 miles 4 furlongs 20 poles, and the other 19 miles 5 furlongs 17 poles: what is the difference?

13. John was born the 22d day of January, 1838; James the 10th day of May, 1844: what is the difference of their ages?

14. A bond was given the 18th day of February, 1835, and paid on the 10th day of June, 1844: for what time must interest be computed thereon?

15. Greenwich Observatory, near London, is 51° 28′ 39″ N., and the Philadelphia High School Observatory 39° 57′ 8″ N. latitude: what is the difference of latitude between the two places?

16. When a planet has moved through 8*s.* 8° 8′ 8″ of its orbit, how much is it short of a complete revolution?

17. From a pile of wood containing 36 cords, was sold 15 cords 108 cubic feet: what was the quantity of wood left?

18. Bought several vessels of cider, containing 7284*gal.* of which 3001*gal.* 3*qt.* 1*pt.* were sold: what quantity is remaining?

19. Bought a silver tankard, weighing 4*lb.* 2*oz.* 16*dwt.* 18*gr.*; the lid alone weighed 10*oz.* 14*dwt.* 23*gr.*: how much did the tankard weigh without the lid?

20. Subtract 2℔ 9ʒ 4ʒ 1℈ 12*gr.* from 7℔ 8ʒ 2ʒ 2℈ 17*gr.*; what is the remainder?

21. A farmer purchased 23*a.* 3*r.* 16*p.* of land, and sold as follows: to A 4*a.* 0*r.* 6*p.*; to B 11*a.* 3*r.* 18*p.*; how much had he remaining?

22. What time elapsed from the battle of Lexington, April 19th, 1775, to the death of Fisher Ames, July 4, 1808? to the death of John Adams and Thomas Jefferson, July 4, 1826?

23. On the 23d of September, 1779, Commodore John Paul Jones captured the British frigate Serapis. How many years, months, and days elapsed from that date to the 10th of September, 1813, when Commodore Perry gained the battle of Lake Erie?

24. What time elapsed from the surrender of General Hull, August 16, 1812, to General Jackson's victory at New Orleans, January 8, 1815?

25. What was the duration of the Revolutionary war, allowing it to have commenced at Lexington, April 19, 1775, and to have terminated in the treaty of peace between the United States and Great Britain, April 19, 1782?

26. How many years was it from the close of the Revolution, April 19, 1782, to the treaty of peace between the United States and Great Britain, signed at Ghent, December 24th, 1814?

27. A and B start from the same point on a journey. During the first day, A travels 33*mi.* 1*fur.* 29*p.*; B, 25*mi.* 6*fur.* 33*p.* How much further has A travelled than B?

28. The time occupied by the planet Vesta in making a revolution round the Sun is 3 years and 330 days; by Pallas, 4 years and 222 days. What is the difference in the times of revolution?

29. The diurnal rotation of the Earth, which produces day and night, is performed in 23 hours, 56 minutes and 4 seconds; that of the planet Jupiter, occupies 9 hours, 55 minutes and 26 seconds. How much more time does the Earth require for a daily revolution than Jupiter?

30. The height of a mountain on the surface of the Moon, to which the name of Newton has been given, is 4*mi.* 4*fur.* 4*p.* 2*yds.* 1*ft.*; that of another mountain, Blancanus, is 3*mi.* 0*fur.* 10*p.* 2*yds.* 2*ft.* What is the difference in the height of the two?

MULTIPLICATION OF FEDERAL MONEY.

§ **49.** RULE. Multiply as in common multiplication, and point off one figure to the right hand of the product for mills, the next two for cents; the remainder will be dollars. If there are no mills in the quantity multiplied, point off two places to the right; the remainder will be dollars.

EXAMPLES.

1. What will 306 yards of tape cost, at 17 mills per yard?
2. What will 72 yards of cloth cost, at $2.25 per yard?

(1.)	(2.)
306	2.25
17	72
2142	450
306	1575
$5,20,2	$162.00

3. What will 144 reams of paper cost, at $1.80 per ream?
4. What will 117*cwt.* of iron cost, at $2.67 per *cwt.*?
5. What will 198 bushels of wheat cost, at $1.60 per bushel?
6. Bought a piece of cloth, containing 48 yards, at $1.83 per yard: what does it all cost?
7. Multiply one dollar six mills by 378.
8. What will a chest of tea cost which weighs 98*lb.*, at 76 cents per *lb.*?
9. What does a labourer earn in a year, at sixty cents per day, working six days in each week?
10. If a person's income be 90 cents per day, how much will he receive in a year?
11. What will 15 tons of coal cost, at $3.85 per ton?
12. What will 9 hogsheads of molasses cost, at $33.25. per hogshead?
13. What will 192 feet of boards cost, at 3 cents and 5 mills per foot?
14. What will 17 boxes of lemons cost, at $2.25 per box?
15. What will 47 barrels of cider cost, at $3.82 per barrel?
16. What will 108 kegs of nails cost, at $4.75 per keg?

17. What will 25 barrels of flour cost, at $6.25 per barrel?

18. Sold 1844*lbs.* of tobacco, at 23 cents per pound: what is the value?

19. If 147 acres of land be let at $2.33 per acre, what is the yearly rent?

20. What is the value of 1784*lbs.* of sugar, at 8 cents per pound?

21. Bought 21 tons of hay, at $17.60 per ton: what was the whole cost?

22. What will 45*lbs.* of beef cost, at 7 cents per pound?

23. Bought 19 horses, for $65.50 each: what was the whole cost?

24. Bought 49 pairs of shoes, at $1.50 per pair: what was the whole cost?

25. What sum will pay for 138 hats, at $3.75 each?

26. If a bankrupt should pay 57 cents in the dollar, what sum will he pay to a debt of $4875?

27. Bought a gallon of oil for $2.18: what will 112 gallons come to at that rate?

28. Sold 27 cords of wood, at $4.75 per cord: what is the amount?

29. How much is the expense of painting a room which measures 219 yards, at 21 cents a yard?

30. What is the value of 20 boxes of candles, each containtaining 51*lbs.*, at 16 cents 5 mills per pound?

31. Find the cost of 15 boxes of white soap, each 70*lbs.*, at 8 cents and 2 mills per pound.

32. Required the capital of a trading company, consisting of 220 shares, each $166.25.

33. What is the value of 3648 bushels of potatoes, at 37 cents 5 mills per bushel?

34. What will 232*lbs.* of butter cost, at 17 cents per *lb.*?

35. If 1*cwt.* of cheese cost $7.33, what will 36*cwt.* cost?

36. What sum was divided among 12 brothers and sisters, so that each had but $983.25?

37. Bought 124 gallons of cider, at 32 cents per gallon: what did the whole cost me?

38. Suppose a person to gain by trade $392 per year on an average, for 25 years, what would be the amount?

39. Bought 1124 gallons of oil, at $1.25 per gallon: what did it cost?

40. Gave $37.40 per acre, for 211 acres of land: wha did the estate cost?

41. Bought 14 dozen pairs of men's hose, at 33 cents per pair: to what sum did they amount?

42. What will 978 yards of carpeting cost, at $2.25 per yard?

43. What will 36 lamps cost, at 67 cents each?

44. If 1 dozen brushes cost, $17.38, what will 72 dozen cost?

45. What will 119*lbs.* of wire cost, at 13 cents a pound?

46. What will 8 bags of coffee, each containing 78*lbs.*, cost, at 11 cents per pound?

47. If one yard of flannel cost 49 cents, what will be the value of 879 yards?

48. Bought 18 bags of salt, each containing 3 bushels, for 98 cents a bushel: what did it cost?

49. If one half-eagle is worth $5, what is the value of 75 half-eagles?

50. What will 87 boxes of block tin cost, at $8.75 per box?

51. What will 24650 bricks cost, at 5 mills each?

52. Bought 75 yards of linen, at 75 cents a yard: what does it amount to?

53. If one silk handkerchief cost $1.23, what will 7 dozen cost?

54. What will 148 yards of damask cost, at $1.11 per yard?

55. What will 121 ounces of plate come to, at $1.08 per ounce?

56. What is the value of 221 tuns of Port wine, at $112.18 per tun?

57. If one barrel of shad cost $11.25, what is the value of 64 barrels?

58. If one pair of scissors cost 13 cents, what will 860 gross cost?

59. If it cost $47.86 to clothe a sailor one year, how much will it cost to clothe a ship's crew, consisting of 627 men, for the same time?

Find the amount of the following bills:

Philadelphia, September 30, 1844.

Mr. John Snyder,

Bought of Thomas Smithson,

10 yards of broadcloth	at	$7.25 per yard . . .	$72.50
17 do. of velvet,	"	2.75 do.	46.75
		(*Carried over,*)	$119.25

		(*Brought over,*)		$119.25
8 yards of satin,	at	$1.50	per yard	12.00
56 do of linen,	"	.75	do.	42.00
12 do. of cambric muslin,		.56	do.	6.72
18 do. of flannel, . .		.40	do.	7.20
16 pairs of hose, . . .		.50	do.	8.00
28 yards of sheeting, . .		.16	do.	4.48
				$199.65

Received payment,

THOMAS SMITHSON.

Baltimore, October 10, 1844.

Mr. George Baker,

Bought of Peter Haines,

12 pairs of shoes, . .	at	$1.12½
4 pairs of boots, . .	at	3.10
42*lb.* of flour, . . .	at	.05
36*lb.* of ginger, . . .	at	.13
14 pairs of men's gloves,	at	1.06
18*lb.* of coffee, . . .	at	.11
16 pairs of stockings, .	at	.20
10*lb.* of pepper, . . .	at	.18

$54.50

Received payment,

PETER HAINES.

New York, October 1, 1844.

Mr James Lawson,

Bought of Henry Quay,

1 cheese, wt. 28*lb.* . .	at	$0.12½	*per lb.*
1 bag of coffee, 96*lb.* .	at	.13	"
12*lb.* of sugar, . . .	at	.13	"
10 gallons Holland gin, .	at	1.50	*per gal.*
10 do. French brandy,	at	2.00	"
1 box of candles, 42*lb.*	at	.22	*per lb.*
11*lb.* of black tea, . .	at	.35	"
18*lb.* of rice, ...	at	.04	"
2*lb.* pepper . . .	at	.12½	"

$66.60

Received payment,

HY. QUAY.

Boston, October 11, 1844.

Messrs. South & Jones,

Bought of William Hays,

3	pieces superfine cloth, each 25 yds.,	at	$7.00	*per yd.*	
5	do. do. do. do. 20 "	at	3.00	"	
10	pairs 3-point blankets, ...	at	4.00	*per pair.*	
7	pairs 8-4 do.	at	5.00	"	
4	pieces super welsh flannel, 160 yds.	at	.50	*per yd.*	
4	do. fine do. do. 140 "	at	.30	"	
5	do. do. do. do. 150 "	at	.20	"	
3	do. furniture chintz, . 90 "	at	.30	"	
3	do. mourning do. . . 90 "	at	.40	"	
5	do. fancy prints, . . . 150 "	at	.50	"	
4	do. cambric muslins, . 40 "	at	.50	"	
					$1210.00

Received payment,

Wm. Hays.

REDUCTION.

§ **50.** Reduction teaches to change quantities from one name or denomination to another, still retaining the same value.

It consists of two parts, descending and ascending.

REDUCTION DESCENDING.

§ **51.** Reduction Descending teaches to bring quantities from a higher denomination to a lower.

RULE.

1. Multiply the quantity in the highest denomination, by that number of the next lower which will make an integer, or one in that higher, and write down the product.

2. To this product add the number, if any, which was in the lower denomination before, and multiply the sum by the number of the next lower denomination, which makes an integer in the present one.

3. Proceed in the same manner through all the denominations to the lowest, and the sum last found will be the value of all the quantities which were in the higher denominations taken together.

Q. What is reduction?
Q. Of how many parts does it consist?
Q. Give the rule for reduction descending?

EXAMPLES.

1. Reduce 241*l.* 3*s.* 4*d.* 1*qr.* to farthings.
2. Reduce 27*lb.* 6*oz.* and 12*dwts.* to grains, troy weight.

(1.)

£.	s.	d.	qr.
241	3	4	1
20			
4823			
12			
57880			
4			
231521 *far.* Ans.			

(2)

lb.	oz.	dwt.
27	6	12
12		
330		
20		
6612		
24		
26448		
13224		
158688 *gr.* Answer.		

3. Reduce 47*hhd.* 16*gal.* 1*qt.* to quarts, wine measure.
4. Reduce 28*lb.* 11*oz.* 7*dr.* 2*scr.* to grains.
5. Reduce 1*cwt.* 3*qr.* 21*lb.* 14*oz.* to ounces.
6. Reduce 20*r.* 4*yd.* 2*ft.* 6*in.* to inches.
7. Reduce 1*yd.* 3*qr.* 2*n.* 1*in.* to inches.
8. Reduce 1*A.* 3*R.* 20*p.* 14*Sq. yd.* to square feet.
9. Reduce 56*c.* 20*S. ft.* to solid inches.
10. Reduce 1*t.* 3*hhd.* 40*gal.* 1*qt.* to quarts, wine measure.
11. Reduce 2*mo.* 3*wk.* 5*d.* 13*hr.* to hours.
12. Reduce 2*chal.* 1*bu.* 2*pk.* 7*qt.* to quarts, dry measure.
13. Reduce 7*M.* 112*A.* 3*R.* 2*p.* to perches.
14. Reduce 36*t.* 8*cwt.* 3*qr.* 15*lb.* to ounces avoirdupois.
15. Reduce 98*lb.* 9*oz.* 18*dwt.* to grains, troy weight.
16. How many grains of rhubarb are there in 18*lb.* 7*oz.* 3*dr.* 2*scr.* 16*gr.*?
17. How many grains of gold are there in a cup weighing 4*oz.* 2*dwt.*?
18. How many pounds are there in 75*t.* 12*cwt.* 3*qr.* 12*lb.*?
19. In 73*yd.* 3*qr.* 2*n.* how many nails?
20. How many barleycorns will reach 100 miles?
21. How many yards in 9*mi.* 2*fur.* 18*p.*?
22. How many perches in 368*a.* 1*r.* 26*p.*?
23. In 350*l.* 16*s.* 8¼*d.* how many farthings?

24. In 81 guineas, how many pence?
25. In a silver teapot, weighing 15*oz.* 10*dwt.*, how many grains?
26. In 2532 ells Flemish, how many quarters?
27. In 48*mi.* 4*fur.*, how many inches?
28. In 246 acres, how many perches?
29. In a pipe of Sherry wine, how many pints?
30. In 208 hogsheads of beer, how many pints?
31. How many seconds in 10 years 3 months 21 days?
32. In 11*s.* 14° 18′ 22″, how many seconds?
33. In 18 tons of round timber, how many inches?
34. In 36 chaldrons, how many quarts?
35. How many nails in 70 *E. E.* 2*qr.*?
36. In 456 cents, how many mills?
37. In 1878 dimes, how many mills?
38. In 41 dollars, how many cents?
39. How many mills in 12 eagles?
40. In 32 half-eagles, how many dollars?

REDUCTION ASCENDING.

§ **52.** Reduction Ascending teaches to bring quantities from a lower denomination to a higher.

RULE.

1. Divide the given quantity by as many of that denomination as make one of the next higher, and write down what remains.

2. Divide the quotient by as many of this as make one of the next higher, and write down what remains, as before.

3. Proceed in the same manner through all the denominations to the highest, and the quotient last found, together with the several remainders, if there be any, will be of the same value as the first quantity proposed.

EXAMPLES.

1. In 9184 pennyweights, how many pounds Troy?

20)9184
12)459 . . 4
38lbs. 3oz. 4dwt. Ans.

2. Reduce 100003 farthings to pounds.
3. How many pounds Troy in 6530 dwts.?

4. How many ounces and pounds in 3456 scruples?
5. How many yards in 11616 nails?
6. In 2000 nails, how many ells English?
7. In 95040 poles, how many leagues?
8. In 6272640 square inches, how many square yards?
9. In 63113858 seconds, how many days?
10. How many guineas in 37254 pounds?
11. How many pounds in 8870 guineas?
12. How many pounds avoirdupois are there in 98765 drachms?
13. In 19680 perches, how many acres?
14. In 21880 grains of rhubarb, how many pounds?
15. In 3798 quarters, how many Flemish ells?
16. How many boxes of raisins, each containing 24 pounds avoirdupois, can be filled out of 67584 ounces?
17. In 3072960 inches, how many miles?
18. In 6048 pints of wine, how many puncheons?
19. In 373248 pints of beer, how many butts?
20. In 10108 pints of wheat, how many bushels?
21. In 1296000 cubic inches, how many tons of hewn timber?
22. In 1920 cubic feet, how many cords?
23. In 10800 minutes, how many signs?
24. In 3058560 cubic inches of round timber, how many tons?
25. In 80640 gills, how many tuns of wine?
26. In 1712 gallons of wine, how many hogsheads?
27. Reduce 413280 minutes into weeks.
28. In 190080 barleycorns, how many miles?
29. In 7094 pecks, how many chaldrons?
30. How many cents in 7100 mills?
31. How many dimes in 8264500 mills?
32. How many dollars in 4700 cents?
33. How many eagles in 630000 mills?
34. In 17064 quarts of ale, how many hogsheads?
35. In 2160 pints of oats, how many bushels?
36. In 31556929 seconds, how many years?
37. In 36481 halfpence, how many pounds?

MISCELLANEOUS EXERCISES.

1. In 45768 pence, how many crowns, at 64 pence each?
2. In a piece of linen, measuring 21 English ells, how many shirts can be cut, of 3 yards 3 quarters each?

3. How many hours elapsed from the passage of the first bankrupt law of the United States, April, 4, 1800, to the expiration of the same, April 21, 1806?

4. In 9 pounds of silk, each 16 ounces, how many drachms?

5. How many packages of 10 ounces, are there in 56 pounds of bark?

6. In 27 bales, each of 14 pieces, and each piece 21 yards, 2 quarters, how many Flemish ells?

7. In 3 fields, measuring respectively, 3 acres, 2 roods, 14 perches; 7 acres, 3 roods, 27 perches; and 10 acres, 1 rood, 36 perches: how many square inches?

8. In a butt of beer, how many tankards of three pints each?

9. From March 8 to December 29, how many hours?

10. How many half-dollars would extend from the earth to the sun, which is 95 millions of miles distant, 5 half-dollars being equal to 6 inches?

11. How many minutes elapsed from the passage of the second bankrupt law of the United States, August 19, 1841, to its repeal, March 3, 1843?

COMPOUND MULTIPLICATION.

§ **53.** Compound Multiplication teaches to find the amount of any given number of different denominations repeated a certain proposed number of times.

GENERAL RULE.

1. Place the multiplier under the lowest denomination of the given quantity.

2. Multiply the number in the lowest denomination by the multiplier, and find how many integers of the next higher denomination are contained in the product, and write down what remains.

3. Carry the integers thus found to the product of the next higher denomination, with which proceed as before, and so on through all the denominations to the highest; and this product, together with the several remainders, taken as one number, will be the whole amount required.

The proof is the same as in simple multiplication.

§ 54. CASE 1. When the multiplier is less than 12.

RULE.

Begin by multiplying the lowest denomination, and carry from one denomination to another, as above directed.

EXAMPLES.

(1.)			(2.)				(3.)		
£	s.	d.	lb.	oz.	dwt.	gr.	yd.	qr.	n.
16	17	$4\frac{3}{4}$	7	11	19	23	49	3	3
		2				3			4
33	14	$9\frac{1}{2}$	23	11	19	21	199	3	0

4. £21 14*s.* $4\frac{1}{4}$*d.*×2.
5. £87 11*s.* 6*d.*×3.
6. 5*lb.* 7*oz.* 16*dwt.*×4.
7. 12*lb.* 5*oz.* 15*dwt.* 13*gr.*×5.
8. 2℔ 9℥ 6ʒ 2℈×2.
9. 4℔ 7℥ 4ʒ 1℈ 18*gr.*×3.
10. 12*T.* 15*cwt.* 3*qr.*×7.
11. 7*T.* 11*cwt.* 1*qr.* 12*lb.* 13*oz.* 7*dr.*×6.
12. 8*L.* 2*m.* 5*fur.* 17*p.* 3*yd.* 2*ft.* 3*in.*×3.
13. 12*L.* 0*m.* 6*fur.* 33*p.* 1*yd.* 1*ft.* 8*in.* 2*bc.*×2.
14. 17*A.* 2*R.* 14*p.* 11*ft.*×9.
15. 112*A.* 1*R.* 34*p.* 91*ft.* 116*in.*×2.
16. 41*T.* 32*ft.* 834*in.*×8.
17. 25*Cd.* 118*ft.* 671*in.*×6.
18. 11*yd.* 3*qr.* 2*n.*×10.
19. 6*E. E.* 1*qr.* 2*n.*×11.
20. 8*E. Fl.* 0*qr.* 1*n.*×12.
21. 12*T.* 1*hhd.* 7*gal.* 3*qt.*×7.
22. 6*P.* 1*hhd.* 8*gal.* 1*qt.* 1*pt.*×5.
23. 8*hhd.* 51*gal.* 2*qt.* 1*pt.*×9.
24. 23*T.* 1*bt.* 1*hhd.* 42*gal.* 1*qt.*×11.
25. 14*bu.* 2*p.* 7*qt.* 1*pt.*×10.
26. 19*chal.* 16*bu.* 3*p.* 6*qt.*×8.
27. 13*yr.* 11*mo.* 2*w.* 3*d.* 6*h.*×4.
28. 6*w.* 6*d.* 14*h.* 11*min.* 8*sec.*×12.
29. 2*s.* 27° 52′ 33″×5.
30. 41*lb.* 3*oz.* 19*dwt.*×9.
31. 1*T.* 3*cwt.* 2*qr.* 18*lb.* 15*oz.*×6.
32. 16*T.* 12*cwt.* 1*qr.* 14*lb.* 13*oz.*×7.
33. 2℔ 5℥ 5ʒ 1℈ 6*gr.*×4.

§ **55.** CASE 2. When the multiplier exceeds 12, and is a composite number.

RULE.

Multiply successively by its factors, instead of the whole number at once.

EXAMPLES.

1. Multiply £0 19*s.* 4*d.* by 14.

(1.)

£	*s.*	*d.*	
0	19	4	
			2×7=14
1	18	8	
		7	
13	10	8	

(2.)

lb.	*oz.*	*dwt.*	*gr.*	
5	8	10	11	
				3×5=15
17	1	11	9	
			5	
85	7	16	21	

3. 9*lb.* 7*oz.* 3*dr.* 2*scr.*×21.
4. 6*T.* 12*cwt.* 2*qr.* 17*lb.* 15*oz.* 12*dr.*×27.
5. 8*L.* 2*m.* 6*fur.* 12*p.* 0*yd.* 2*ft.* 4*in.* 1*bc.*×36.
6. 11*A.* 1*R.* 5*p.*×144.
7. 6*T.* 21*ft.* 687*in.*×132.
8. 17*Cd.* 91*ft.* 402*in.*×48.
9. 32*yd.* 2*qr.* 2*n.*×18.
10. 7*E. Fr.* 5*qr.* 2*n.*×72.
11. 18*T.* 0*hhd.* 59*gal.* 3*qt.*×84.
12. 6*P.* 1*hhd.* 8*gal.* 1*qt.* 1*pt.*×96.
13. 8*T.* 1*bt.* 1*hhd.* 53*gal.* 2*qt.* 1*pt.*×16.
14. 41*bu.* 1*p.* 5*qt.* 1*pt.*×64.
15. 7*chal.* 19*bu.* 2*p.* 1*qt.*×81.
16. 3*yr.* 3*mo.* 1*w.* 4*d.* 11*h.* 30*min.* 21*sec.*×108.
17. 8*yr.* 12*mo.* 2*w.* 0*d.* 6*h.*×121.
18. 3*s.* 0° 11′ 17″×28.
19. £37 0*s.* 7½*d.*×99.
20. 18*yd.* 3*qr.* 3*n.*×77.
21. 28*A.* 1*R.* 17*p.*×110.
22. 7*T.* 15*cwt.* 1*qr.* 12*lb.*×192.

§ **56.** CASE 3. When the multiplier is a prime number, and more than 12.

RULE.

1. Find the composite number nearest to it, either greater or less, and multiply by its factors, as before.

2. Then multiply the multiplicand by the difference between this number and the multiplier, and add or subtract the product from that before found, according as the given number is greater or less than the assumed one.

EXAMPLES.

1. Multiply £41 19*s.* 4*d.* by 47.

£	*s.*	*d.*	
41	19	4	4×2
			5×9+2=47
209	16	8	Product by 5
		9	
1888	10	0	" 9
83	18	8	" 2 to be added.
1972	8	8	" 47

Or thus:

41	19	4	4×1
			6×8—1=47
251	16	0	Product by 6
		8	
2014	8	0	" 48
41	19	4	" 1 to be subtracted.
1972	8	8	" 47

2. 8*s.* 1¾*d.*×13.
3. 3*s.* 11*d.*×59.
4. 12*s.* 2*d.*×47.
5. £1 9*s.* 3*d.*×89.
6. £18 16*s.* 8½*d.*×67.
7. 9*cwt.* 2*qr.* 14*lb.*×23.
8. 6*s.* 8*d.*×137.
9. £1 3*s.* 2¼*d.*×41.
10. £2 7*s.* 8½*d.*×73.
11. £7 14*s.*×29.

§ **57.** CASE 4. When the multiplier is expressed by three or more figures.

RULE.

1. Multiply by as many tens less one, as there are figures in the multiplier.

2. Multiply the last product by the left hand figure in the multiplier.

3. Then multiply the first line by the units, the first product by the tens, and the like for the 100's, &c. The products then added together will be the answer required.

EXAMPLES.

1. Multiply £2 1*s.* 4*d.* by 4842.

£	*s.*	*d.*	
2	1	4×2	
		10	
20	13	4×4	
		10	
206	13	4×8	
		10	
2066	13	4	
		4	
8266	13	4	product by 4000
1653	6	8	" " 800
82	13	4	" " 40
4	2	8	" " 2
10006	16	0	" " 4842

2. £1 1*s.* 8½*d.*×3475.
3. £173 6*s.* 8*d.*×2286.
4. 19*s.* 6½*d.*×4302.
5. 18*s.* 7¼*d.*×4880.
6. 9*s.* 3*d.*×359.
7. 9¾*d.*×129.

Q. How do you multiply when the multiplier is less than 12?

Q. How do you multiply when the multiplier exceeds 12, and is a composite number?

Q. How do you multiply when the multiplier is a prime number, and exceeds 12?

Q. How do you multiply when the multiplier consists of three or more figures?

MISCELLANEOUS EXERCISES.

1. What sum will pay for 12 hats, at 15*s.* 10*d.* each?
2. How many pounds sterling in 29 purses, each purse containing 12 guineas 10*s.*?
3. What will 50 loads of corn cost, at 8*l.* 12*s.* per load?
4. If a bankrupt should pay 13*s.* 7*d.* in the pound, what sum will he pay to a debt of 4572*l.*?
5. What is the value of 1476*oz.* 10*dwt.* of gold, at 3*l.* 15*s.* 4½*d.* per ounce?
6. How much cloth will it require to give 950 soldiers new coats, allowing to each 2 yards and 3 nails?

7. How much is the yearly income of a person who has 17*s.* 6*d.* each week?

8. A ship's company, consisting of 450 men, had each 1*l.* 1*s.* of prize money: how many pounds were distributed?

9. Bought 40 hogsheads of sugar, each containing 10*cwt.* 2*qr.* 12*lb.*: what was the weight of the whole?

10. If one spoon weigh 2*oz.* 18*dwt.* 20*gr.*, what is the weight of 60 spoons?

11. If a man travel 15*m.* 2*fur.* 20*p.* in one day, how far will he go in 25 days?

12. What does a labourer earn in a year, at 2*s.* 6*d.* per day, working six days in each week?

13. 793 Spaniards obtained from the Peruvians, on an average, 6742*l.* 12*s.* 10½*d.* each: what was the whole amount of which they gained possession?

14. What will 1*cwt.* of sugar cost, at 11½*d.* per pound?

15. Bought a gallon of wine for 13*s.* 9*d.*: what will a pipe (containing 126 gallons) come to at that rate?

16. Bought 21 tracts of land, each containing 111*A.* 2*R.* 10*p.*: what were the contents of the whole?

17. There are 12 sticks of round timber, each containing 42*ft.* 1002*in.*: what is the whole quantity?

18. There are 19 piles of wood, each containing 5 cords 127 feet: what is the whole quantity?

19. What is the product of 5℔ 3ℨ 6ℨ 1℈ by 27?

20. Multiply 2*s.* 29° 32′ by 96.

21. Find the amount of 4572 at 173*l.* 6*s.* 8*d.* each.

22. In the lunar circle of 19 years, of 365*d.* 5*h.* 48*min.* 48*sec.* each, how many days?

23. What is the value of 119 yards of cloth, at 2*l.* 4*s.* 3*d.* per yard?

24. What is the weight of 6 hogsheads of sugar, each weighing 8*cwt.* 1*qr.* 16*lb.*?

25. What is the weight of 12 chests of tea, each weighing 3*cwt.* 1*qr.* 12*lb.*?

26. What will 17 bushels 1 peck 4 quarts of wheat cost, at 7*s.* 6*d.* per bushel?

27. What does a painter charge for 1500 yards, who is paid 6¾*d.* per square yard?

28. Bought 1*cwt.* of tobacco for 16 guineas: required the worth of 64*cwt.*

29. Bought 2000 feet of mahogany boards, at 11*d.* per foot: what did they cost?

30. A merchant bought 8 hogsheads of whiskey, at $18.75 each; and 5 barrels of molasses, at $13.80 per barrel: what did he pay for the whole?

31. What will 17 tons of coal cost, at £1 8*s*. 3*d*. per ton?

32. A merchant intends to purchase fourteen pieces of silk, at $18.38 per piece: what will he give for the whole number of pieces?

33. A captain of a packet ship carries 218 passengers from Limerick to New York, receiving 35 dollars from each for the passage: what sum will he receive from them all?

34. If one hundred operatives in a factory receive $5.37 each, how much will they all receive?

35. Five partners in business receive upon a settlement, $13987.75 each: what is the whole amount shared between them?

36. Four schooners, each laden with 500 bushels of corn, are sent to a merchant: how much will he receive for their cargoes, at 43 cents per bushel?

37. If one soldier receive 9 dollars per month, how much must be paid to 838 soldiers for 11 months' service?

38. If one acre of land be worth $58.50, what will a farm sell for which contains 131 acres, 2 roods?

39. During the Revolutionary war, the patriotic ladies of Philadelphia purchased the material necessary for the manufacture of 2107 shirts, which were placed in the hands of General Washington for the use of the soldiers. Allowing 3 yards and 1 quarter to each shirt, what was the whole number of yards required?

DIVISION OF FEDERAL MONEY

§ **58.** RULE. Reduce the quantity to be divided to mills, and proceed to divide as in common division, separating the three right hand figures in the quotient. The left hand figures will then represent the dollars, the first two on the right the cents, and the last the mills.

EXAMPLES.

1. Bought a piece of cloth, containing 48 yards, for $414: how much is that per yard?

$ cts. m. $ cts. m.
48)414. 00. 0(8.62.5. Answer.
384
300
288
120
96
240
240

2. If a man earn $430.77.3 in 21 months, how much does he earn in one month?

3. If 420 pounds of butter cost $52.50, what is the cost of 1 pound?

4. If $135 is paid for 18 barrels of flour, how much is that per barrel?

5. Sold 138 bushels of wheat for $125: what was the price per bushel?

6. Paid $329 for 140 reams of paper: what did it cost per ream?

7. Paid $252 for 288 pounds of tea: what was the price of 1 pound?

8. Paid $97.40 for 54 pairs of boots: what was the price per pair?

9. Paid $16.40 for 205 pounds of beef: what was the price per pound?

10. Bought 48 pounds of sugar for $6: what is it per pound?

11. A tradesman gained $15200 in 16 years: what was the average gain per year?

12. $150 were paid for 25000 bricks: what was that per thousand?

13. A father divided an estate of $20000 equally among 9 children: what was the portion of each?

14. When 465 bushels of corn cost $209.25, what is the price per bushel?

15. Bought 19 hats for $80.75: what was the price of each?

16. Sold 144 bales of silk for $14097.60: what is that per bale?

17. Sold 120 chests of tea for $3464.40: what is that per chest?

18. Bought 108*cwt.* of tobacco for $1393.20: what was that per *cwt.*?

19. Sold 256 tuns Curacoa, for $6248: how much was received per tun?

20. If 132 gallons of brandy cost $250.14, what will 1 gallon cost?

21. At a public dinner, where 48 gentlemen dined, the bill amounted to $72: how much did each pay?

22. The capital of a commercial house amounts to $40000, and is divided into 120 shares: what does a single share amount to?

23. A gentleman purchased 40 dozen of Port wine, for $144: what did it cost him per bottle?

24. What is the value of one ounce of opium, if 19 lb Av. are sold for $140.

25. An estate of 630 acres 1 rood is let for $1543.50: what rent does each acre yield?

26. Bought 48*cwt.* of tobacco for $1081.44: what is the worth of 1*cwt.*?

27. What is sugar per pound, at $10.64 per *cwt.*?

28. If 21 tons of hay cost $283.50, what is the value of 1 ton?

29. A man divided $215.25 equally among 205 boys: how much did each receive?

30. If you buy cheese weighing 128 pounds for $10.24, what is the price per pound?

31. A merchant bought 111 barrels of flour for $610.50: how much did he pay per barrel?

32. Sold 625 bushels of oats for $137.50: how much is that per bushel?

33. If 126 ells of Holland cost $42, what will 1 ell cost?

34. Bought a pipe of wine for $346.50: what is the worth of 1 gallon?

35. Bought 230 bushels of salt, for $71.50: wha was it per bushel?

36. If my income be $1901.65 per year, how much is it per day?

37. What does a painter charge per square yard, who is paid $145 for 500 yards?

38. What is wool per pound, if 77 pounds cost $28.49?

39. What is wood per cord, if 97 cords cost $412.25?

40. Purchased 28 yards of broadcloth for $201.88: what is 1 yard worth?

41. What is soap per pound, if 112 pounds cost $9.52?

42. If 1 cask of raisins, weighing 234 pounds, cost $27.91, what is 1 pound worth?

43. Sold 1728 pounds of coffee, for $232.28: what is a single pound worth?

44. Laid out $50.40 for 672 yards of muslins: required the price paid per yard.

45. A merchant purchased 25 bales of cotton cloth, each bale containing 49 yards, for $196: how much did he pay per yard?

46. If 114 drums of figs cost $256.25, what will be the value of 1 drum?

47. Bought 416 pounds of sheet lead, for $25.79: how much is that per pound?

48. Purchased 617 pounds of coffee, for $64.78½: how much is that per pound?

49. If 115 boxes of oranges cost $244.37½, what will be the price of 1 box?

50. Bought a drove of 107 sheep, for $204.37: what did I pay for each?

51. A piece of silk, containing 25 yards, cost $45.75: what is the value of 1 yard?

52. Four pieces of cloth, each containing 23 yards, cost $333.50: what was the price of a yard?

53. If a prize of $10350 be equally divided among 23 persons, what will each receive?

54. Bought 238 tons of coal, for $928.20: what is the cost of 1 ton?

55. Paid $81.27, for 21 dozen pairs of men's gloves: how much is that per pair?

56. Bought 360 tuns of Madeira, for $16804.80: what is the value of one tun?

57. Bought 207 yards of velvet, for $126.27: what was the cost per yard?

58. Sold 241 tons of potash, for $2922.25: how much is that per ton?

59. If you pay $125.10, for 834 pounds of leather, how much is that per pound!

60. A person left property to the amount of $3942, and directed that $749 should be given to the poor, and that the remainder be equally divided among his 5 children: what is the portion of each child?

COMPOUND DIVISION.

§ **59.** This rule is the reverse of Compound Multiplication. It shows how often a given quantity is contained in another of different denominations.

GENERAL RULE.

Divide the first denomination on the left hand by the divisor; multiply the remainder, if any, by the number of the second denomination contained in a unit of the first; and add the second to the product; divide the sum as before, &c.

§ **60.** CASE 1. When the divisor is less than 12.

RULE.

Divide the dividend by said divisor, as above directed.

EXAMPLES.

1. Divide £2 10*s.* 2*d.* by 7.
2. Divide 19*D.* 7*h.* 14*min.* 6*sec.* by 9.

(1.)				(2.)				
£	*s.*	*d.*		*D.*	*h.*	*min.*	*sec.*	
7)2	10	2		9)19	7	11	6	
0	7	2	Ans.	2	3	27	54	Ans.

3. £46 8*s.* 6½*d.*÷2.
4. 9*lb.* 6*oz.* 10*dwt.* 20*gr.*÷4.
5. 10*lb.* 2*oz.* 0*dr.* 1*scr.* 8*gr.*÷3.
6. 16*T.* 18*cwt.* 2*qr.* 12*lb.* 12*oz.* 14*dr.*÷5.
7. 64*L.* 2*m.* 6*fur.* 36*p.* 2*yd.* 2*ft.* 10*in.* 2*bar.*÷6.
8. 80*A.* 2*R.* 28*p.* 0*ft.* 24*in.*÷8.
9. 99*T.* 36*ft.* 459*in.*÷9.
10. 28*Cd.* 112*ft.* 1449*in.*÷7.
11. 118*yd.* 3*qr.* 3*n.*÷11.
12. 28*E. E.* 2*qr.* 0*n.*÷4.
13. 21*hhd.* 54*gal.* 2*qt.*÷10.
14. 16*T.* 0*bt.* 1*hhd.* 52*gal.* 2*qt.*÷2.
15. 132*bu.* 1*p.* 4*qt.*÷12.
16. 36*chal.* 18*bu.* 2*p.* 2*qt.*÷6.
17. 22*mo.* 2*w.* 6*d.* 18*h.* 24*min.* 57*sec.*÷3.
18. 9*s.* 21° 51′ 45″÷9.

§ **61.** CASE 2. When the divisor is a composite number, and is greater than 12.

RULE.

Divide by the factors successively.

EXAMPLES.

1. Divide 241*cwt.* 2*qr.* 24*lb.* 0*oz.* by 192.

$4\times6\times8=192.$

	cwt.	*qr.*	*lb.*	*oz.*
8)	241	2	24	0
6)	30	0	24	0
4)	5	0	4	0
	1	1	1	0

2. Divide £65 19*s.* 4*d.* by 24.
3. £114 7*s.* 2*d.*÷35.
4. 530*cwt.* 3*qr.*÷28.
5. £67 4*s.*÷63.
6. 56*T.* 13*cwt.* 3*qr.* 10*lb.*÷45.
7. 78*lb.* 7*oz.* 7*dr.* 0*scr.* 0*gr.*÷72.
8. 1246*cwt.* 3*qr.* 27*lb.*÷132.
9. 882*cwt.* 3*qr.* 16*lb.*÷120.

§ **62.** CASE 3. When the divisor is a prime number greater than 12.

RULE.

Divide by the whole divisor at once as in Long Division.

EXAMPLES.

1. Divide £475 19*s.* 11*d.* by 59.

£	*s.*	*d.*
59)475	19	11(£8 1*s.* 4*d.* 1*qr.*
472		
3		
20		
79		
59		
20		
12		
251		
236		
15		
4		
60		
59		
1 remainder.		

2. 571*yd.* 2*qr.* 1*n.*÷47.
3. 34*A.* 1*R.* 15*p.*÷17.
4. 125*m.* 0*fur.* 29*p.* 0*yd.* 2*ft.* 4*in.*÷13.
5. 80*m.* 1*w.* 4*d.* 11*h.* 33*min.* 20*sec.*÷37.
6. 2764*cwt.* 2*qr.* 21*lb.*÷571.
7. 7142*cwt.* 0*qr.* 8*lb.*÷223.

§ 63. CASE 4. When the divisor is a compound number.

RULE.

Reduce the dividend and the divisor to the lowest denomination named in either, and divide one by the other, as in simple numbers.

EXAMPLES.

1. How much cloth, at £1 6*s.* 8*d.* per yard, can I buy for £41 6*s.* 8*d.*?

(1.)

	£41 6*s.* 8*d.*	by £1 6*s.* 8*d.*
	20	20
	826	26
	12	12
8×8×5=320.	5)9920	320
	8)1984	
	8)248	
	31 Answer.	

2. Divide 103*yd.* 2*qr.* by 4*yd.* 1*qr.* 1*n.*
3. How much tea, at 5*s.* 6*d.* per pound, can be bought for £35 4*s.*?
4. Divide 5280*ft.* by 2*ft.* 6*in.*
5. A piece of cloth, at 12*s.* 9*d.* per yard, cost £47 16*s.* 3*d.*: how many yards did it contain?

Q. What is the rule when the divisor is less than 12?
Q. What is the rule when the divisor is a composite number, and greater than 12?
Q. How do you proceed when the divisor is a prime number greater than 12?
Q. How do you proceed when the divisor is a compound number?

MISCELLANEOUS EXERCISES.

1. If 48*cwt.* of cheese cost 106*l.* 14*s.*, what will 1*cwt.* cost?
2. Divide 11*l.* 11*s.* 11¼*d.* by 12.

3. Gave 216*l.* for 48 pieces of cloth: what is the value per piece?

4. If 50 reams of paper cost 25*l.*, what is the cost per sheet, 480 to the ream?

5. How many chests will be required to hold 41*cwt.* 2*qr.* 14*lb.* of tea, each to contain 1*cwt.* 14*lb.*?

6. A person bought 24 silver spoons, weighing 6*lb.* 5*oz.* 7*dwt.*: what was the weight of each spoon?

7. What is the value of 1 yard of cloth, when 84 yards of the same cost 36*l.* 13*s.* 3*d.*?

8. The planet Mercury revolves round the sun in 88 days: how many revolutions will he perform in 17 years and 219 days?

9. How many gallons of train oil, each gallon weighing 7½*lbs.*, are there in 14*cwt.* 2*qr.*?

10. How many guineas, each weighing 5*dwt.* 9*gr.*, may be coined from a lump of gold weighing 5*lb.* 4*oz.* 10*dwt.*?

11. How many reapers will cut down a field of wheat, containing 12*A.* 2*R.* 24*p.*, allowing each to reap 2*R.* 12*p.* a day?

12. In what time will the interest of a sum, which yields 91*l.* 15*s.* yearly, amount to 367*l.*?

13. Bought at public sale a quantity of cloth for 84*l.* 7*s.* 6*d.*, at the price of 11*s.* 3*d.* per yard: required the number of pieces, each containing 25 yards.

14. What is the value of a grain of gold, when a pound is worth 46*l.* 14*s.* 6*d.*?

15. In walking 16 miles, how many times does a walking stick touch the ground, supposing it do so at every third step, and each step to be 2 feet 8 inches?

16. A merchant bought a pipe of brandy, containing 105 gallons, which cost him 160*l.* 4*s.*; he has drawn off 16 gallons for his own use: at how much should he sell the remainder per gallon to cover the cost?

17. A merchant bought 74 hogsheads of tobacco, weighing 31 *T.* 4*cwt.* 1*qr.* 14*lb.*: what was the weight of each hogshead?

18. If 112 ingots of gold are worth 77878*l.* 5*s.* 4*d.*, what is the value of each?

19. Divide 10416*l.* 13*s.* 4*d.* among one million persons: what will each receive?

20. What sum added to the 20th part of 2590*l.* will make it 600*l.*?

FRACTIONS.

§ **64.** Fractions are expressions for any assignable part or parts of a unit, and are represented by two numbers, the one placed above the other, with a line drawn between. The number above the line is called the numerator, and that below the line the denominator. The denominator shows into how many parts the unit is divided, and the numerator shows how many of those parts are expressed by the fraction.

Fractions are either proper, improper, simple, compound, mixed, or complex.

Q. What are fractions?

A. Fractions are parts of a unit.

Q. How are they represented?

A. By two numbers placed one above the other, with a line drawn between them.

Q. What is the number above the line called?

A. The numerator.

Q. What is the number below the line called?

A. The denominator.

Q. What do the denominator and numerator point out?

A. The denominator expresses the number of parts into which the unit is divided, and the numerator points out the number of parts used. Thus, $\frac{3}{4}$ denotes that the unit is divided into 4 equal parts, and that 3 of these are used.

Q. What are the terms of a fraction?

A. The denominator and numerator of a fraction, are called its terms.

Q. What is a proper fraction?

A. A proper fraction is one whose numerator is less than the denominator, or whose value is less than unity, as $\frac{1}{5}$, $\frac{6}{7}$, $\frac{7}{8}$, &c.

Q. What is an improper fraction?

A. An improper fraction has its numerator equal to or greater than its denominator, and is in value equal to or greater than a unit; as, $\frac{5}{5}$, $\frac{8}{7}$.

Q. What is a simple fraction?

A. A simple fraction has but one numerator, and one denominator, and may be either proper or improper; as, $\frac{5}{7}$, or $\frac{8}{7}$

Q. What is a compound fraction?

A. A compound fraction consists of two or more simple fractions, and is a fraction of a fraction; as, $\frac{3}{5}$ of $\frac{4}{5}$, or $\frac{1}{2}$ of $\frac{4}{7}$ of $\frac{8}{9}$.

Q. What is a mixed number?

A. A mixed number is composed of a whole number and a fraction; as, $7\frac{1}{3}$, $29\frac{11}{17}$, &c.

Q. What is a complex fraction?

A. It is a fraction that has a fraction for its numerator, or for its denominator, or for both; as, $\frac{\frac{1}{3}}{8}$, $\frac{9}{\frac{2}{7}}$, $\frac{\frac{3}{5}}{\frac{1}{2}}$, $\frac{6\frac{1}{2}}{14}$, &c.

Q. When are fractions equal to unity?

A. All fractions whose numerators and denominators are the same; as, $\frac{3}{3}$, $\frac{6}{6}$, $\frac{241}{241}$, &c., are equal to unity, and also to each other.

Q. When are fractions greater than unity?

A. When their numerators are greater than their denominators; as, $\frac{4}{3}$, $\frac{13}{8}$, &c.

Q. Is the value of a fraction altered, if both terms be multiplied by the same number, or divided by a number which measures both?

A. Its value is not altered in either case; because multiplying both terms of $\frac{2}{6}$ by 2, gives $\frac{4}{12}$, the terms of which, when divided by 4, produce $\frac{1}{3}$, which is equal to $\frac{2}{6}$, or $\frac{4}{12}$.

Q. How is a whole number expressed in the form of a fraction?

A. A whole number may be expressed in the form of a fraction, by placing 1 for its denominator; as, $\frac{6}{1}$.

Q. What is meant by the common measure of two or more numbers?

A. The common measure of two or more numbers is that number which will divide each of them without a remainder. Thus, 4 is the common measure of 12, 16, and 20; and the greatest number that will do this, is called the greatest common measure.

Q. What is meant by the common multiple of two or more numbers?

A. A number which can be measured by two or more numbers is called their common multiple; and if it be the least number that can be so measured, is called the least common multiple: thus, 40, 60, 80, 100, are multiples of 4 and 5; but their least common multiple is 20.

Q. When is a fraction said to be inverted?

A. When the denominator and numerator change places.

Q. When is a fraction said to be in its lowest terms?

A. When its terms are prime to each other; that is, when no number but a unit will measure them both.

Q. Upon what does the value of a fraction depend?

A. The value of a fraction depends on the proportion which the numerator bears to the denominator.

Q. What is a perfect number?

A. A number which is equal to the sum of all its aliquot parts. The first perfect number is 6, the aliquot parts of which are 3, 2, 1=6; the second is 28, whose aliquot parts are 14, 7, 4, 2, 1=28. Also 496, 8128, 33550336, &c.

Q. What is the reciprocal of a number?

A. That number inverted, or unity divided by it. Thus, the reciprocal of $\frac{3}{8}$ is $\frac{8}{3}$, and the reciprocal of 7 is $\frac{1}{7}$.

MENTAL EXERCISES.

1. 5 is $\frac{1}{4}$ of what number? 6 is $\frac{1}{8}$ of what number? 4 is $\frac{1}{7}$ of what number? Of what number is 9 the $\frac{1}{10}$ part? 15 is $\frac{3}{14}$ of what number? What is $\frac{1}{5}$ of 35? $\frac{1}{6}$ of 30? $\frac{3}{5}$ of 45? $\frac{1}{8}$ of 48? $\frac{1}{6}$ of 42? $\frac{3}{8}$ of 56? $\frac{4}{7}$ of 63? $\frac{1}{8}$ of 72? $\frac{4}{9}$ of 27? $\frac{4}{6}$ of 54? $\frac{2}{5}$ of 45? $\frac{1}{7}$ of 49? $\frac{5}{6}$ of 66? $\frac{1}{6}$ of 40? $\frac{1}{8}$ of 30? $\frac{1}{9}$ of 48? $\frac{1}{12}$ of 62? $\frac{1}{7}$ of 45? $\frac{1}{2}$ of $\frac{1}{3}$ of 12? $\frac{1}{3}$ of $\frac{1}{4}$ of 24? $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of 72?

§ **65.** CASE 1. To find the greatest common measure of two or more numbers.

RULE.

1. Divide the greater number by the less, and this divisor by the remainder, and so on, till nothing remains; then the last divisor is the greatest common measure.

2. When there are more than two numbers, find the greatest common measure of two of them, as before, and then of that common measure and one of the other numbers; and so on through all the numbers to the last.

3. If 1 is found to be the common measure, the numbers are prime to each other, and are incommensurable.

ELUCIDATION. The reason of this rule may be shown from the first example. For since 27 measures 54 it also measures 54+27, or 81. Again, since 27 measures 54 and 81, it also measures (5×81)+54 or 459. In the same manner it will

measure $(2\times459)+81$ or 999, and so on. Therefore 27 measures both 459 and 999.

It is also the greatest common measure; for suppose there be a greater, then since the greater measures 459 and 999, it also measures the remainder 81; and since it measures 81 and 459, it also measures the remainder 54; in the same manner it will be found to measure the remainder 27; that is, the greater measures the less, which is absurd; therefore 27 is the greatest common measure.

EXAMPLES.

1. Required the greatest common measure of 459, 999 and 261.

459)999(2
918
81)459(5
405
54)81(1
54
27)54(2
54

So 27 is the greatest common measure of 459 and 999.

Hence 27)261(9
243
18)27(1
18
9)18(2
18

Therefore 9 is the greatest common measure.

2. What is the greatest common measure of 612 and 540?
3. What is the greatest common measure of 918 and 1998?
4. What is the greatest common measure of 202 and 182?
5. What is the greatest common measure of 1825 and 2555?
6. What is the greatest common measure of 42 and 534?
7. What is the greatest common measure of 375 and 408?
8. What is the greatest common measure of 296 and 407?
9. What is the greatest common measure of 720, 336 and 1736?
10. What is the greatest common measure of 278, 336, and 420?

Q. How do you find the greatest common measure of two or more numbers?

§ **66.** CASE 2. To find the least common multiple of two or more numbers.

1. When the least common multiple of two numbers only is to be found, find the greatest common measure of both; divide one of them by it, and multiply the quotient by the other. The product will be the least common multiple of the two given numbers.

2. If there are more than two numbers given, first find the least common multiple of two of them, then of that multiple and a third, and so on through all the numbers.

3. Or, divide by any prime number that will measure two or more of the given numbers, and write the quotients and undivided numbers in a line beneath. Divide this line as before, and so on, until there are no two numbers that can be divided, when the continued product of the divisors and last found quotients will give the multiple required.

EXAMPLES.

1. Find the least common multiple of 12, 16 and 24.

4 is the greatest common measure of 12 and 16.

$12 \div 4 = 3$, and $3 \times 16 = 48$.

24 is the greatest common measure of 24 and 48.

$24 \div 24 = 1$, and $1 \times 48 = 48$.

Or,

3)	12.	16.	24
2)	4	16	8
2)	2	8	4
2)	1	4	2
	1	2	1

then $3 \times 2 \times 2 \times 2 \times 2 = 48$.

therefore 48 is the least common multiple of 12, 16 and 24.

2. What is the least common multiple of 3, 5, 8 and 10?

3. What is the least number that 3, 4, 8 and 12 will measure?

4. What is the least common multiple of 3, 5, 7, 12, and 15?

5. What is the least common multiple of 7, 13, 39 and 84?

6. What is the least common multiple of 5, 7, 8 and 10?

7. What is the least common multiple of 15, 18, and 35?

8. What is the least common multiple of 10, 18, 24 and 25?

9. What is the least common multiple of 3, 4, 5, 9, 10, and 12?

10. What is the least number that can be divided by the nine digits, without a remainder?

Q. How do you find the least common multiple of two or more numbers?

REDUCTION OF FRACTIONS.

§ 67. CASE 1. To reduce fractions to their lowest terms.

RULE.

Divide both terms by any number that will divide them without a remainder, and these quotients again in the same manner; and so on, till it appears that there is no number greater than one which will divide them, when the fraction will be in its lowest terms. Or,

Divide both terms by their greatest common measure.

ELUCIDATION. That dividing the numerator and denominator by any number that will divide both without a remainder will give a fraction equal to the former, is clear. And if these divisions are performed as often as can be done, or the common divisor be the greatest possible, the terms of the resulting fraction must be the least possible.

Any number ending with an even number, or a cipher, is divisible by 2; as, 44, 82, 250.

Any number ending with 5, or 0, is divisible by 5; as, 25, 245, 370.

If the right hand place of any number be 0, the whole is divisible by 10.

If the two right hand figures of any number are divisible by 4, the whole is divisible by 4.

If the three right hand figures of any number are divisible by 8, the whole is divisible by 8.

If the sum of the digits constituting any number be divisible by 3 or 9, the whole is divisible by 3 or 9.

If the right hand digit be even, and the sum of all the digits be divisible by 6, the whole is divisible by 6.

A number is divisible by 11, when the sum of the 1st, 3d, 5th, &c. digits is equal to the sum of the 2d, 4th, and 6th.

All prime numbers, except 2 and 5, have 1, 3, 7, or 9, in the place of units; and all other numbers are composite.

EXAMPLES.

1. Reduce $\frac{288}{480}$ to its lowest terms.

$8)\frac{288}{480}=6)\frac{36}{60}=2)\frac{6}{10}=\frac{3}{5}$, the answer.

2. Reduce $\frac{48}{272}$ to its lowest terms.

3. Reduce $\frac{192}{576}$ to its lowest terms.

4. Reduce $\frac{825}{960}$ to its lowest terms.
5. Reduce $\frac{252}{364}$ to its lowest terms.
6. Reduce $\frac{1344}{1536}$ to its lowest terms.
7. Reduce $\frac{5194}{6012}$ to its lowest terms.
8. Reduce $\frac{57}{456}$ to its lowest terms.
9. Reduce $\frac{384}{1152}$ to its lowest terms.
10. Reduce $\frac{784}{952}$ to its lowest terms.
11. Reduce $\frac{1800}{4320}$ to its lowest terms.
12. Reduce $\frac{356}{1000}$ to its lowest terms.
13. Reduce $\frac{270}{300}$ to its lowest terms.
14. Reduce $\frac{468}{1184}$ to its lowest terms.
15. Reduce $\frac{344}{2236}$ to its lowest terms.
16. Reduce $\frac{3480}{5600}$ to its lowest terms.
17. Reduce $\frac{6689680}{36700016}$ to its lowest terms.

Q. How do you reduce fractions to their lowest terms?

§ **68.** CASE 2. To reduce a mixed number to an improper fraction.

RULE.

Multiply the whole number by the denominator of the fraction, and add the numerator to the product; and place their sum over the denominator of the fraction.

ELUCIDATION. All fractions represent a division of the numerator by the denominator, and are taken altogether as proper and adequate expressions for the quotient. Thus, the quotient of 3, divided by 7, is $\frac{3}{7}$; from whence the rule is evident, for if any number be multiplied and divided by the same, it is manifest the quotient must be the same as the number first proposed.

EXAMPLES.

1. Reduce $54\frac{4}{9}$ to an improper fraction.

54
9
486
4
490
9

Or: $(54\times9)+4=\frac{490}{9}$ Answer.

2. Reduce $84\frac{2}{3}$ to an improper fraction.
3. Reduce $96\frac{11}{13}$ to an improper fraction.

4. Reduce $27\frac{9}{27}$ to an improper fraction.
5. Reduce $100\frac{5}{18}$ to an improper fraction.
6. Reduce $346\frac{7}{120}$ to an improper fraction.
7. Reduce $183\frac{5}{21}$ to an improper fraction.
8. Reduce $514\frac{5}{16}$ to an improper fraction.
9. Reduce $100\frac{19}{59}$ to an improper fraction.
10. Reduce $127\frac{4}{17}$ to an improper fraction.
11. Reduce $653\frac{3}{19}$ to an improper fraction.
12. Reduce $473\frac{147}{400}$ to an improper fraction.

Q. How is a mixed number reduced to an improper fraction?

§ **69.** CASE 3. To reduce an improper fraction to its equivalent whole or mixed number.

RULE.

Divide the numerator by the denominator, and the quotient will be the whole or mixed number required.

EXAMPLES.

1. Reduce $\frac{379}{14}$ to its mixed number.

$$14)379(27\tfrac{1}{14}$$
$$\underline{28}$$
$$99$$
$$\underline{98}$$
$$1$$

2. Reduce $\frac{64}{8}$ to its equivalent whole or mixed number.
3. Reduce $\frac{1245}{27}$ to its proper terms.
4. Reduce $\frac{3843}{21}$ to its proper terms.
5. Reduce $\frac{11712}{11}$ to its proper terms.
6. Reduce $\frac{10117}{12}$ to its proper terms.
7. Reduce $\frac{1259}{13}$ to its proper terms.
8. Reduce $\frac{1816}{18}$ to its proper terms.
9. Reduce $\frac{41527}{120}$ to its proper terms.
10. Reduce $2\frac{693}{17}$ to its equivalent whole or mixed number.
11. Reduce $\frac{12412}{10}$ to its proper terms.
12. Reduce $\frac{6216133}{514}$ to its proper terms.

Q. How do you reduce an improper fraction to its equivalent whole or mixed number?

§ 70. CASE 4. To reduce a whole number to an equivalent fraction having a given denominator.

RULE.

Multiply the whole number by the given denominator, and place the product over that denominator, to form the fraction required.

EXAMPLES.

1. Reduce 8 to a fraction whose denominator shall be 11.

$8 \times 11 = 88$, and $\frac{88}{11}$ Ans.

2. Reduce 7 to a fraction whose denominator shall be 9.
3. Reduce 13 to a fraction whose denominator shall be 12.
4. Reduce 19 to a fraction whose denominator shall be 14.
5. Reduce 100 to a fraction whose denominator shall be 90.

§ 71. CASE 5. To reduce compound to simple fractions.

RULE.

Multiply all the numerators together for a new numerator, and all the denominators for a new denominator, and they will form the simple fraction required; which should then be reduced to its lowest terms. If part of the compound fraction be a whole or mixed number, it must be reduced to an improper fraction. And when any two terms of the fraction can be divided by the same number, the quotients may be used instead of them; or, cancel such factors as are common to both numerators and denominators.

EXAMPLES.

1. Reduce $\frac{4}{5}$ of $\frac{8}{9}$ of $\frac{10}{12}$ of $\frac{25}{32}$ to a simple fraction.

$$\frac{4}{5} \times \frac{8}{9} \times \frac{10}{12} \times \frac{25}{32} = \frac{8000}{17280} = \frac{25}{54} \text{ Ans.}$$

Or thus: $\frac{\cancel{4}}{\cancel{5}} \times \frac{\cancel{8}}{9} \times \frac{\cancel{10}^{5}}{\cancel{12}_{3}} \times \frac{25}{\cancel{32}_{4}} = \frac{50}{108} = \frac{25}{54}$ as before.

2. Reduce $\frac{5}{7}$ of $\frac{11}{12}$ of $\frac{7}{11}$ of $\frac{36}{40}$ to a simple fraction.

$$\frac{\cancel{5}}{\cancel{7}} \times \frac{\cancel{11}}{\cancel{12}} \times \frac{\cancel{7}}{\cancel{11}} \times \frac{\cancel{36}^{3}}{\cancel{40}_{8}} = \frac{3}{8}$$

3. Reduce $\frac{3}{7}$ of $\frac{5}{8}$ of $\frac{6}{10}$ to a simple fraction.
4. Reduce $\frac{3}{11}$ of $\frac{6}{8}$ of $\frac{1}{2}$ of $\frac{3}{4}$ to a simple fraction.

5. Reduce $\frac{3}{4}$ of $\frac{4}{6}$ of $\frac{8}{12}$ of $\frac{24}{36}$ to a simple fraction.
6. Reduce $\frac{2}{6}$ of $\frac{6}{11}$ of $\frac{8}{9}$ of $\frac{9}{12}$ to a simple fraction.
7. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of $1\frac{3}{4}$ to a simple fraction.
8. Reduce $\frac{11}{12}$ of $\frac{7}{13}$ of $\frac{8}{19}$ of 10 to a simple or improper fraction.
9. Reduce $\frac{3}{12}$ of $\frac{13}{15}$ of $\frac{8}{17}$ of 20 to a simple or improper fraction.
10. Reduce $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{3}{4}$ of $12\frac{1}{2}$ to a simple fraction.
11. Reduce $\frac{2}{4}$ of $2\frac{3}{4}$ of 8 of $\frac{10}{4}$ of $6\frac{2}{3}$ to a simple fraction.
12. Reduce $\frac{5}{6}$ of $\frac{8}{5}$ of $3\frac{4}{5}$ of 9 to a simple fraction.
13. Reduce $\frac{9}{10}$ of 7 of $5\frac{3}{5}$ of 12 to a simple fraction.

Q. How do you reduce a compound fraction to a simple one?

Q. When the compound fraction is a whole or mixed number, how do you proceed?

Q. How do you abbreviate the operation?

§ **72.** CASE 6. To reduce fractions to a common denominator.

RULE 1.

Multiply each numerator by all the denominators but its own, for a new numerator; and all the denominators together for a common denominator.

EXAMPLES.

1. Reduce $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{5}{9}$ to a common denominator

$3\times8\times9=216$ the new numerator for $\frac{3}{4}$.
$7\times4\times9=252$ " " " $\frac{7}{8}$.
$5\times4\times8=160$ " " " $\frac{5}{9}$.
$4\times8\times9=288$ the common denominator.

Therefore the fractions are $\frac{216}{288}$, $\frac{252}{288}$, and $\frac{160}{288}$.

Reduce to common denominators the following fractions:

2. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{8}$.
3. $\frac{1}{8}$, $\frac{3}{4}$ of $\frac{4}{5}$, $5\frac{1}{2}$, and $\frac{9}{10}$.
4. $\frac{3}{5}$, $\frac{1}{4}$, $\frac{2}{3}$, and 4.
5. $\frac{3}{5}$, $\frac{2}{6}$, $\frac{5}{7}$, and 5.
6. $\frac{2}{9}$, $\frac{1}{5}$, 7, $\frac{6}{8}$, and 3.
7. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{7}{8}$.
8. $\frac{1}{2}$, $\frac{2}{3}$ of $\frac{5}{8}$, $7\frac{3}{4}$, and $\frac{3}{13}$.
9. $\frac{11}{15}$, $\frac{2}{4}$ of $1\frac{1}{4}$, $\frac{9}{11}$, and $\frac{5}{7}$.
10. $\frac{11}{13}$, $\frac{3}{4}$ of $1\frac{1}{2}$, $\frac{7}{12}$, and $\frac{5}{8}$.
11. $\frac{7}{11}$, $\frac{13}{49}$ of $\frac{49}{17}$, and $\frac{41}{43}$.

§ **73.** RULE 2. To reduce any given fractions to others, which shall have the least common denominator.

1. Find the least common multiple of all the denominators, for the common denominator.

2. Divide the common denominator by the denominator of each fraction, and multiply the quotient by the numerator, and the products will be the numerators required.

EXAMPLES.

1. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$, to fractions having the least common denominator.

$$\begin{array}{rcccc} 2) & 2 & 3 & 4 & 6 \\ 3) & 1 & 3 & 2 & 3 \\ & 1 & 1 & 2 & 1 \end{array}$$

$2\times3\times1\times1\times2\times1=12$, least common denominator; $(12\div2)\times1=6$, the first numerator; $(12\div3)\times2=8$, the second numerator; $(12\div4)\times3=9$, the third numerator; $(12\div6)\times5=10$, the fourth numerator; whence the fractions are $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$, and $\frac{10}{12}$.

2. Reduce $\frac{7}{12}$, and $\frac{11}{18}$, to fractions having the least common denominator.

3. Reduce $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{9}$, and $\frac{7}{10}$, to fractions having the least common denominator.

4. Reduce $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{7}{8}$, to fractions having the least common denominator.

5. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$, to fractions having the least common denominator.

6. Reduce $\frac{1}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{16}$, and $\frac{17}{24}$, to fractions having the least common denominator.

Q. How do you reduce fractions to a common denominator?

Q. How do you reduce fractions to others which shall have the least common denominator?

§ **74.** CASE 7. To reduce a fraction of one denomination to that of another which shall have the same value.

RULE.

Find how many of the less denomination make one of the greater; and multiply the *numerator* by that number, if the reduction be to a less denomination, or the *denominator*, if to a greater.

EXAMPLES.

1 Reduce $\frac{7}{8}$ of a farthing to the fraction of a pound.

$\frac{7}{8}\times\frac{1}{4}\times\frac{1}{12}\times\frac{1}{20}=\frac{7}{7680}$ Answer.
And, $\frac{7}{7680}\times\frac{4}{1}\times\frac{12}{1}\times\frac{20}{1}=\frac{6720}{7680}=\frac{7}{8}$, the proof.

2. Reduce $\frac{1}{7}$ of a pound to the fraction of a penny.

$\frac{1}{7}\times\frac{20}{1}\times\frac{12}{1}=\frac{240}{7}$ Answer.
And, $\frac{240}{7}\times\frac{1}{20}\times\frac{1}{12}=\frac{240}{1680}=\frac{1}{7}$, the proof.

3. Reduce $\frac{7}{8}$ of a pound to the fraction of a *cwt.*
4. Reduce $\frac{2}{3}$ of a grain to the fraction of a *lb.* troy.
5. Reduce $\frac{1}{2}$ of a pint of wine to the fraction of a *hhd.*
6. Reduce $\frac{3}{4}$ of a yard to the fraction of a nail.
7. Reduce $\frac{2}{4}$ of a second to the fraction of a week.
8. Reduce $\frac{11}{36}$ of a pound to the fraction of a penny.
9. Reduce $\frac{2}{13}$ of a month to the fraction of a day.
10. Reduce $\frac{3}{7}$ of a pound to the fraction of a guinea.
11. Reduce $\frac{9}{6552}$ of a hogshead of wine to the fraction of a pint.
12. Reduce $\frac{1}{36}$ of a yard to the fraction of a nail.
13. Reduce $4\frac{1}{2}$ furlongs to the fraction of a mile.
14. Reduce $\frac{7}{1920}$ of a pound troy to the fraction of a pennyweight.
15. Reduce $\frac{3}{392}$ of an cwt. to the fraction of a pound.
16. Reduce $\frac{53}{168}$ of a gill to the fraction of a gallon.
17. Reduce $\frac{1}{56}$ of a gallon to the fraction of a gill.
18. What part of a yard is $\frac{2}{3}$ of an Ell Flemish?
19. What part of a yard is $\frac{2}{17}$ of a rod?

Q. How do you reduce a fraction of a lower denomination to a higher?

Q. How do you reduce a fraction of a higher denomination to a lower?

§ **75.** CASE 8. To find the value of a fraction in the known parts of the integer.

RULE.

Multiply the numerator by the next lower denomination, and divide the product by the denominator. And if any thing remains, multiply it by the next lower denomination, and divide by the denominator as before, and so continue as far as may be required; and the quotients placed in order will be the answer.

EXAMPLES.

1. What is the value of $\frac{7}{9}$ of a hundred weight?

$\frac{7}{9}cwt.=\frac{7}{9}\times\frac{4}{1}=\frac{28}{9}=3\frac{1}{9}qr.$
$\frac{1}{9}qr.=\frac{1}{9}\times\frac{28}{1}=\frac{28}{9}=3\frac{1}{9}lb.$
$\frac{1}{9}lb.=\frac{1}{9}\times\frac{16}{1}=\frac{16}{9}=1\frac{7}{9}oz.$
$\frac{7}{9}oz=\frac{7}{9}\times\frac{16}{1}=\frac{112}{9}=12\frac{4}{9}dr.$

Ans. $3qr.\ 3lb.\ 1oz.\ 12\frac{4}{9}dr.$

2. What is the value of $\frac{7}{13}$ of a day?
3. What is the value of $\frac{4}{7}$ of a pound avoirdupois?
4. What is the value of $\frac{7}{8}$ of a tun of wine?
5. What is the value of $\frac{161}{480}$ of a £?
6. What is the value of $\frac{7}{12}$ of a guinea?
7. What is the value of $\frac{5}{8}$ of a ton?
8. Reduce $\frac{9}{16}$ of a pound troy to its proper quantity.
9. Reduce $\frac{9}{12}$ of a yard to its proper quantity.
10. Reduce $\frac{11}{38}$ of a bushel to its proper quantity.
11. Reduce $\frac{5}{12}$ of a chaldron to its proper quantity.
12. Reduce $\frac{5}{8}$ of an acre to its proper quantity.
13. Reduce $\frac{5}{9}$ of an ell English to its proper quantity.
14. What is the value of $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{5}{8}$ of one mile?
15. What is the value of $\frac{7}{8}$ of $\frac{1}{3}$ of $\frac{3}{4}$ of an acre?

Q. How do you find the value of a fraction in the known parts of the integer?

§ **76.** CASE 9. To reduce a compound number to the fraction of the integer.

RULE.

Reduce the given number to the lowest denomination it contains, for a numerator; and then reduce the integers to the same denomination for a denominator.

EXAMPLES.

1. What part of a pound is 9*s*. 10*d*.?

s.	*d*.	*s*.
9	10	20
12		12
118		240

Therefore, $\frac{118}{240}=\frac{59}{120}$£

2. Reduce 12*s.* 8½*d.* to the fraction of a £.
3. Reduce 6*oz.* 15*dwt.* to the fraction of a *lb.* troy.
4. Reduce 12*cwt.* 2*qr.* to the fraction of a ton.
5. Reduce 16*bu.* 2*p.* to the fraction of a chaldron.
6. Reduce 16*cwt.* 3*qr.* to the fraction of a ton.
7. Reduce 8*oz.* 6½*dr.* to the fraction of a *lb.* avoirdupois.
8. Reduce 2*qr.* 3½*n.* to the fraction of a yard.
9. Reduce 6*d.* 6*h.* 15⅚*min.* to the fraction of a week.
10. What part of an acre is 2 roods 16¾ poles?
11. Express 6*fur.* 16*p.* in the fraction of a mile.
12. What part of a *hhd.* is 17*gal.* 1*qt.*?
13. What part of a day is 7*h.* 12*min.*?
14. What part of a dollar is 35 cents?

§ **77.** CASE 10. To reduce a complex fraction to a simple one.

RULE.

If the numerator or denominator be whole or mixed numbers, reduce them to improper fractions; then multiply the numerator of the upper fraction into the denominator of the lower, for a new numerator, and the denominator of the upper into the numerator of the lower, for the new denominator: which reduce to its lowest terms.

EXAMPLES.

1. Reduce $\frac{24\frac{3}{4}}{39}$ to a simple fraction.

$$\frac{24\frac{3}{4}}{39} = \frac{\frac{99}{4}}{39} = \frac{99}{39\times 4} = \frac{33}{52} \text{ Answer.}$$

2. Reduce $\frac{14}{30\frac{5}{6}}$ to a simple fraction.

$$\frac{14}{30\frac{5}{6}} = \frac{\frac{14}{1}}{\frac{185}{6}} = \frac{14\times 6}{1\times 185} = \frac{84}{185} \text{ Answer.}$$

3. Reduce $\frac{4\frac{1}{3}}{6\frac{1}{6}}$ to a simple fraction.

$$\frac{4\frac{1}{3}}{6\frac{1}{6}} = \frac{\frac{13}{3}}{\frac{37}{6}} = \frac{13\times 6}{3\times 37} = \frac{78}{111} = \frac{26}{37} \text{ Answer}$$

Simplify the following complex fractions:

4. $\frac{16}{24\frac{4}{5}}$ | 6. $\frac{8\frac{1}{4}}{12\frac{2}{3}}$ | 8. $\frac{3\frac{9}{10}}{4}$ | 10. $\frac{\frac{3}{4}\text{ of }\frac{1}{2}}{\frac{2}{3}\text{ of }\frac{1}{4}}$

5. $\frac{12\frac{3}{5}}{18}$ | 7. $\frac{4\frac{1}{3}}{7\frac{1}{8}}$ | 9. $\frac{2\frac{1}{2}}{\frac{1}{2}\text{ of }\frac{1}{3}}$ | 11. $\frac{\frac{13}{17}\text{ of }\frac{1}{8}}{\frac{5}{8}\text{ of }\frac{9}{16}}$

Q. How do you reduce a complex fraction to a simple one?

ADDITION OF FRACTIONS.

§ **78.** CASE 1. To add fractions that have a common denominator.

RULE.

Place the sum of the numerators over the common denominator.

EXAMPLES.

1. Add $\frac{341}{842}$, $\frac{284}{842}$, and $\frac{73}{842}$ together.

$$341+284+73=\frac{698}{842}=\frac{349}{421}=\text{Answer.}$$

2. $\frac{5}{19}+\frac{7}{19}+\frac{11}{19}+\frac{1}{19}$.

3. $\frac{4}{87}+\frac{14}{87}+\frac{79}{87}+\frac{86}{87}$.

4. $\frac{1}{99}+\frac{36}{99}+\frac{91}{99}$.

5. $\frac{666}{777}+\frac{333}{777}+\frac{746}{777}$.

6. $\frac{1728}{1842}+\frac{1642}{1842}+\frac{1806}{1842}$.

7. $\frac{461}{576}+\frac{376}{576}+\frac{444}{576}$.

8. $\frac{3784}{8340}+\frac{2471}{8340}+\frac{6789}{8340}$.

9. $\frac{1828}{2856}+\frac{1654}{2856}+\frac{1778}{2856}$.

Q. How do you add fractions that have a common denominator?

§ **79.** CASE 2. To add fractions that have different denominators.

RULE.

1. Reduce compound fractions to simple ones, mixed numbers to improper fractions; and then reduce all of them to a common denominator.

2. Add all the numerators together, and place the sum over the common denominator, and it will be the sum of the fractions required.

3. When mixed numbers only are to be added, reduce the

fractional parts to a common denominator, and add the whole numbers separately.

EXAMPLES.

1. Add $\frac{4}{5}$ and $\frac{3}{7}$ together.

$4\times7=28$ } Numerators.
$3\times5=15$ }

$5\times7=35$ Denominator.

Therefore $\frac{28}{35}+\frac{15}{35}=\frac{43}{35}=1\frac{8}{35}$ sum required.

2. What is the sum $2\frac{2}{5}$, $\frac{3}{8}$ and $\frac{1}{2}$ of $\frac{4}{7}$?

First $2\frac{2}{5}=1\frac{12}{5}$ and $\frac{1}{2}$ of $\frac{4}{7}=\frac{2}{7}$

Therefore the fractions are $1\frac{12}{5}$, $\frac{3}{8}$ and $\frac{2}{7}$.

$12\times8\times7=672$ }
$3\times5\times7=105$ } Numerators.
$2\times5\times8=\ 80$ }

$5\times8\times7=280$ Denominator.

$\frac{672}{280}+\frac{105}{280}+\frac{80}{280}=\frac{857}{280}=3\frac{117}{280}$ Ans.

3. Add $12\frac{1}{2}$, $16\frac{2}{3}$, and $26\frac{3}{4}$ together. The fractional parts are $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$.

Therefore $1\times3\times4=12$
$2\times2\times4=16$
$3\times2\times3=18$
$\overline{46}$
$2\times3\times4=\overline{24}$

$$= \begin{array}{r} 12 \\ 16 \\ 26 \\ \hline 1\frac{11}{12} \\ \hline 55\frac{11}{12} \end{array}$$

4. Add $\frac{3}{7}$, $2\frac{3}{4}$, $\frac{9}{14}$ and $\frac{2}{5}$ together.
5. Add $\frac{6}{7}$ of $\frac{8}{9}$, $3\frac{1}{3}$, $\frac{4}{5}$, and $\frac{1}{2}$ of $\frac{2}{3}$ together.
6. Add $\frac{3}{5}$ of $\frac{1}{2}$, $\frac{2}{3}$ of 6, $1\frac{1}{5}$, $\frac{3}{7}$ of $\frac{3}{2}$ and $\frac{2}{5}$ together.
7. Add $\frac{5}{6}$, $7\frac{1}{2}$ and $\frac{1}{3}$ of $\frac{3}{4}$ together.
8. Add $\frac{7}{8}$ of $\frac{4}{7}$, $3\frac{1}{2}$ and $\frac{3}{16}$ together.
9. Add $6\frac{7}{8}$ of $\frac{9}{10}$, $\frac{4}{7}$ of $\frac{1}{2}$ and $7\frac{1}{2}$ together.
10. Add $\frac{1}{2}$ of $\frac{7}{8}$ and $\frac{2}{3}$ of $\frac{19}{20}$ together.
11. Add $\frac{3}{5}$, $\frac{4}{5}$ of $\frac{1}{3}$ and $9\frac{5}{20}$ together.
12. What is the sum of $18\frac{3}{10}$, $24\frac{2}{5}$, $99\frac{8}{15}$ and $141\frac{1}{30}$?
13. Required the sum of $\frac{24\frac{3}{4}}{38}$, $\frac{12\frac{2}{3}}{18}$, $\frac{16}{24\frac{4}{5}}$ and $\frac{8\frac{1}{4}}{12\frac{2}{3}}$.

§ **80.** CASE 3. When the fractions are of several denominations.

RULE.

Reduce them all to their proper quantities, and add them together.

EXAMPLES.

Required the respective amounts of

1. $\frac{3}{4}$ of a £ + $\frac{2}{3}$ of a shilling.

	£	s.	d.
The value of $\frac{3}{4}$ of a £ =	0	15	0
do. $\frac{2}{3}$ of a s. =	0	0	8
Sum required	0	15	8

2. $\frac{1}{3}$ of a week + $\frac{1}{4}$ of a day + $\frac{1}{2}$ of an hour.

	d.	h.	m.
The value of $\frac{1}{3}$ of a week =	2	8	0
$\frac{1}{4}$ of a day =	0	6	0
$\frac{1}{2}$ of an hour =			30
Sum required	2	14	30

3. $\frac{3}{10}$ of a ton + $\frac{4}{9}$ of 1 *cwt.*
4. $\frac{4}{11}$ of a chaldron of coals + $\frac{2}{5}$ of a bushel.
5. $\frac{2}{3}$ of a pound of troy + $\frac{3}{8}$ of an ounce + $\frac{4}{5}$ of a dwt.
6. $\frac{1}{4}$ of a mile + $\frac{1}{3}$ of a yard + $\frac{3}{4}$ of a foot.
7. $\frac{3}{7}$ of a guinea + $\frac{4}{5}$ of a guinea.
8. $3\frac{2}{5}$ miles + $\frac{8}{9}$ of a mile.
9. $9\frac{6}{17}$ acres + $27\frac{13}{17}$ acres.
10. $3\frac{2}{29}$ cwt. + $14\frac{25}{29}$ cwt. + $\frac{9}{29}$ of a *cwt.*
11. $3\frac{4\frac{1}{3}}{7}$ furlongs + $1\frac{2\frac{3}{7}}{5}$ furlongs + $\frac{1}{3}$ of a furlong.
12. $13\frac{1\frac{1}{2}}{11}$ shillings + $6\frac{\frac{2}{5}}{7}$ shillings + $1\frac{\frac{6}{7}}{5}$ shillings.
13. $25\frac{3}{7}$ miles + $3\frac{8}{9}$ furlongs.
14. $2\frac{4}{7}$ acres + $\frac{3}{11}$ of a rood + $5\frac{7}{9}$ poles.
15. $13\frac{2}{5}$ cwt. + $3\frac{4}{5}$ quarters + $\frac{3}{5}$ of a quarter + $13\frac{6}{7}$ pounds.
16. $1\frac{4\frac{6}{11}}{9}$ furlong + $4\frac{2\frac{3}{5}}{6}$ poles + $20\frac{\frac{2}{3}}{4}$ yards.
17. $6\frac{\frac{3}{4}}{7}$ sq. yards + $5\frac{\frac{4}{7}}{6}$ sq. feet + $27\frac{3\frac{4}{8}}{9}$ sq. inches + $\frac{\frac{2}{11}}{13}$ of a sq. inch.

Q. How do you add fractions that have different denominators.

Q. How do you add compound fractions?

Q. How do you add mixed numbers?

Q. How do you add fractions of different denominations?

SUBTRACTION OF FRACTIONS.

§ 81. CASE 1. To subtract fractions that have a common denominator.

RULE.

Subtract the less numerator from the greater, and under the remainder place the common denominator, and reduce the fraction to its lowest terms.

EXAMPLES.

1. Subtract $\frac{25}{49}$ from $\frac{39}{49}$.

$$39-25=14,\ \tfrac{14}{49}=\tfrac{2}{7}\text{ Ans.}$$

2. $12\frac{5}{8}-7\frac{3}{8}$.
3. $24\frac{1}{4}-\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$.
4. $\frac{18}{72}-\frac{8}{72}$.
5. $\frac{381}{688}-\frac{117}{688}$.
6. $\frac{1681}{2568}-\frac{1428}{2568}$.
7. $\frac{34567}{48873}-\frac{29869}{48873}$.

§ 82. CASE 2. To subtract fractions whose denominators are different.

RULE.

Prepare the fractions as in Case 2d of Addition of Fractions, and the difference of the numerators placed above the common denominator will give the difference of the fraction required.

EXAMPLES.

1. What is the difference of $\frac{5}{9}$ and $\frac{4}{11}$?

$$\left.\begin{aligned}5\times11&=55\\4\times\ 9&=36\end{aligned}\right\}\text{Numerators.}$$
$$9\times11=99\text{ Denominator.}$$
$$\text{Therefore }\frac{55-36}{99}=\frac{19}{99}\text{ Ans.}$$

2. What is the difference between $\frac{5}{6}$ and $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{2}{5}$?

$\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{2}{5}=\frac{6}{108}=\frac{1}{18}$, and $\frac{5}{6}$

$$\left.\begin{aligned}5\times18&=90\\1\times\ 8&=\ 8\end{aligned}\right\}\text{Numerators.}$$
$$18\times\ 8=144\text{ Denominator.}$$
$$\text{Therefore }\frac{90-8}{144}=\frac{82}{144}=\frac{41}{72}\text{ Ans.}$$

3. $\frac{87}{100}-\frac{3}{7}$.
4. $\frac{5}{11}-\frac{14}{87}$.
5. $\frac{20}{59}-\frac{59}{60}$.
6. $\frac{2}{3}-\frac{2}{9}$ of $\frac{3}{7}$.
7. $2\frac{4}{7}-\frac{3}{5}$ of $\frac{4}{7}$.
8. $14\frac{1}{4}-\frac{2}{3}$ of 19.
9. $\frac{36}{99}-\frac{1}{18}$ of $\frac{8}{9}$.
10. $19\frac{4}{5}-\frac{1}{5}$ of $8\frac{1}{4}$.
11. $\frac{4\frac{1}{3}}{6\frac{1}{6}}-\frac{14}{30\frac{4}{6}}$.

§ 83. CASE 3. When the fractions are of different denominations.

RULE.

Find their simple values, and subtract one from the other.

EXAMPLES.

Required the respective results of

1. $\frac{2}{13}$ of a ton—$\frac{4}{5}$ of a cwt.

The value of $\frac{2}{13}$ of a ton=	3cwt.	0qr.	8lb.	$9\frac{11}{13}$oz.
" $\frac{4}{5}$ of a cwt.=	0	3	5	$9\frac{3}{5}$
Difference required,	2	1	3	$0\frac{16}{65}$

2. $\frac{5}{9}$ of a £—$\frac{2}{7}$ of $\frac{1}{3}$ of a £.
3. $\frac{4}{7}$ of a ton—$\frac{1}{2}$ of $\frac{3}{4}$ of a cwt.
4. $\frac{2}{4}$ of a league—$\frac{7}{10}$ of a mile.
5. $\frac{3}{6}$ of an oz.—$\frac{7}{8}$ of a dwt.
6. $\frac{5}{9}$ of a £—$\frac{1}{8}$ of a guinea.
7. $\frac{2}{3}$ of a chaldron—$\frac{4}{8}$ of a bushel.
8. $18\frac{4}{15}$ acres—$9\frac{1}{15}$ acres.
9. 7 miles—$\frac{8}{25}$ mile.
10. $7\frac{1}{5}$ cwt.—$\frac{11}{15}$ of a cwt.
11. $\frac{8}{9}$ of a mile—$\frac{4}{7}$ of a furlong.
12. $5\frac{4}{11}$ furlongs—$\frac{8}{13}$ of a pole.
13. $\frac{5}{6}$ of an acre—$2\frac{3}{5}$ roods.
14. $13\frac{6\frac{2}{3}}{7}$ shillings—$9\frac{4\frac{3}{8}}{13}$ shillings.
15. $2\frac{3\frac{4}{7}}{5}$ furlongs—$\frac{1\frac{2}{3}}{9}$ of a mile.
16. $\frac{\frac{1}{100}}{9}$ of an acre—$\frac{\frac{1}{139}}{131}$ of a square yard.

Q. How do you subtract fractions that have a common denominator?

Q. How do you subtract fractions that have different denominators?

Q. How do you subtract compound fractions?

Q. How do you subtract mixed numbers?

Q. How do you subtract fractions of different denominations?

MULTIPLICATION OF FRACTIONS.

§ 84. CASE 1. To multiply one fraction by another.

RULE.

Reduce compound fractions to simple ones, and mixed numbers to improper fractions, then multiply the numerators together for a new numerator, and the denominators for a new denominator, which will form the product required. The operation may be sometimes abridged by cancelling.

EXAMPLES.

1. Required the product of $\frac{7}{8}$, $\frac{6}{9}$, $\frac{9}{12}$, and $\frac{8}{4}$.

$\frac{7}{8}\times\frac{6}{9}\times\frac{9}{12}\times\frac{8}{4}=\frac{1}{3}\frac{1}{4}\frac{3}{5}\frac{4}{6}=\frac{21}{64}$ Answer.

Or thus: $\frac{7}{8}\times\frac{\cancel{6}}{\cancel{9}}\times\frac{\cancel{9}}{\cancel{12}}\times\frac{\cancel{8}}{\cancel{4}}=\frac{21}{64}$ as before.

2. Multiply $2\frac{1}{2}$, $\frac{1}{8}$, $\frac{1}{2}$ of $\frac{4}{5}$, and 3 together.

$2\frac{1}{2}=\frac{5}{2}$, $\frac{1}{2}$ of $\frac{4}{5}=\frac{2}{5}$, and $3=\frac{3}{1}$.

$\frac{5}{2}\times\frac{1}{8}\times\frac{2}{5}\times\frac{3}{1}=\frac{30}{80}=\frac{3}{8}$ Answer.

Or thus: $\frac{\cancel{5}}{\cancel{2}}\times\frac{1}{8}\times\frac{\cancel{2}}{\cancel{5}}\times\frac{3}{1}=\frac{3}{8}$ as before.

3. Multiply $\frac{4}{15}$ by $\frac{5}{24}$.
4. Multiply $\frac{2}{9}$ of $\frac{3}{4}$ by $\frac{5}{8}$ of $3\frac{2}{7}$.
5. Multiply $4\frac{1}{2}$, $\frac{3}{4}$ of $\frac{1}{7}$ and $18\frac{4}{5}$ together.
6. Multiply $\frac{2}{3}$, $3\frac{1}{4}$, 5, and $\frac{8}{4}$ of $\frac{3}{5}$ together.
7. Multiply $\frac{2}{5}$, $\frac{3}{5}$, $\frac{1}{2}$, and $\frac{2}{7}$, together.
8. Multiply $4\frac{2}{3}$, $1\frac{1}{13}$, $3\frac{1}{2}$ of 8, together.
9. Multiply $\frac{4}{7}$, $\frac{3}{8}$, $6\frac{2}{9}$, $5\frac{4}{5}$ and 12 together.
10. Multiply $1\frac{7}{10}$, $\frac{7}{7}$, $\frac{13}{14}$ and $\frac{1}{11}$ together.
11. Multiply $42\frac{3}{10}$ by $\frac{1}{2}$ of $\frac{5}{6}$ of 4.
12. Multiply $\frac{4\frac{1}{3}}{6\frac{1}{6}}$, $\frac{16}{24\frac{4}{5}}$ and $\frac{12\frac{2}{3}}{18}$ together.

Q. How do you multiply a fraction by a fraction?
Q. How do you multiply a mixed number by a fraction?
Q. How do you multiply compound fractions?

§ 85. CASE 2. To multiply a fraction by a whole number.

RULE.

Multiply the numerator or divide the denominator by the whole number.

EXAMPLES.

Multiply $\frac{9}{16}$ by 8.

$$\frac{9}{16}\times 8 = \frac{72}{16} = 4\frac{1}{2}$$

Or divide the denominator by 8.

Thus $\frac{9}{16}\times \frac{8}{8)16} = 4\frac{1}{2}$.

2. $\frac{4}{7}\times 42$. | 4. $17\frac{5}{9}\times 16$. | 6. $341\frac{6}{7}\times 41$

3. $\frac{4}{19}\times 38$. | 5. $240\frac{8}{11}\times 19$. | 7. $15\frac{1}{5}\times 5$.

MISCELLANEOUS EXERCISES.

Required the respective products of

1. $\frac{9}{217}$ of an acre $\times$ 11.
2. $\frac{3}{13}$ of a hundred weight $\times$ 5.
3. $\frac{14}{15}$ of a mile $\times$ 17.
4. $\frac{13}{15}$ of a guinea $\times \frac{9}{19}$.
5. $\frac{33}{49}$ of a square pole $\times \frac{7}{24}$.
6. $\frac{4}{23}$ of a square yard $\times 1\frac{7}{8}$.
7. $\frac{8}{25}$ of a cubic yard $\times$ 5, 7, $\frac{2}{4}$, continually.
8. $6\frac{1}{3}$ pence $\times 4\frac{2}{3}$, 30, 8, $\frac{12}{13}$, $\frac{1}{8}$, continually.
9. $1\frac{2\frac{10}{11}}{3}$ cwt. $\times$ 55.
10. $5\frac{\frac{11}{12}}{5}$ shillings $\times 27\frac{5}{6}$.
11. $\frac{\frac{6}{7}}{9}$ of a furlong $\times \frac{\frac{2}{4}}{3}$, 21, $\frac{3}{5}$, $\frac{1}{6}$, continually.
12. $\frac{\frac{10}{11}}{99}$ of a mile $\times$ 33, $1\frac{3}{4}$, 17, $\frac{\frac{2}{4}}{5}$, continually.
13. $4\frac{3}{16}$ acres $\times 2\frac{3}{4}$, $3\frac{5}{7}$, 32, $\frac{1}{3}$, continually.
14. $6\frac{\frac{3}{4}}{5}$ square yards $\times 10\frac{1\frac{1}{2}}{13}$.
15. $\frac{3\frac{1}{2}}{11}$ of a furlong $\times 6\frac{1\frac{3}{5}}{12}$.
16. What will $23\frac{7}{8}$ pounds of cheese cost, at 6 cents a pound?
17. What will $15\frac{11}{16}$ pounds of coffee cost, at 12 cents a pound?
18. What will $17\frac{5}{12}$ pounds of sugar cost, at 16 cents a pound?
19. What will $9\frac{7}{8}$ yards of broadcloth cost, at 6 dollars per yard?
20. Bought 29 bales of muslins each bale containing $32\frac{3}{8}$ yards: what will be the cost, at 16 cents per yard?

DIVISION OF FRACTIONS.

§ 86. CASE 1. To divide one fraction by another.

RULE.

Prepare the fractions as in case 1, of multiplication of fractions; then invert the divisor, and proceed as in multiplication.

ELUCIDATION. The reason of the rule may be explained thus: Suppose it were required to divide $\frac{3}{4}$ by $\frac{2}{3}$. Now $\frac{3}{4} \div 2$ is evidently $\frac{1}{2}$ of $\frac{3}{4}$ or $\frac{3}{4 \times 2}$, but $\frac{2}{3} = \frac{1}{3}$ of 2, or $\frac{2}{3}$ must be contained 3 times as often in $\frac{3}{4}$ as 2 is; that is, $\frac{3 \times 3}{4 \times 2} = \frac{9}{8}$, which is according to the rule.

EXAMPLES.

1. Divide $\frac{1}{2}$ of 27 by $\frac{3}{4}$ of $\frac{16}{5}$.

$\frac{1}{2}$ of $27 = \frac{27}{2}$, and $\frac{3}{4}$ of $\frac{16}{5} = \frac{48}{20} = \frac{12}{5}$:

Therefore, $\frac{27}{2} \div \frac{12}{5} = \frac{27}{2} \times \frac{5}{12} = \frac{135}{24} = \frac{45}{8}$ Answer.

2. Divide $\frac{43}{54}$ by $\frac{24}{25}$.

$\frac{43}{54} \times \frac{24}{25} = \frac{1032}{1350} = \frac{172}{225}$ Answer.

Or by cancelling: $\frac{43}{\cancel{54}_{9}} \times \frac{\cancel{24}^{4}}{25} = \frac{172}{225}$ the sum as before.

3. $8\frac{3}{5} \div \frac{1}{2}$ of $\frac{4}{5}$ of 5.
4. $\frac{1}{7}$ of $\frac{2}{3}$ of $18 \div \frac{5}{6}$ of $\frac{7}{10}$.
5. $\frac{3}{5}$ of $\frac{5}{7} \div \frac{2}{7}$ of $\frac{3}{5}$.
6. $\frac{4}{7}$ of $\frac{7}{12} \div \frac{3}{12}$ of $\frac{6}{7}$.
7. $4\frac{5}{9} \div \frac{5}{9}$ of 4.
8. $\frac{5}{9}$ of $4 \div 4\frac{5}{9}$.
9. $\frac{7}{8}$ of $6 \div \frac{3}{4}$ of $\frac{6}{7}$ of $\frac{11}{12}$.
10. $\frac{14}{24} \div \frac{7}{10}$.
11. $\frac{7}{8} \div 4\frac{2}{3}$.
12. $\frac{16}{24\frac{4}{5}} \div \frac{12\frac{2}{3}}{18}$.

Q. How do you divide a fraction by a fraction?

§ 87. CASE 2. To divide a whole number by a fraction.

RULE.

Multiply the whole number by the denominator of the fraction, and divide by its numerator.

EXAMPLES.

1 Divide 48 by $\frac{8}{9}$.

$$\begin{array}{r} 48 \\ 9 \\ \hline 8)\overline{432} \\ \underline{54} \end{array}$$ Answer.

2. $54 \div \frac{3}{5}$ of 9.	5. $167 \div \frac{5}{6}$.	8. $47 \div \frac{1}{2}$ of $\frac{4}{5}$.
3. $72 \div \frac{2}{3}$.	6. $281 \div \frac{11}{12}$.	9. $63 \div \frac{3}{4}$ of $\frac{5}{6}$.
4. $19 \div \frac{7}{11}$.	7. $38 \div \frac{4}{17}$.	10. $98 \div \frac{2}{3}$ of 4.

Q. How do you divide a whole number by a fraction?

§ 88. CASE 3. To divide a fraction by a whole number.

RULE.

Multiply by the denominator of the fraction; and place the product under the numerator.

EXAMPLES.

1. Divide $\frac{7}{8}$ by 24.

$8 \times 24 = 192$. $\frac{7}{192}$ Answer.

2. $\frac{8}{11} \div 16$.	4. $\frac{15}{24} \div 18$.	6. $\frac{3}{4}$ of $\frac{6}{7} \div 81$
3. $\frac{11}{18} \div 36$.	5. $\frac{27}{4} \div 7$.	7. $\frac{2}{15}$ of $\frac{3}{4} \div 250$.

Q. How do you divide a fraction by a whole number?

§ 89. CASE 4. To divide a mixed fraction by a whole number.

RULE.

Divide as in whole numbers, and if any thing remains, multiply it by the denominator of the fraction, and to the product add the numerator, and place it over the product of the divisor multiplied by the denominator.

EXAMPLES.

1. Divide $41\frac{1}{3}$ by 9.

$9)41\frac{1}{3}$

$4\frac{16}{27}$ Answer.

2. $17\frac{3}{8} \div 8$.	4. $193\frac{1}{3} \div 11$.	6. $28\frac{4\frac{1}{3}}{6\frac{1}{6}} \div 6$.
3. $37\frac{1}{2} \div 9$.	5. $116\frac{11}{12} \div 24$.	7. $\frac{1}{2}$ of $61\frac{9}{11} \div 8$

Q. How do you divide a mixed fraction by a whole number?

MISCELLANEOUS EXERCISES.

1. In 7 acres how many $\frac{6}{7}$ of an acre?
2. In 13 miles how many $\frac{3}{14}$ of a mile?
3. In 9 guineas how many $\frac{2}{13}$ of a guinea?
4. In $13\frac{5}{7}$ of a cwt. how many $\frac{8}{13}$ of a cwt.?

5. In $17\frac{6}{11}$ acres how many $\frac{14}{15}$ of an acre?
6. In $\frac{8}{9}$ of a furlong how many $\frac{3}{7}$ of a furlong?
7. Divide $\frac{3}{11}$ by $\frac{5}{7}$.
8. Divide $\frac{6}{19}$ by $\frac{99}{100}$.
9. In $27\frac{5}{8}$ cwt. how many $3\frac{4}{5}$ cwt.?
10. Divide $9\frac{3}{11}$ square yards by 51.
11. Divide $17\frac{8}{15}$ furlongs by 24.
12. In $76\frac{3}{4}$ miles how many $9\frac{6}{11}$ miles?
13. Divide $31\frac{3}{4}$ acres by $62\frac{5}{7}$.
14. Divide $47\frac{4\frac{2}{5}}{9}$ cwt. by $15\frac{1\frac{1}{4}}{7}$.
15. Divide $147\frac{\frac{6}{7}}{11}$ cubic yards by $13\frac{1\frac{2}{3}}{5}$.
16. Sold $4\frac{3}{4}$ cords of wood for $\$45$: what was this a cord?
17. Bought $14\frac{3}{4}$ yards of broadcloth, for $\$47\frac{1}{4}$: what was this a yard?
18. Sold $18\frac{3}{8}$ tons of iron, for $\$220\frac{1}{2}$: what was this a ton?
19. Paid $\$1476$ for a lot of ground, measuring $22\frac{3}{4}$ feet by $15\frac{1}{2}$: what was this a square foot?
20. What is the 36th part of $\$101\frac{7}{9}$?

DECIMAL FRACTIONS.

§ **90.** A decimal fraction is one whose denominator is an unit, with as many ciphers annexed as the numerator has places. It is expressed by writing the numerator only with a point before it, thus, $\frac{4}{10}$, $\frac{23}{100}$, $\frac{871}{1000}$, &c. are decimal fractions, and are expressed by .4, .23, and .871, respectively.

The point prefixed is called the separatrix. Ciphers annexed to the right hand of decimals make no alteration in their value, for .6, .60, .600, &c., are decimals having the same value, each being $=\frac{6}{10}$ or $\frac{3}{5}$; but if they are placed on the left hand they decrease their value in a tenfold proportion. Thus, .3 is $\frac{3}{10}$ or three tenths, but .03 is $\frac{3}{100}$ or three hundredths; and .003 is $\frac{3}{1000}$, or three thousandths.

A mixed number consists of a whole number and a decimal; as 114.892, 16.72, &c.

In notation of whole numbers, the values of figures increase in a tenfold proportion, from the right hand to the left; so in decimals their values decrease from the left hand

to the right. Thus .2 expresses 2 tenth parts of the integer, .02, 2 hundredths parts.

Unity may be considered as a fixed point, from whence whole numbers proceed, infinitely increasing toward the left hand, and decimals infinitely decreasing toward the right hand, which is exemplified in the following

TABLE.

Hundreds of millions.	Tens of millions.	Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	Ten thousandths.	Hundred thousandths.	Millionths.	Ten millionths.	Hundred millionths.
9	8	7	6	5	4	3	2	1.	2	3	4	5	6	7	8	9
Integers.									Decimals.							

Write the following numbers in words.

1. 33.49.	6. 34.005.	11. .001001.
2. 86.73.	7. 79.00009.	12. .98467.
3. 34.825.	8. 4.00101.	13. .776776.
4. 947.67.	9. .78543.	14. .10001.
5. 89.676.	10. .12387.	15. .00009.

Q. What is a decimal fraction?
Q. How are decimals distinguished from integers?
Q. What effect have ciphers on a decimal?
Q. How are decimals distinguished?
Q. What does a mixed number consist of?

ADDITION OF DECIMALS.

§ 91. RULE 1. Write down the numbers so that the decimal points, and like places, may be under each other.

2. Find their sum as in whole numbers, and point off as many places for decimals, as are equal to the greatest number of decimal places in any of the given numbers.

EXAMPLES.

1. Find the sum of 24.025+0.8724+136.+.00053+1843.213.

24.025
0.8724
136.
.00053
1843.213
2004.11093 the sum.

2. Add .0084+30.67+.4085+2.081+3.15.
3. Add 40.40+5.862+307.5+.4075+37.+.8.
4. Add 9.99+.415+.00001+.847+.0083+375.375.
5. Add 3+.3+.7+7+.9841+81.18+189.432+.009.
6. Add .0001+23.1817+84621.5+38.472+8.816.
7. Add 500.5001+7.903+46.731+73.143+16.32.

Q. How do you write decimals to be added?

SUBTRACTION OF DECIMALS.

§ **92.** RULE. Place the less number under the greater, so that like places stand under each other; then subtract as in whole numbers, and point off the decimals as in addition.

EXAMPLES.

1. Find the difference of 3149.42 and 842.31416.

3149.42
842.31416
2307.10584 the difference.

2. 275.—48.71653.
3. 14.675—9.784651.
4. .3205—.27981.
5. 4900.—.0007.
6. 7.008—.0008.
7. .8764—.465.
8. 81.5—41.092.
9. 365.—.365.

Q. How do you place decimals to be subtracted?

MULTIPLICATION OF DECIMALS.

§ **93.** RULE 1. Place the factors, and multiply them, as in whole numbers.

2. Point off as many figures from the product as there are

decimal places in both the factors; and if there are not so many places in the product, supply the deficiency by prefixing ciphers.

3. To multiply by 10, 100, 1000, &c., we only have to remove the point as many places to the right, as there are ciphers in the multiplier.

The reason of pointing off as many decimal places in the product, as there are in both factors, is that the operation is the same as in multiplication of vulgar fractions; for since each decimal place in either factor diminishes its value ten times, it must equally diminish the value of the product. Thus, $.75 \times .33 = .2475$, and $\frac{75}{100} \times \frac{33}{100} = \frac{2475}{10000}$, which is the same.

EXAMPLES.

1. Multiply .07854
by .08437
54978
23562
31416
62832
.0066264198 the product.

2. $.571 \times .27$.	7. $.00082 \times 9.03$.
3. $.00163 \times .123$.	8. $.852 \times 4.003$.
4. $.6938 \times .072$.	9. 1209.6×6.032.
5. $.1843 \times .37$.	10. $7.0001 \times .001$.
6. $.03412 \times 182.1$.	11. $1008 \times .1008$.

Q. How do you multiply decimals?

Q. How do you proceed if the product has not so many places as are necessary to be pointed off?

Q. Why do you point off as many decimal places in the product as there are in both factors?

Q. How do you multiply by 10, 100, 1000, or the like?

DIVISION OF DECIMALS.

§ **94.** RULE. 1. Divide as in whole numbers, and from the right hand of the quotient point off as many places for decimals as the decimal places in the dividend exceed those in the divisor.

2. If the places in the quotient are not so many as the rule requires, supply the defect by prefixing ciphers.

3. If at any time there be a remainder, or the decimal places in the divisor be more than those in the dividend, ciphers may be affixed to the dividend, and the quotient carried on to any degree of exactness.

4. To divide by 10, 100, 1000 or the like, move the point as many places to the left as there are ciphers in the divisor.

The quotient figure is always of the same value with that figure of the dividend, under which the units place of its product stands. Or, the decimal parts in the divisor and quotient must always be equal in number to those of the dividend.

When the decimal places in the divisor and dividend are equal, and there is no remainder after dividing, the quotient will be whole numbers. When the places of decimals in the dividend exceed those of the divisor, the decimal parts in the quotient must be equal to that excess.

If the divisor exceed the dividend in decimal places, annex ciphers to make them equal, and if there is no remainder after dividing, then will the quotient be integers.

5. The reason of pointing off as many decimal places in the quotient as those in the dividend exceed the divisor, will easily appear; for since the number of decimal places in the dividend is equal to those in the divisor and quotient taken together, by the nature of multiplication it therefore follows that the quotient contains as many as the dividend exceeds the divisor.

EXAMPLES.

1. Divide 754.34738 by 12.269.

```
12.269)754.34738(61.48
       73614
       ------
       18207
       12269
       -----
        59383
        49076
        -----
       103078
        98152
        -----
         4926.
```

2. 72.3456÷54.32.
3. 11269.8÷1.116.
4. 76.÷.2508.
5. 12.54÷6789.

6. 14÷.7854	10. 100÷.001
7. .8727587÷162	11. .382÷.347
8. .001÷6	12. .1÷1000
9. 1.÷.1	13. 785.÷.008

Q. How do you divide decimals?

Q. How do you fix the decimal point in the quotient?

Q. How do you proceed, if, after the division is finished, the quotient has not so many decimals as the rule requires?

Q. How do you proceed when there are more decimals in the divisor than in the dividend?

Q. Why do you point off as many places in the quotient, as the number in the dividend exceeds that in the divisor?

Q. How do you divide by 10, 100, 1000, or the like?

REDUCTION OF DECIMALS.

§ **95.** CASE 1. To reduce a vulgar fraction to its equivalent decimal.

RULE.

Annex ciphers to the numerator, and divide by the denominator, and the quotient will be the decimal required.

EXAMPLES.

Reduce to equivalent decimals the following fractions:

1. $\frac{1}{5}$, $\frac{2}{7}$ and $\frac{3}{8}$.	5)1.0 .2	7)2.000 .2857+	8)3000 .375
2. $\frac{4}{9}$, $\frac{5}{8}$ and $\frac{7}{15}$.	9)4.0000 .4444+	8)5.000 .625	15)7.0000 .4666+

3. $\frac{19}{20}$ and $\frac{15}{17}$.	7. $\frac{284}{527}$.	11. $\frac{109423}{846821}$.
4. $\frac{5}{0}$, $\frac{7}{18}$, and $\frac{18}{27}$.	8. $\frac{542}{897}$.	12. $\frac{1924231}{2487794}$.
5. $\frac{13}{36}$, and $\frac{16}{23}$.	9. $\frac{1758}{2407}$.	13. $\frac{1}{4}$ of $\frac{11}{16}$.
6. $\frac{148}{278}$.	10. $\frac{1767}{3978}$.	14. $158\frac{13}{151}$.

§ **96.** CASE 2. To reduce a decimal to its equivalent vulgar fraction.

RULE.

Write down the given decimal as a numerator, and for a denominator write 1 with as many ciphers as there are places in the decimal for a denominator, and then reduce the fraction to its lowest terms.

EXAMPLES.

Reduce the following decimals to their equivalent vulgar fractions.

1. .440. $8)\frac{440}{1000}=5)\frac{55}{125}=\frac{11}{25}$ Answer.
2. 496825. $25)\frac{496825}{1000000}=\frac{19873}{40000}$ Answer.

3. .6845	6. .8215	9. .72846
4. .5492	7. .74226	10. .83497
5. .5625	8. .90435	11. .786476

§ **97.** CASE 3. To find the value of a decimal in lower denominations.

RULE.

1. Multiply the decimal by the number required to reduce it to the next lower denomination, and cut off as many places for the remainder, to the right hand, as there are places in the given decimal.

2. Multiply the remainder by the next lower denomination, and cut off for a remainder as before.

3. Proceed in the same manner through all the denominations, and the figures standing on the left hand of the several points will be the answer required.

EXAMPLES.

What is the value of the following decimals?

1. .67 of a league?

.67
3
2.01
8
0.08
40
3.20
5½
1.10
3
0.30
12
3.60
3 *m. fur. p. yd. ft. in. bar*
1.80 Answer: 2. 0. 3. 1. 0. 3. 1.

2. .8625 of a ton?
3. .305 of a bushel?
4. .6789 of a mile?
5. .58 of a year?
6. .83289 of a £?
7. .6725 of a cwt.?
8. .0087 of a pound troy?
9. .4712 of a yard?
10. .0923 of a guinea?
11. .387 of an acre?
12. .0962$\frac{19}{35}$ of a sq. yard?
13. .7854 of a sign?

§ **98.** CASE 4. To find the value of compound numbers in decimals of higher denominations.

RULE.

1. Write down the several denominations one above the other, proceeding regularly from the least to the greatest.

2. Opposite to each dividend, on the left hand, place such a number as will bring it to the next superior name, and draw a line between them.

3. Begin with the lowest denomination, and write the quotient of each division, as decimal parts, on the right hand of the dividend next below it; and so on, till they are all used, and the last quotient will be the decimal required.

ELUCIDATION.—The reason of this rule may be explained from the first example; thus, three farthings is $\frac{3}{4}$ of a penny, which brought to a decimal, is .75; therefore $6\frac{3}{4}d.$ may be expressed $6.75d.$; but 6.75 is $\frac{675}{100}$ of a penny$=\frac{675}{1200}$ of a shilling, which brought to a decimal is .5625; and $16s.$ $6\frac{3}{4}d.$ may be expressed $16.5625s.$ In the same manner $16.5625s.$ $\frac{16562\,5}{10000}$ of a shilling$=\frac{16562\,5}{200000}$ of a pound$=.828125$, as by the rule.

EXAMPLES.

1. Reduce $16s.$ $6\frac{3}{4}d.$ to the decimal of a £.

4	3.
12	6.75
20	16.5625
	.828125 the decimal required.

2. Reduce 3 quarts 1 pint to the decimal of a gallon.

2	1
4	3.5
	.875 decimal required.

3. Reduce $5cwt.$ $2qr.$ $10lb.$ to the decimal of a ton.
4. Reduce $12dwt.$ $10gr.$ to the decimal of a pound troy
5. Reduce $3lb.$ $5oz.$ to the decimal of a hundred weight.
6. Reduce 45 days to the decimal of a year.

7. Reduce 3 furlongs 4 poles to the decimal of a mile.
8. Reduce 2 feet 4 inches to the decimal of a yard.
9. Reduce $7\frac{1}{2}$ inches to the decimal of a foot.
10. Reduce 1 rood 14 poles to the decimal of an acre.
11. Reduce 6 ounces 11.80672 drachms to the decimal of a hundred weight.
12. Reduce 1 shilling 11 pence 1.1384 farthings to the decimal of a guinea.

Q. How do you reduce a vulgar fraction to a decimal?
Q. How do you reduce a decimal to a vulgar fraction?
Q. How do you find the value of a decimal in lower denominations?
Q. How do you find the value of compound numbers in decimals of higher denominations?

MISCELLANEOUS EXERCISES.

1. Required the sum of 4.8, .764, .091, 27, 5.2614, 1.935 furlongs.
2. The sum of 15*cwt.* $3.147\frac{19}{101}qr.$ $\frac{5}{11}$ of a pound, and and $14.2761\frac{1}{7}$ pounds.
3. The sum of $.0\frac{7}{13}$ of a square yard, $4.51632\frac{8}{17}$ square yards, $2.35\frac{4}{7}$ square feet, and .0007 of a square foot.
4. The sum of 2.7416 acres, $\frac{8}{13}$ of an acre, $3.135\frac{4}{11}$ roods, $2\frac{3}{5}$ roods, and $19.2\frac{1}{742}$ poles.
5. The sum of $4\frac{7\frac{2}{3}}{9}$ shillings, $.6271\frac{8\frac{1}{2}}{11}$ shillings, $10.2\frac{1}{7}$ pence, and $3\frac{7}{5}$ pence.
6. From 18.57 acres subtract 19.7358 poles.
7. From 63.7 guineas subtract 29.85 pounds.
8. From $11\frac{5\frac{2}{3}}{6}$ cwt. subtract $3.91\frac{6}{11}$ quarters.
9. From 74 square feet subtract 59.716 square inches.
10. From 13.84 miles subtract $7.631\frac{8}{11}$ furlongs.
11. Multiply .0076 of a hundred weight by 3.1.
12. Multiply .64 of a week by 6.001.
13. Multiply .3421 of a degree of a circle by 14.9.
14. Multiply 6.4251 acres by 17.01.
15. Multiply 74.1 cwt. by 2.002.
16. Divide 51.474 acres by 94.87623.
17. In .74261 of a rood how many .1073 of a rood?
18. In 37.546 shillings how many .17489 of a shilling?
19. Divide .000546 of a cwt. by 13.
20. Divide .0974 of a mile by 39.35.

DUODECIMALS, OR CROSS MULTIPLICATION.

§ **99.** This rule is used by artificers and workmen to measure their work, the dimensions being taken in feet, inches, and twelfths. A foot is divided into 12 parts, called inches (*in.*), each inch into 12 parts, called seconds ($''$), each second into 12 parts, called thirds ($'''$), and each third into 12 parts, called fourths ($''''$), according to the following table:

12 fourths ($''''$) make 1 third.
12 thirds ($'''$) make 1 second.
12 seconds ($''$) make 1 inch.
12 inches or primes make 1 foot.
Feet multiplied by feet give feet.
Feet multiplied by inches give inches.
Feet multiplied by seconds give seconds.
Inches multiplied by inches give seconds.
Inches multiplied by seconds give thirds.
Seconds multiplied by seconds give fourths.

RULE 1st. Under the multiplicand write the corresponding denominations of the multiplier; that is, place feet under feet, inches under inches, &c.

2d. Multiply each term in the multiplicand, beginning at the lowest, by the feet in the multiplier, write each product under its respective term; observing to carry one for every 12, from each lower denomination, to its next superior.

3d. Multiply in the same manner with the inches; and set the product of each term one place further to the right hand, and carry one for every 12 as before.

4th. Work in like manner with the seconds, &c., and the sum of the lines will be the product required.

EXAMPLES.

1. Multiply $3ft.\ 4in.$ by $6ft.\ 8in.$

ft.	*in.*		
3	4		
6	8		
20	0	Upper line × by $6ft.$	
2	2	$8''$	Upper line × by $8in.$
22	2	$8''$	Answer.

2. Multiply $7ft.\ 8in.\ 9''$ by $3ft.\ 5in.\ 6'$

ft.	*in.*	$''$			
7	8	9			
3	5	6			
23	2	3			
3	2	7	9		
	3	10	4	6	
26	8	9	1	$6''''$	Answer.

	ft.	in.		ft.	in.			ft.	in.	″		ft.	in.	″
3. Mul'y	10	4	by	6	8		8. Mul'y	8	7	1	by	3	2	6
4. "	9	7	by	11	6		9. "	10	4	5	by	5	8	4
5. "	29	4	by	7	9		10. "	9	5	7	by	7	4	10
6. "	14	3	6″ by	5	4		11. "	10	0	4	by	9	0	7
7. "	18	5	3″ by	6	3	8″	12. "	7	5	3	by	11	4	7

REDUCTION OF CURRENCIES.

§ **100.** Previous to the act of congress establishing a mint, and regulating the coin of the United States, passed in the year 1792, the currency of the United States was pounds, shillings, pence, and farthings, the same as in England. The following table gives the value of the dollar in the old currency of the several states:

	s.	d.	£
New England, Virginia, Kentucky, Tennessee, and Ohio,	6	0	$=\frac{3}{10}$
New York and North Carolina,	8	0	$=\frac{4}{10}$
New Jersey, Pennsylvania, Delaware, and Maryland,	7	6	$=\frac{3}{8}$
South Carolina and Georgia,	4	8	$=\frac{7}{30}$
Canada and Nova Scotia,	5	0	$=\frac{1}{4}$
England, (sterling,)	4	6	$=\frac{9}{40}$

§ **101.** CASE 1. To change any of the above currencies to federal money.

RULE.

Reduce the shillings, pence, and farthings, if there be any, to the decimal of a pound, and annex them to the pounds.

Multiply the sum thus found by the denominator, and divide the product by the numerator of the fractional part of a pound, as found in the above table.

EXAMPLES.

1. Change £218 7*s.* 4*d.* New England currency to federal money.

7*s.* 4*d.*=.3666 the decimal of a pound, which annex to the pounds, thus 218.366 and 218.3666$\times\frac{10}{3}$=$727.88.8: Answer.

2. Change 90*l.* 17*s.* New England currency to federal money.

3. Change 129*l.* Pennsylvania currency to federal money.

4. Change 172*l.* 12*s.* 10½*d.* New York currency to federal money.
5. Change 140*l.* 9*s.* 6*d.* North Carolina currency to federal money.
6. Change 113*l.* 8*s.* of South Carolina and Georgia currency to federal money.
7. Change 42*l.* 5*s.* 3*d.* New Jersey currency to federal money.
8. Reduce 33*l.* 6*s.* 8*d.* Maryland and Delaware currency to federal money.
9. Reduce 48*l.* Nova Scotia and Canada currency to federal money.
10. Reduce 97*l.* 8*s.* sterling to federal money.

§ **102.** CASE 2. To change federal money to any of the currencies named in the table.

RULE.

Multiply by the numerator, and divide the product by the denominator of the fractional part of a pound as expressed in the above table.

EXAMPLES.

1. Change $182.25 to New England currency.

$182.25 \times \frac{3}{10} = 54.675 = £54\ 13s.\ 6d.$ Answer.

2. Reduce $500 to pounds New England currency.
3. Reduce $50.75 to pounds New York currency.
4. Change $243.20 to pounds Pennsylvania currency.
5. Reduce $237.87½ to pounds North Carolina currency.
6. Reduce $358.16 to pounds South Carolina and Georgia currency.
7. Change $729.60 to pounds New Jersey currency.
8. Reduce $981.33⅓ to pounds Delaware and Maryland currency.
9. Change $171.90 to pounds Nova Scotia and Canada currency.
10. Change $3620 to sterling money.

THE SINGLE RULE OF THREE.

§ **103.** The single rule of three teaches, when three numbers or terms are given, two of which are of one kind, to find a fourth number of the same kind as the other given term.

As the rule of three is only an application of vulgar fractions, it may be useful to state the nature of proportion on which this rule depends.

Thus, 2 is half of 4, and 3 half of 6; and 2 is said to be to 4, as 3 is to 6, or, to be in the same proportion, which is represented thus, as 2 : 4 : : 3 : 6. Therefore $2\times6=4\times3$; so 7 : 10 : : 14 : 20, and $7\times20=10\times14$.

That is, the product of the first and last terms, equals the product of the two middle terms; and if three out of the four are known, the other may be found.

RULE.

1. State the question, that is, put in the third place that term which is of the same kind as the required answer, then if the answer is to be greater than the third term, place the greater of the remaining terms in the second place; but if the answer is to be less than the third term, put the less term in the second place. Write the remaining term in the first place. Bring the first and second terms to the same denomination, and the third to the lowest name mentioned.

2. Multiply the second and third terms together, and divide the product by the first; the quotient will be the answer to the question in the same denomination as the third term.

3. If either of the terms consists of fractions, state the question as in whole numbers, and reduce mixed numbers to improper fractions, compound fractions to simple ones, then invert the first term (being the divisor,) and proceed as in multiplication. Reduce the new fraction to its proper quantity for the answer.

4. To cancel or abridge the terms. When the question is stated, if any of the terms which are to be multiplied together, and the term by which you divide can be measured by any number, divide them by that number, and use the quotients instead of the original terms.

The division of this rule into direct and inverse is now generally rejected.

PROOF.—The operation may be proved by inverting the question.

Q. What does the rule of three teach?
Q. How do you state the question?
Q. How do you solve the question?
Q. How do you abridge the terms?
Q. How do you prove questions in this rule?

EXAMPLES.

1. If 48 yards of broadcloth cost 120 dollars, what will 8 yards cost?

yd. yd. $
As 48 : 8 :: 120
8
48)960
20 Answer.

By cancelling,
As 48 : 8 :: 120
6 1
That is 48÷8=6
And 120÷6=20 Answer.

Again, as 48 : 8 : 120
4 2 10
2
20 Answer.

2. If $\frac{3}{5}$ of a yard cost $\frac{7}{15}$ of a £, what cost $\frac{3}{14}$ yards?

As $\frac{3}{5} : \frac{3}{14} :: \frac{7}{15}$.

Thus $\frac{5}{3}\times\frac{3}{14}\times\frac{7}{15}=\frac{105}{630}=\frac{1}{6}=3s.\ 4d.$ Answer.

Or, cancelled, $\frac{\cancel{5}}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{14}_{2}}\times\frac{\cancel{7}}{\cancel{15}_{3}}=\frac{1}{6}=3s.\ 4d.$

3. How many men can do as much work in .8 of a month, as 32 could do in 1.5 months?

mo. mo. men.
As .8 : 1.5 :: 32
40 40.
60 men. Answer.

4. If the carriage of 6*cwt.* 2*qr.* 14*lb.* cost $2.50, what should be paid for 1 *T.* 19*cwt.* 3*qr.*, at the same rate?

cwt. qr. lb. *T. cwt. qr.* $
As 6 2 14 : 1 19 3 :: 2.50
4 20 6
26 39 $15.00 Answer.
28 4
222 159
52 28
1272
318
4452
6

5. How many bushels of wheat, at $1.12 per bushel, can I buy for $81.76?

6. How many yards of linen may be bought for $38.40, when 8 yards cost $3.20?

7. If 19*doz.* chip hats cost $36.80, what is that per hat?

8. Bought 12 pieces of cloth, each 12 yards, at $1.40 per yard, what will they amount to?

9. What will 518*lbs.* of tea cost, if 90*lbs.* cost $47.70?

10. If 1000 feet of boards cost $5.50, what will 37000 feet come to?

11. What does a man receive in a year, at $2.40 per day?

12. What will be paid for 2 casks of wine, containing 116 gallons, at 90 cents per gallon?

13. What is 1*lb.* of iron worth, at $5.60 per *cwt.*?

14. If 16 gallons of oil are sold for $20.00, what will be the price of 5 barrels, each containing 36 gallons?

15. What are 8 bales worth, if 50 bales cost $2700?

16. How much oil will $180 buy, if 25*gals.* cost $22.50?

17. At 49 miles in 7 days, how far can I travel in 84 days?

18. If I travel 36 miles every 2 days, how long before I shall have travelled 252 miles?

19. What will 76*doz.* of wine cost, at $450 for 50*doz.*?

20. If 74 pieces cost $5476, what are 36 pieces worth?

21. What is the price of 50 boxes of sugar, allowing $20.50 for every 3 boxes?

22. If 450*bu.* of rye cost $490, what are 54*bu.* worth?

23. How much land at $2.50 per acre, should be given in exchange for 360 acres, at $3.75 per acre?

24. A captain of a ship is provided with 18000 pounds of bread, for 150 seamen, of which each man eats 4 pounds per week. how long will it last them?

25. How long will 2295 pounds of beef last 45 seamen, if they each get 1 pound 3 times a week?

26. How many yards of cloth 3 quarters wide, will equal 30 yards 5 quarters wide?

27. Bought 4 pieces of linen, two of which contained 26½ yards each, and each of the other two 25 yards: what will it come to at 44 cents per yard?

28. A bankrupt compounded with his creditors, at 37 cents in the dollar: what will the merchant receive to whom he owes $1324 dollars?

29. If 350 men consume 70 barrels of provision in 9 months, how much will 500 men consume in the same time?

30. What are $7\frac{3}{8}$ yards of cloth worth, at \$5.50 per yard?

31. Suppose 120 seamen are provided with 3600 gallons of water for a cruise of 4 months, each month 30 days: how much will each man's share be per day?

32. How much per day may a person spend, to save \$1200 of his annual income of \$3858.24?

33. If a man's annual income be \$1533, and he expends daily \$2.25, how much will he save at the end of a year?

34. What is a clerk's weekly pay whose salary is 1000 dollars a year, allowing 52 weeks to the year?

35. The earth being 360 degrees in circumference, turns on its axis in 24 hours: how far are the inhabitants of the equator carried in five minutes, allowing a degree to be 60 geographical miles?

36. A ship's crew of 16 men is on allowance of 6 ounces of bread per day, when meeting with a vessel they are supplied with 2 cwt. of bread: what addition will this make to their daily allowance, if their voyage will last 28 days?

37. A merchant bought 5 tuns of wine, cost \$1767, by misfortune of one pipe staving, he lost 120 gallons: how must he sell it per gallon to get the first cost?

38. What is the value of 7 cheeses, each weighing 26*lb.* 12*oz.* at 8 cents per pound?

39. If 1 ounce of fine gold is sold for \$20, what will 15 ingots come to, each weighing 14*lb.* 7*oz.* 3*dwt.*?

40. If 1 ounce of silver be worth \$1.25, what is the price of a tankard that weighs 1*lb.* 1*oz.* 10*dwt.*?

41. If keeping of one horse cost 44 cents per day, what will be the cost of keeping 11 horses for one year?

42. How many yards of paper, 3 quarters wide, will be sufficient for a room of 210 square yards?

43. How much plank of 8 inches wide, will be sufficient to floor a room 21 feet by 12 feet?

44. If 30 men can perform a piece of work in 11 days, how many men will accomplish another piece of work, 4 times as large, in a fifth part of the time?

45. What will be the value of 5 hogsheads of sugar, each weighing 7*cwt.* 3*qr.* 27*lb.* at \$8.96 per hundred weight?

46 A cubic inch of lead weighs nearly 6*oz.* 9*dr.*: how many cubic inches are there in 20 hundred weight?

47. If a traveller go 160 miles in 7 days, when the days are 16 hours long, how many days will he go the same journey, when the days are 12 hours long?

48. If 144*lb.* avoirdupois be equal to 175*lb.* troy, how much troy weight is equal to 120*lb.* avoirdupois?

49. If 60 gallons of water, in one hour's time, fall into a cistern containing 200 gallons; and by a pipe in the same cistern there runs out 45 gallons in an hour: in how many hours will it be filled?

50. If a shadow 6 feet in length is cast by a pole 14 feet high, how high is that steeple which at the same time casts a shadow 90 feet long?

51. If 74 men had provision for 35 days; but after 5 days, 20 men were sent away: how long will the provision last the remaining 54?

52. If a man complete a journey of 25 days at 9 hours' walking each day: how long would he have been, if he had travelled 13 hours a day?

53. If a stick 3 feet high project a shadow 2 feet 8 inches, what must be the height of a tower whose shadow measures 74 feet?

54. At 7½*cts.* per *lb.* what will 10*cwt.* 2*qr.* 16*lb.* of rice come to?

55. Suppose 50 men to finish a piece of work in 48 days, how many are sufficient to finish it in 32 days?

56. If 18 men can perform a piece of work in 28 days, how many men can do it in one-fourth part of the time?

57. Suppose 60 men to complete a piece of work in 7½ months, how many could finish it in 9 months?

58. My year's rent of 350 acres of land is $500, and sundry taxes $27.50: what does my farm bring me per acre?

59. If $200 in 12 months gain $10 interest, what will $750 gain in the same time?

60. How many yards of stuff, 3 quarters wide, are equal in measure to 36 yards of 7 quarters wide?

61. What must be the width of a court yard which is 50 feet long, to be equal in measure to another that is 125 feet long by 20 feet broad?

62. If 12 inches long require 12 inches broad to make a square foot, what length will 8 inches broad require?

63. How many yards of paper, 27 inches wide, will hang a room that measures 60 feet round and 9 feet high?

64. If 220 yards in length, and 22 in breadth, make an acre, what must be the length when the breadth is 36 yards?

65. If the carriage of 37 cwt. for 56 miles come to $26, how far can I have 259 cwt. carried for the same sum?

66. If 5000 yards of cloth, 5 quarters wide, will make coats for 2000 men, how many yards of shalloon, of 3 quarters wide, will line them?

67. If 49*cwt.* 2*qrs.* are carried 150 miles for a certain sum, what distance would 198*cwt.* be carried for the same?

68. If $14\frac{1}{4}$ cwt. be carried 200 miles for $32, how many pounds can I have carried 72 miles for the same money?

69. If $\frac{4}{7}$ ounces cost $\frac{11}{12}$ £, what will 1 ounce cost?

70. If $\frac{6}{13}$ of a sloop cost $420, what is $\frac{5}{12}$ of her worth?

71. If $\frac{5}{8}$ of a *gal.* of wine cost $\frac{5}{8}$ what will $\frac{5}{9}$ of a tun cost?

72. A merchant bought $3\frac{1}{2}$ pieces of silk, each containing $24\frac{1}{3}$ yards, at 45 cents per yard: what does the whole come to?

73. A person having $\frac{3}{5}$ of a coal mine, sells $\frac{3}{4}$ of his share for $342: what is the whole mine worth?

74. If $\frac{1}{8}$ of a ship be worth $560, what part of her may be purchased for $1920?

75. If $3\frac{1}{2}$ times $3\frac{1}{2}$ pound cost $1\frac{1}{2}$ times $1\frac{1}{2}$ dollars, what is the value of $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of 392 pounds?

76. If $3\frac{1}{4}$ yards of cloth, that is $1\frac{1}{3}$ yards wide, be sufficient to make a cloak, how much Persian, which is but $\frac{4}{7}$ yards wide, will be required to line it?

77. In exchanging $20\frac{1}{2}$ yards of cloth, of $1\frac{1}{4}$ yards wide, for some of the same kind of $\frac{3}{4}$ yards wide, what quantity of the latter makes an equal barter?

78. If the penny loaf weigh 14 ounces, when wheat was 4 shillings per bushel, what must it weigh when wheat is at 7 shillings per bushel?

79. Bought 5.8 tuns of oil for $338.64, whereof 50.6 gallons leaked out; how much must the remainder be sold for per gallon, that the purchaser may be no loser?

80. A lends B $100\frac{3}{8}$ dollars for $6\frac{3}{8}$ months; what sum should B lend A for $3\frac{1}{6}$ years to requite his kindness?

81. If 1.47*cwt.* of sugar cost $6.86, what are 1.2*lbs.* worth?

82. What are 7.8*yds.* worth, if .5 of a yard cost $4.50?

83. What is the value of 443.875 barrels of flour, if 5.75 barrels cost $30?

84. If 21 men have built a wall in 1.75 days, how many men must be employed to build it in .25 of a day?

85. There are two numbers to one another as 16 to 17, the lesser is 65.525, what is the greater?

86. If a pasture serve 36 cows for 75 days, how many cows would eat it in 25 days?

87. Of a piece of cloth, containing 20.125 yards, a part was sold for $5.46; what quantity remained, supposing the whole to be worth $18.35?

88. If a ball of 18*lb.* be shot from a cannon with such a force as to cause it to move 100*ft.* in a second, how fast would a 24*lb.* ball move, driven by the same force?

89. If a ship sail 180¾ miles in 15 hours, how many hours will it require to sail 60 miles?

90. If 12.8*yds.* cost $10.125, what cost 5.36 E. English?

91. If the earth performs its diurnal revolution in 24 hours, in what time does it move through 1 degree?

92. If a common field will feed 520 sheep 90 days, how long will it feed 600 sheep?

93. Suppose a haymow to be sufficient for 40 head of cattle 18 weeks, how long would it serve 60 head of cattle?

94. If 1000 men, in a garrison, have provision for 6 months, how long would the same provision last 1500 men?

95. If a certain number of men can throw up an entrenchment in 9 days, when the days are 16 hours long, what time will it take when the days are 12 hours?

96. If a person can perform a journey in 6 days, riding 9 hours each day, how long will it take him if he rides 12 hours a day?

97. How many perches in length, with 12 in breadth, must I receive in exchange for 40 perches in length and 18 in breadth?

98. There are two rooms, the floors of which have an equal number of square feet; the one is 50 feet by 30, the other is 40 in length: what is the breadth?

99. How many yards of paper, 3 quarters wide, will cover a chamber that is 60 feet round, and 10 feet 1½ inches high?

100. How much stuff, 2½ quarters wide, will face 15 yards of silk 3 quarters wide?

101. How many yards of brown drugget that is a yard and half wide, will cover a room that is 15 feet long and 14 broad?

102. If for a certain sum I can have 16*cwt.* 2*qr.* carried fifty miles, what distance would 66*cwt.* be carried for the same money?

103. A hare pursued by a greyhound was 86*yds.* before him at starting; whilst the hare ran 5*yds.* the dog ran 7*yds.* how far had the dog ran when he caught the hare?

104. If 1.9703 acres cost $236.84, for how much can 874.695487 acres be purchased?

THE DOUBLE RULE OF THREE.

§ **104.** The double rule of three teaches to resolve questions which require to be stated more than once by the single rule of three.

RULE.

Put in the third place the term which is of the same kind as the answer.

Proceed with each pair of terms exactly as in the single rule of three. Multiply all the first terms together for a general first term, and all the second terms together for a general second term.

Multiply the second and third terms together, and divide by the first.

EXAMPLES.

1. If 4 men earn $12 in 7 days, how long will 14 men be earning $30?

m.		*m.*	
As 14	:	4	} : : 7*d.*
$		$	
12	:	30	
168	:	120	

$$168\overline{)840}(5 \text{ Ans.}$$
$$840$$

Or by cancelling:

Thus, draw a line and placing all the first terms below it, and all the second and third terms above it, and dividing any of them, of opposite sides, by any number which will divide both without a remainder, as $\frac{4\times30\times7}{14\times12}$, the numbers 14 and 4 being both divisible by 2 without a remainder, will be reduced to 7 and 2; and 12 and 30 being divisible by 6, will be reduced to 2 and 5.

The question then stands $\frac{\not2\times5\times\not7}{\not7\times\not2}=5$ Ans.

The 7 above and the 7 below the line being on opposite sides, destroy each other, as do also the 2 above and the 2 below. The only remaining figure 5 will therefore be the answer.

2. A cellar which is 22.5 feet long, 17.3 wide, and 10.25 deep, being dug in $2\frac{1}{2}$ days, by 6 men, working 12.3 hours a day: how many days of 8.2 hours, should 9 men take to excavate one which measures 45. by 34.6 by 12.3 feet?

As 22.5 feet : 45 feet
17.3 feet : 34.6 feet
10.25 feet : 12.3 feet
9. men : 6 men
8.2 hours : 12.3 hours
} : : 2.5 days.

By cancelling:

$$\text{Thus } \frac{\cancel{4.5}\times\cancel{34.6}\times\cancel{12.3}\times\cancel{6}\times\cancel{12.3}\times\cancel{2.5}}{\cancel{22.5}\times\cancel{17.3}\times\cancel{10.25}\times\cancel{9}\times\cancel{8.2}} = 2\times2\times3=12 \text{ Ans.}$$

3. If 50*l.* in $\frac{5}{12}$ of a year gain 2*l.* 5*s.* 1*d.* $2\frac{2}{3}$*qr.*, in what time will $13\frac{1}{3}l.$ gain $1\frac{1}{12}l.$, and at what rate per cent. per annum?

£		£	
As $13\frac{1}{3}$	:	50	} : : $\frac{5}{12}y.$
3			
40			
£ s. d. qrs.		£	
2 5 1 $2\frac{2}{3}$	:	$1\frac{1}{12}$	
20		12	
45		13	
12		20	
5 41		260	
4		12	
2166		3120	
3		4	
6500		12480	
3		12	

Thus $\frac{3}{40}\times\frac{3}{6500}\times\frac{50}{1}\times\frac{12480}{12}\times\frac{5}{12}=\frac{2808000000}{3744000000}=\frac{3}{4}y.$ Ans.

Or cancelled,

$$\frac{\cancel{3}}{\cancel{40}}\times\frac{\cancel{3}}{\cancel{6500}}\times\frac{50}{1}\times\frac{\cancel{12}\cancel{48}\cancel{0}}{\cancel{12}}\times\frac{\cancel{5}}{\cancel{12}}=\frac{50\times78}{1300\times4}=\frac{3900}{5200}=\frac{3}{4}y. \text{ Ans.}$$

1300 78 4

Again, as £ $13\frac{1}{3}$: £ 100 } : : $1\frac{1}{12}$

$\frac{3}{4}$ y. : 1 y.

Thus, $\frac{\not{3}}{\not{4}\not{0}} \times \frac{\not{4}}{\not{3}} \times \frac{\not{1}\not{0}\not{0}}{1} \times \frac{1}{1} \times \frac{13}{\not{1}\not{2}} = \frac{130}{12} = 10\frac{1}{6}$ per ct. Answer.

4. If 15 men eat 36 cents' worth of bread in 6 days, when wheat is sold at 90 cents per bushel, how many days will 30 men require to eat 160 cents' worth, when wheat is sold at 60 cents per bushel?

As *m.* 30 : *m.* 15
cts. 36 : *cts.* 160
cts. 60 : *cts.* 90 } : : 6*d.*

Thus, $\frac{\not{1}\not{5} \times \not{1}\not{6}\not{0} \times \not{9}\not{0} \times \not{6}}{\not{3}\not{0} \times \not{3}\not{6} \times \not{6}\not{0}} = \frac{40}{2} = 20$ days. Answer.

5. If 12 cows produce 240 lbs. of butter in 4 weeks, how many cows may be supposed to yield 20 lbs. in 1 week?

6. If 30 acres of grass be mown by 6 men in 5 days, how many acres can be mown by 50 men in 20 days.

7. A park wall, 1236 yards in length, was to have been built by 60 men in 21 days; but at the end of 15 days, finding only 824 yards completed, how many more must be employed to finish it in the given time?

8. If 9 carpenters in 12 days lay 36 squares of flooring, how many squares will 12 men lay in 10 days?

9. If 25 reapers cut down a field of oats of 50 acres in 6 days, how many acres will 16 reapers reap in 14 days?

10. If 20 reapers cut down in 5 days 36 acres of wheat, in what time will 50 reapers cut down 180 acres?

11. If the freight of a canal boat, of 30 tons burden, for 7

months be $175, what ought to be paid for a vessel of 50 tons, for 4 months, at the same rate?

12. If 34 men perform a piece of work in 27 days, working 7 hours a day, in what time will 27 men do the same, when they work 17 hours a day?

13. If 260 stones, 15 inches by 10, pave 30 square yards, how many stones, 24 inches by 18, will pave 50 square yards?

14. Suppose 4 men in 12 days mow 48 acres, how many acres can 8 men mow in 16 days?

15. If 12 oxen in 16 days eat 20 acres of grass, how many acres will serve 24 oxen 48 days?

16. If 10 bushels of oats be sufficient for 18 horses 20 days, how many bushels will serve 60 horses 36 days, at that rate?

17. If 2 men can do 12 rods of ditching in 6 days, how many rods may be done by 8 men in 24 days?

18. If the carriage of 8 cwt., 128 miles, cost $6.40, what must be paid for the carriage of 4 cwt. 32 miles?

19. If 200 lbs. be carried 40 miles for 40 cents, how much must be paid at that rate for the carriage of 20200 lbs. 60 miles?

20. If the freight of 9 hogsheads of sugar, each weighing 12 cwt., for 20 miles, cost $18, what must be paid for the freight of 50 hogsheads of sugar, each weighing $2\frac{1}{2}$ cwt. 100 miles?

21. If a family of 9 persons spend £120 in 8 months, how much will serve a family of 24 persons, 16 months?

22. If 8 men can dig 24 yards of earth in 6 days, how many men must there be to dig 18 yards in 3 days?

23. If a person travel 300 miles in 10 days, when the days are 12 hours long, in how many days may he travel 600 miles, when the days are 16 hours long?

24. If 2 men can do $12\frac{3}{4}$ rods of ditching in $6\frac{1}{2}$ days, how many rods may be done by 18 men in 14 days?

25. If 5 persons drink $7\frac{4}{5}$ gallons of beer in a week, what quantity will serve 8 persons $22\frac{1}{2}$ weeks?

26. Fourteen persons, upon examining into their expenses for 20 weeks past, found they had laid out £$40\frac{4}{5}$; in what time, at the same rate, would £$20\frac{3}{7}$ be expended by 46 persons?

27. When 12 persons use $1\frac{1}{8}$ lb. of tea per month, how much should a family of 8 persons provide for $\frac{1}{2}$ a year?

28. A person put out $186 to receive interest for the same; and when it had continued 8 months, he received for principal and interest $193.44: Required the rate per cent. per annum of interest received?

29. When 12 oxen graze down 16.25 acres in 20 days, how much of like pasture would serve 24 such cattle 100 days?

30. If 3 men receive $8.90 for 19.5 days' labour, how much must 20 men have for 100.25?

31. How many men should reap 417.6 acres in 12 days, when 5 men cut down 52.2 acres in 6 days?

32. Suppose the interest of $76.94, for 9.5 months, to be $15.25, what sum will gain $6 in 12.75 months?

33. If 180 men, in 6 days of 12 hours each, build a wall 600 feet long, 2 wide, and 3 high, in how many days will 100 men, working at the same rate, but only 8 hours every day, build a wall 1080 feet long, 3 wide, and 4 high?

34. If a canal 1500 yards long, 6 yards wide, and 3 yards deep, be dug in 30 days of 12 hours each, by 280 men, how many men would be required to dig another canal 3 miles long, 4 yards wide, and 2 yards deep, in 125 days of 10 hours each?

35. If 30 bricklayers build a garden wall in 4 months, working 10 hours a day, how long will 40 take to build the same, working only 8 hours a day?

36. A garrison of 1500 men have provision for 12 weeks, at the rate of 20*oz.* per day to each man: how many men will the same provisions maintain for 20 weeks, allowing each man only 8 ounces per day?

37. An individual contracted to make 240 yards of a road in 4 weeks, and for that purpose hired 30 men, but at the end of 8 days he found he had executed only 60 yards: how many additional men must be employed to finish the work in the stipulated time?

38. If 15 lads of 18 years of age do a piece of work in 60 days, in what time will 9 men, 24 years old, do the same, supposing their strength to increase in the same proportion as their age?

39. If the expenses of 9 horses for oats be $45 for $4\frac{1}{2}$ months, when the price of oats is $36\frac{2}{3}$ cents per bushel, how many horses will, at that rate, consume the value of $216 in 6 months, when oats sell at 32 cents?

40. If 30 men in 40 hours dig 80 cubic yards, how many yards will 80 men, stronger in the proportion of 5 to 4, dig in 90 hours, supposing the ground in this case harder than the former, in the ratio of 9 to 8?

41. It was computed that 200 men would fortify a camp, with a ditch and rampart, in 60 days, working 14 hours a day; but the general, fearing an attack from the enemy sooner than he expected, ordered 150 men more to be employed, and to work 16 hours a day, after the others had wrought 14 days: in what time will the fortification be finished?

42. If a footman, when the days are 14 hours long, can travel 276 miles in 16 days, in how many days can he travel 852 miles, when the days are but 12 hours long?

43. A person having engaged to remove 8000*cwt.* a certain distance in 9 days, with 18 horses, in 6 days he removed 4500*cwt.*: how many horses will be required to remove the remainder in the remaining 3 days?

44. A farmer having sown 48 bushels, found that it produced 576 bushels the first year; now, supposing he sows 240 bushels of grain each year for 6 years successively, what will be his whole increase at the expiration of the last year?

45. If a regiment of soldiers, consisting of 939 men, can eat 351 bushels of wheat in 7 months, how many soldiers will eat 1464 bushels in 5 months, at that rate?

46. If 248 men in 5 days, of 11 hours each, can dig a trench 230 yards long, 3 wide, and 2 deep, in how many days, of 9 hours long, will 24 men dig a trench of 420 yards long, 5 wide, and 3 deep?

47. A person undertook to perform a piece of work in 8 days with 12 men, but at the end of the 6th day, found $\frac{2}{5}$ of it unfinished: how many more must he employ to finish it at the set time?

48. Suppose $240 will defray the expenses of 5 men for 22 weeks and 6 days: how long will 12 men be spending $360?

49. If $240 principal, in 12 months gain $19.20 interest, what principal will gain $20.64 in 5 months?

50. If 12 inches in length, 12 in breadth, and 12 in thickness, make a solid foot, what length of a plank, that is 6 inches broad and 4 thick, will make the same?

§ 105. QUESTIONS TO BE PERFORMED BY ANALYSIS.

All questions in the rule of three resolve themselves into cases of the multiplication of fractions.

By the following method of proceeding all the difficulties of the rule of three, direct and inverse, are avoided, and the first question is to find the unit, the price of one, after which the whole operation becomes clear.

1. Thus, if 9 lbs. of sugar cost 80 cents, how much will 7 lbs. cost?

By the conditions of the question 1 lb. of sugar cost $\frac{80}{9}$.

Therefore $\frac{80}{9}$ = the price of 1 lb.:

and $\frac{80\times7}{9}$ = the price of 7 lbs.:

consequently $\frac{560}{9}=62\frac{2}{9}$ cents. Answer.

2. If 13 men could do a piece of work in 6 days, how long would it take 16 men to do it?

In how many days could 1 man do this work? In 13×6.

Therefore 16 men could do it in $\frac{1}{16}$ part of the time.

Then $\frac{13\times6}{16}=4d.\ 10h.\ 30m.$ Answer.

3. If I could buy 3 boxes for 8 cents, how many can I get for 14 cents?

How many boxes can I get for 1 cent? $\frac{3}{8}$ of a box.

Therefore for 14 cents I can get $\frac{42}{8}$.

which $=5\frac{1}{4}$. Answer.

4. If $\frac{9}{10}$ of a pound of tea cost $\frac{2}{5}$ of a dollar, how much would $\frac{3}{8}$ of a pound cost?

One pound cost $\frac{2\times10}{5\times9}=\frac{4}{9}$.

Therefore $\frac{3}{8}$ of it cost $\frac{4}{9}\times\frac{3}{8}=\frac{1}{6}$ of a dollar $=16\frac{2}{3}$ cents

5. If $\frac{3}{5}$ of a house is worth $\frac{7}{8}$ of a farm, what portion of the house would half the farm be worth?

As the answer is in terms of the house, the farm must be the unit sought.

Therefore, what is the farm worth in terms of the house? $\frac{24}{35}$. Thus $\frac{7}{8}$ farm $=\frac{3}{5}$ of house.

Multiply both by $\frac{8}{7}$:

Therefore $\left(\frac{8\times7}{7\times8}\right)$ one farm $=\frac{8\times3}{7\times5}$ house.

Consequently $\frac{1}{2}$ farm $=\frac{24}{35}\times\frac{1}{2}=\frac{12}{35}$ of the house.

6. A fort, which had been provisioned for 9 months, was besieged, and on taking account of the state of the troops and provisions, on a certain day, the general found that $\frac{2}{5}$ of the men had been killed, and $\frac{1}{2}$ the provisions were expended: how much longer would the provisions last?

The provisions left $=\frac{9}{2}$ months for the whole garrison.

The men $=\frac{3}{5}$ of the whole garrison.

And the provisions divided among them would be

$$\frac{9}{2}\times\frac{5}{3}=\frac{45}{6}=7\frac{1}{2} \text{ months.}$$

7. If 4 yards of cloth cost $12.80, what will 96 yards cost?

8. How many yards of cloth may be bought for $307.20, when 4 yards cost $12.80?

9. If 38 yards of silk cost $51.50, how much will 435.5 yards come to?

10. At $6.5 per 100 lbs., what is 2728 lbs. worth?

11. If 2728 lbs. of beef cost $177.32, what will 200 lbs. cost?

12. If $6\frac{1}{4}$ cents buy 1 pound of sugar, how many pounds can be purchased with $\$76\frac{1}{16}$?

13. Bought $27\frac{3}{4}$ yards, at $.355 per yard, what sum must I pay?

14. If 365 men consume 150 barrels of beef in 9 months, how much will 1000 men consume in the same time?

15. If $1\frac{1}{4}$ yards of silk cost $1.08, what is the value of $16\frac{1}{4}$ yards?

16. A person having $\frac{4}{7}$ of a ship, sells $\frac{3}{5}$ of his share for $1531.2, what is the whole ship worth?

17. If $\frac{7}{16}$ of a vessel cost $3489, what is the whole worth?

18. If $2\frac{1}{2}$ cwt. of cheese cost $30, what will 15 cwt. cost?

19. If 8 men can do a piece of work in 16 days, in what time will 12 men do it?

20. How much in length, that is 8 perches in breadth, must be taken to contain an acre?

21. How much in length, that is $4\frac{1}{2}$ inches broad, will make a square foot?

22. How many yards of carpeting, $2\frac{1}{2}$ feet broad, will cover a floor 27 feet long and 20 broad?

23. How many yards of cloth, 3*qrs.* wide, are equal in measure to 30 yards, 5*qrs.* wide?

24. How many yards of paper, 1.25 yards wide, will paper a room 20 yards in circumference and 4 in height?

25. If $\frac{3}{8}$ of a yard cost $\frac{5}{8}$ of a dollar, what will $\frac{3}{5}$ of a yard cost?

26. What part of $\frac{12}{37}$ is $\frac{7}{11}$?

27. If 3 men do a piece of work in $4\frac{1}{2}$ hours, in how many hours shall 10 men do the same work?

28. If $2\frac{3}{4}$ yards cost $2.16, what is 1 yard worth?

29. If $\frac{2}{3}$ of $\frac{3}{5}$ of a yard of cloth is worth 84 cents, what is 1 yard worth?

30. If 1*lb.* of spice cost $\frac{7}{9}$ of a dollar, what will $\frac{3}{4}$ of an ounce cost?

31. Bought $9\frac{1}{2}$ pieces of cloth, each containing $44\frac{3}{4}$ yards, at $7\frac{1}{2}$ per yard: what do they come to?

32. How much sugar, at $4 per *cwt.*, can I buy for $3.16?

33. If 9*lb.* of lead cost 11.5 cents, what will 3 fothers, each 19.5*cwt.* come to?

34. If 2.625 tons cost $36.75, what is the value of 1.5*lb.*?

35. If 5 men earn $38.525 in 1 week, what will 18 men earn in the same time?

36. If 6 men reap 18 acres of wheat in 5 days, how many acres will 10 men reap in 12 days?

37. If 8 persons spend $100 in 4 months, how much will 20 persons spend in 6 months?

38. If the carriage of 5*cwt.* for 48 miles be 7*s.* 6*d.*, what will be the carriage of 15*cwt.* for 24 miles?

39. If $144 be the wages of 36 men for 9 days, what will be earned by 12 men in 90 days.

40. If a person travel 240 miles in 7 days, when the days are 14 hours long, in how many days of 7 hours each will he travel 120 miles?

41. If 14 men can dig 360 cubical yards of earth in 5 days, how many men can dig 144 cubical yards in 7 days?

42. If 75 men can throw up an entrenchment in 5 days, when the days are 12 hours long, in what time will 50 men do it when the days are 18 hours long?

43. If a barrel of ale will last a family of 6 persons 2 months, how many persons would drink 9 barrels in a year?

44. Suppose 84 gallons of brandy will serve 220 seamen 8 days, how much will 380 seamen drink in 12 days?

45. If $150 in 12 months gain $6\frac{3}{4}$ dollars, what will be the interest of $700 for 7 years?

46. If 7 horses eat 25 bushels of oats in 10 days, how many horses will consume 100 bushels in 14 days?

47. Lent a friend $800 for 9 months, at 5 per cent., how long ought he to lend me $1250 at 4 per cent. to requite my kindness?

48. If 6 horses plough 10 acres of land in 5 days, how many horses will plough 16 acres in 12 days?

49. How many bushels of wheat will serve 54 persons 14 days, when 3 bushels will serve 6 persons 21 days?

50. If 756 bricks, 14 inches long and 10 broad, will pave a floor, how many bricks would it take, 15 inches long and 2 inches broad?

51. If $14\frac{3}{4}$ rods of wall, $2\frac{1}{2}$ feet deep, can be built by a certain number of men in $25\frac{1}{2}$ days, how long will the same number of men be in building $34\frac{4}{5}$ rods of wall $3\frac{1}{8}$ feet deep?

52. If $2\frac{1}{4}$ yards of cloth $1\frac{3}{5}$ yards wide, cost $\$3\frac{3}{5}$, what is the value of $38\frac{1}{4}$ yards, 2 yards wide?

53. How many men will reap 417.6 acres in 12 days, if 5 men reap 52.2 acres in 6 days?

PRACTICE.

§ 106. Practice, so called from its general use among merchants and tradesmen, is a short method of ascertaining the value of any quantity of goods or other commodities.

All questions in this rule might be worked by multiplication or the rule of three; but they are here more readily performed by taking aliquot parts.

Aliquot means a part which will divide without a remainder; thus, 25 cents is an aliquot part of a dollar.

TABLE OF ALIQUOT PARTS.

Of a dollar.		Of a shilling.		Of a pound.		Of a ton.		Of a cwt.	
Cts.	$	6*d.* =	$\frac{1}{2}$	10*s.* =	$\frac{1}{2}$	10*cwt.* =	$\frac{1}{2}$	2*qr.* =	$\frac{1}{2}$
$6\frac{1}{4}$ =	$\frac{1}{16}$	4	$\frac{1}{3}$	6 8*d.*	$\frac{1}{3}$	5	$\frac{1}{4}$	1	$\frac{1}{4}$
$8\frac{1}{3}$	$\frac{1}{12}$	3	$\frac{1}{4}$	5	$\frac{1}{4}$	4	$\frac{1}{5}$	0 16*lb.*	$\frac{1}{7}$
$12\frac{1}{2}$	$\frac{1}{8}$	2	$\frac{1}{6}$	4	$\frac{1}{5}$	2 2 *qr.*	$\frac{1}{8}$	0 14	$\frac{1}{8}$
$16\frac{2}{3}$	$\frac{1}{6}$	$1\frac{1}{2}$	$\frac{1}{8}$	3 4	$\frac{1}{6}$	2	$\frac{1}{10}$	0 8	$\frac{1}{14}$
20	$\frac{1}{5}$	1	$\frac{1}{12}$	2 6	$\frac{1}{8}$	1	$\frac{1}{20}$	0 7	$\frac{1}{16}$
25	$\frac{1}{4}$	Of a penny.		2	$\frac{1}{10}$	Of a pound.		0 4	$\frac{1}{28}$
$33\frac{1}{3}$	$\frac{1}{3}$	2 *far.*	$\frac{1}{2}$	1 8	$\frac{1}{12}$	8*oz.* =	$\frac{1}{2}$	Of a qr.	
50	$\frac{1}{2}$	1	$\frac{1}{4}$	1 4	$\frac{1}{15}$	4	$\frac{1}{4}$	14*lb.* =	$\frac{1}{2}$
Of a year.		Of a month.		1 3	$\frac{1}{16}$	2	$\frac{1}{8}$	7	$\frac{1}{4}$
6*m.* =	$\frac{1}{2}$	15*d.* =	$\frac{1}{2}$	1	$\frac{1}{20}$	1	$\frac{1}{16}$	4	$\frac{1}{7}$
4	$\frac{1}{3}$	10	$\frac{1}{3}$	Of an acre.		Of a yard.		$3\frac{1}{2}$	$\frac{1}{8}$
3	$\frac{1}{4}$	6	$\frac{1}{5}$	2*R.* =	$\frac{1}{2}$	2*qr.* =	$\frac{1}{2}$	2	$\frac{1}{14}$
2	$\frac{1}{6}$	5	$\frac{1}{6}$	1	$\frac{1}{4}$	1	$\frac{1}{4}$		
$1\frac{1}{2}$	$\frac{1}{8}$	3	$\frac{1}{10}$	20*p.*	$\frac{1}{8}$	2*n.*	$\frac{1}{8}$		
1	$\frac{1}{12}$	2	$\frac{1}{15}$	16	$\frac{1}{10}$	1	$\frac{1}{16}$		
		1	$\frac{1}{30}$						

§ **107.** CASE 1. When the price is less than a penny.

RULE. Divide the given number by the aliquot parts of a penny, and then by 12 and 20 for the answer.

EXAMPLES.

1. 6156 at $\frac{3}{4}d.$

$\frac{1}{2}$ $\frac{1}{2}$	6156
$\frac{1}{4}$ $\frac{1}{2}$	3078
	1539
12)	4617
20)	38 4 9
	19 4 9 Ans.

2. 1397 at $\frac{1}{4}d.$
3. 4657 at $\frac{1}{2}d.$
4. 6120 at $\frac{3}{4}d.$
5. 39874 at $\frac{1}{2}d.$
6. 60455 at $\frac{1}{2}d.$
7. 83517 at $\frac{1}{4}d.$
8. 99876 at $\frac{1}{2}d.$
9. 664793 at $\frac{3}{4}d.$

§ **108.** CASE 2. When the price is cents and parts of a cent.

RULE. Multiply the quantity by the cents, and take parts for the parts of a cent.

EXAMPLES.

1. 3484 at $6\frac{3}{4}$ cents.

	3484
	6
$\frac{1}{2}$ $\frac{1}{2}$	20904
$\frac{1}{4}$ $\frac{1}{2}$	1742
	871
	235 17
	$ cts.

2. 3456 at $11\frac{1}{4}$ cents
3. 12684 at $6\frac{1}{2}$.
4. 12875 at $17\frac{1}{2}$.
5. 24872 at $18\frac{3}{4}$.
6. 16784 at $12\frac{1}{2}$.
7. 28764 at $9\frac{1}{4}$.
8. 21294 at $11\frac{3}{4}$.
9. 42247 at $18\frac{3}{4}$.

§ **109.** CASE 3. When the price is less than a shilling.

RULE. Divide the given number by the aliquot part or parts of a shilling, add them together, and divide by 20 for the answer required.

4*d.*	$\frac{1}{3}$	5518
$\frac{1}{2}$	$\frac{1}{6}$	1839 4
$\frac{1}{4}$	$\frac{1}{2}$	229 11
		114 $11\frac{1}{2}$
20)		2184 $2\frac{1}{2}$ *s*
		109 4 $2\frac{1}{2}$
		£ *s. d.*

2. 2345 at $2\frac{3}{4}d.$
3. 1342 at $3d.$
4. 4320 at $3\frac{1}{4}d.$
5. 5627 at $3\frac{1}{2}d.$
6. 4396 at $3\frac{3}{4}d.$
7. 5069 at $4d.$
8. 6908 at $4\frac{1}{4}d.$
9. 8005 at $4\frac{1}{2}d.$

10. 2759 at $4\frac{3}{4}d.$	16. 2468 at $6\frac{3}{4}d.$	22. 8765 at $10\frac{1}{4}d.$
11. 7952 at $5d.$	17. 5321 at $7\frac{1}{2}d.$	23. 6213 at $10\frac{3}{4}d.$
12. 6327 at $5\frac{1}{4}$d.	18. 6019 at $7\frac{3}{4}d.$	24. 5986 at $11d.$
13. 3254 at $5\frac{1}{2}d.$	19. 5068 at $8\frac{1}{2}d.$	25. 7328 at $11\frac{1}{4}d.$
14. 4968 at $5\frac{3}{4}d.$	20. 8271 at $9d.$	26. 4537 at $11\frac{1}{2}d.$
15. 3219 at $6d.$	21. 4564 at $9\frac{3}{4}d.$	27. 9765 at $11\frac{3}{4}d.$

§ **110.** CASE 4. When the price is any number of shillings under 20.

RULE. 1. When the price is an even number, multiply the given number by one half of it, doubling the first figure to the right hand for shillings, and the rest will be the pounds.

2. When the price is an odd number find for the greatest even number as before, to which add one-twentieth of the given number for the odd shillings, and the sum is the answer: or multiply the given quantity by the price, and divide by 20 for the answer; or work by aliquot parts.

EXAMPLES.

1. 3643 at 4*s.*

2

£728 12*s.* Answer.

3. 3731 at 7*s.*

7

2|0)2611|7

£1305 17*s.* Answer.

2. 2129 at 15*s.*

7

1490 6

106 9

£1596 15*s.* Answer.

4. 1954 at 12*s.*

10|½|1954

2|⅕|977

195 8

£1172 8*s.* Answer.

5. 6543 at 6*s.*	9. 3908 at 12*s.*	13. 1069 at 19*s.*
6. 3264 at 9*s.*	10. 4258 at 15*s.*	14. 12346 at 18*s.*
7. 7134 at 8*s.*	11. 8372 at 16*s.*	15. 98754 at 16*s.*
8. 4689 at 11*s.*	12. 3258 at 17*s.*	16. 96852 at 19*s.*

§ **111.** CASE 5. When the price is shillings and pence, or dollars and cents.

RULE. Multiply the given quantity by the shillings or dollars, and take parts for the pence or cents.

EXAMPLES.

1. 7150 yards at 1*s.* 8*d.* per yard.

		7150
		1
6*d.*	$\frac{1}{2}$	7150
2	$\frac{1}{3}$	3575
		1191 8
	2 | 0	1191 | 6 8

£595 16 8*d.* Ans.

2. 154 yards at $\$2.16\frac{2}{3}$.

$16\frac{2}{3}$	$\frac{1}{6}$	154
		2
		308
		25 $66\frac{2}{3}$

$\$333\ 66\frac{2}{3}$ Answer.

3. 6234 at 6*s.* 8*d.*	7. 6432 at 4*s.* 8*d.*	11. 317 at $\$3.33\frac{1}{3}$
4. 4327 at 5*s.* 9*d.*	8. 3000 at $\$1.12\frac{1}{2}$	12. 1864 at $\$1.12\frac{1}{2}$
5. 7890 at 3*s.* 4*d.*	9. 1478 at $\$2.62\frac{1}{2}$	13. 12345 at $1.56\frac{1}{4}$
6. 6974 at 9*s.* 6*d.*	10. 156 at $\$1.56\frac{1}{4}$	14. 20863 at $4.93\frac{3}{4}$

§ **112.** CASE 6. When the price is pounds, shillings, pence and farthings, or dollars and cents.

RULE. Multiply the quantity by the pounds or dollars, and take aliquot parts for the shillings, pence and farthings, or cents; then the sums added together will give the answer.

EXAMPLES.

1. 2496 at $\$9.08\frac{1}{3}$.

		2496
		9
$8\frac{1}{3}$	$\frac{1}{12}$	22464
		208
		$\$22672$

2. 4188 at £1 19*s.* $11\frac{3}{4}$*d.*

10*s.*	$\frac{1}{2}$	4188
5*s.*	$\frac{1}{2}$	2094
4*s.*	$\frac{1}{5}$	1047
6*d.*	$\frac{1}{8}$	837 12
3*d.*	$\frac{1}{2}$	104 14
2*d.*	$\frac{1}{3}$	52 7
$\frac{1}{2}$*d.*	$\frac{1}{4}$	34 18
$\frac{1}{4}$*d.*	$\frac{1}{2}$	8 14 6
		4 7 3

£8371 12 9*d.*

3. 299 at £12 2*s.* 6*d.*	8. 1645 at $\$6.85\frac{1}{2}$.
4. 4685 at £6 9*s.* 4*d.*	9. 3230 at $\$4.66\frac{2}{3}$.
5. 2397 at £10 15*s.* 10*d.*	10. 5230 at $\$9.62\frac{1}{2}$.
6. 3256 at £1 15*s.* $5\frac{1}{2}$*d.*	11. 2425 at $\$5.16\frac{2}{3}$.
7. 1436 at £5 5*s.* $5\frac{3}{4}$*d.*	12. 8624 at $\$8.08\frac{1}{3}$.

§ **113.** CASE 7. When both the price and quantity are of several denominations.

RULE. Multiply the price by the highest denomination, and take aliquot parts for the lower denominations. Add the results together for the answer.

EXAMPLES.

1. Bought 29*cwt.* 1*qr.* 11*lb.* of tea, at £5 9*s.* 4*d.* per *cwt.*

		£	*s.*	*d.*				
1*qr.*	¼	5	9	4×1				
				4×7+1=29				
		21	17	4				
				7	*cwt.*	*qr.*	*lb.*	
		153	1	4	=28	0	0	
		5	9	4	= 1	0	0	
7*lb*	¼	1	7	4	= 0	1	0	
4	¼	0	6	10	= 0	0	7	
		0	3	10¾	= 0	0	4	
		£160	8	8¾	=29	1	11	

2. Bought 16*A.* 2*R.* 25*p.* of land at $46.25 per acre: how much does it come to?

3. At £1 19*s.* 3*d.* per *cwt.* what is the value of 36*cwt.* 1*qr.* 7*lb.* of sugar?

4. 36*cwt.* 1*qr.* 17*lb.* at £3 18*s.* 6*d.* per *cwt.*

5. 29*cwt.* 1*qr.* 11*lb.* at £10 18*s.* 8*d.* per *cwt.*

6. 17*T.* 5*cwt.* 2*qr.* at £12 10*s.* 4*d.* per *ton.*

7. 17*gal.* 2*qt.* 1*pt.* at £1 6*s.* 4*d.* per *gallon.*

8. 34*A.* 2*R.* 20*poles* at £2 11*s.* 6*d.* per *acre.*

9. 25*T.* 14*cwt.* 2*qr.* 14*lb.* at £34 6*s.* 8*d.* per *ton.*

10. 476*A.* 3*R.* 28 *perches* at £3 7*s.* 11*d.* per *acre.*

11. 11*cwt.* 0*qr.* 14*lb.* at £4 16*s.* 9*d.* per *cwt.*

12. 45*T.* 17*cwt.* 2*qr.* 14*lb.* at $75.33⅓ per *ton.*

13. 25*cwt.* 3*qr.* 16*lb.* at $8.66⅔ per *cwt.*

14. 644*A.* 2*R.* 20*per.* at $10.50 per *acre.*

15. 36*cwt.* 2*qr.* 20*lb.* at $7.58⅓ per *cwt.*

16. 46*yd.* 3*qr.* 2*na.* at $4.56¼ per *yd.*

17. 184*yd.* 1*qr.* 3*na.* at $1.12½ per *yd.*

Q. What is practice?

Q. When the price is less than a penny how do you proceed?

Q. When the price is in cents and parts of a cent, how do you proceed?

Q. How do you proceed when the price is less than a shilling?

Q. How do you calculate when the price is any number of shillings under 20?

Q. How do you calculate when the price is shillings and pence, or dollars and cents?

Q. How do you proceed when the price is in pounds, shillings, pence and farthings?

Q. How do you calculate when both the price and quantity are of several denominations?

MISCELLANEOUS EXERCISES.

1. What cost 2348 yards, at $\frac{3}{4}d.$ per yard?
2. What is the value of 284 lbs. at $\frac{1}{2}d.$ per lb.?
3. What will 1234 lbs. of beef come to, at $6\frac{1}{2}$ cents per lb.?
4. What will 375 lbs. of coffee come to, at $13\frac{1}{2}$ cents per lb.?
5. What will 349 lbs. of sugar come to, at $6d.$ per lb.?
6. What will 324 yards come to, at $11d.$ per yard?
7. What cost 384 yards of cloth, at 19 shillings per yard?
8. What cost 249 lbs. of tea, at $4s.$ $8d.$ per lb.?
9. What cost 211 gallons of wine, at $\$1.17\frac{1}{2}$ per gallon?
10. What cost 984 yards, at £1 $3s.$ $4d.$ per yard?
11. What will 360 sheep come to, at $\$1.62\frac{1}{2}$ each?
12. Bought 420 ozs. 15 dwt. 16 grains of gold, at $10.25 per ounce?
13. How much will 25 acres, 2 roods, 25 poles of land cost, at $29 per acre?
14. Bought 9 tons, 19 cwt. 3 qrs. $27\frac{3}{4}$ lbs. of iron, at £39 $19s.$ $11\frac{1}{2}d.$ per ton.
15. What will 12 ounces of silk cost, if 1 lb. cost £3 $10s.$?
16. Sold a pair of silver buckles, weighing 50 dwt. 20 grs., at $17s.$ $6d.$ per ounce?
17. Bought 8 cwt. 1 qr. 16 lbs of tobacco, at £5 $17s.$ $9d.$ per cwt.
18. Bought sundry pieces of cloth, containing 1157 yds. 2 qrs., at $29s.$ $4\frac{1}{2}d.$ per yard.
19. What will 279 yds. 2 qrs. of scarlet cloth cost, at £3 $18s.$ $8d.$ per yard.
20. Bought 7 casks of wine, each containing 84 galls. 1 qt., at $11s.$ $3d.$ per gallon.

TARE AND TRET.

§ **114.** This rule teaches the method of deducting such allowances as are usually made by merchants and tradesmen in selling their goods; and the terms in general use are gross weight, tare, tret, suttle, and neat weight.

Gross weight is the whole weight of the goods, together with the box, barrel, bag, chest, &c. that contains them.

Tare is an allowance made to the buyer, for the weight of the box, barrel, &c. This is charged either at so much per box, &c., or at so much in the whole.

Tret is an allowance of 4*lb.* in every 104*lb.* (*i. e.* a 26th part of the whole) for waste, dust, &c.

Suttle weight is when part of the allowance is deducted from the gross.

Neat or net weight is that which remains after all allowances are made.

§ **115.** CASE 1. When the tare is so much in the whole.

RULE. Subtract the tare from the gross, and the remainder will be the neat weight.

EXAMPLES.

1. In 17 barrels of rice, weighing in the whole 36*cwt.* 3*qr.* 6*lb.* gross, and tare in the whole 3*cwt.* 0*qr.* 19*lb.*, how much neat weight, and what is the amount, at $4.25 per *cwt.*?

cwt.	*qr.*	*lb.*	
36	3	6	gross.
3	0	19	tare.
33	2	15	neat weight.

		4.25
		33
2	½	1275
14	¼	1275
1	1/14	140.25
		2125
		531
		37
		$142.94.3 Ans

2. If the gross weight of several bags of coffee be 31*cwt.* 1*qr.* 10*lb.*, and the tare 3*cwt.* 1*qr.* 16*lb.*, what is the neat weight, and the amount thereof, at $11.60 per *cwt.*

3. What is the neat weight of a quantity of sugar, if the gross is 19*cwt.* 2*qr.* 8*lb.*, and the tare 6*cwt.* 3*qr.* 18*lb.*, and what does it come to at $9.83 per *cwt.*

4. The gross weight of 3 loads of hay is 3 *T.* 2*cwt.* 1*qr.*, and tare in the whole 640*lb.*, how much neat weight, and what is the amount, at $8.92 per ton?

5. What is the neat weight of 5 casks of sugar, weighing as follows: No. 1, 4*cwt.* 2*qr.* 14*lb.* gross, tare 21*lb.* No. 2, 3*cwt.* 0*qr.* 17*lb.* gross, tare 18*lb.* No. 3, 5*cwt.* 3*qr.* 10*lb.* gross, tare 1*qr.* 11*lb.* No. 4, 6*cwt.* 1*qr.* 16*lb.* gross, tare 27*lb.* No. 5, 3*cwt.* 3*qr.* 18*lb.* gross, tare 19*lb.*? And what would be the value of the whole, the first three being worth $9.44 per *cwt.* neat, and the other two $9.64 per *cwt.* neat?

§ **116.** CASE 2. When the tare is at so much per box, bag, barrel, &c.

RULE. Multiply the number of boxes, &c. by the tare, and subtract the product from the gross, and the remainder will be the neat weight.

EXAMPLES.

1. What will be the neat weight of 36 bales of silk, weighing in the whole 74*cwt.* 0*qr.* 16*lb.* tare 17*lb.* per bale?

cwt.	*qr.*	*lb.*	
74	0	16	
5	1	24	tare.
68	2	20	neat weight.

17
36
102
51
28)612(21*qr.* = 5 1 24 tare.
56
52
28
24

cwt. qr. lb.

2. What is the neat weight and value of 14 bags of pepper, weighing in the whole 9*cwt.* 2*qr.* 13*lb.* gross, tare per bag 4*lb.* 8*oz.*, at $12.50 per *cwt.*?

3. What is the neat weight and value of 25 barrels of sugar, each weighing gross 5*cwt.* 3*qr.* 7*lb.*, and tare per barrel 2*qr.* 12*lb.*, at $9.10 per *cwt.*?

4. What is the neat weight and value of 21 hogsheads of tobacco, each weighing 3*cwt.* 1*qr.* 8*lb.* gross, and tare per hogshead 3*qr.* 10*lb.*, at $8.44 per *cwt.*?

5. Sold 7 casks of indigo, each weighing 4*cwt.* 3*qr.* 16*lb.* gross, and the tare per cask 1*qr.* 21*lb.*: what is the neat weight and value thereof, at 52 cents per pound?

§ 117. CASE 3. When the tare is at so much per cwt.

RULE. Divide the gross weight by the aliquot part or parts of a cwt., which subtract from the gross, and the remainder will be the neat.

EXAMPLES.

1. In 68 barrels of figs, each 126 lbs. gross; tare 12 lbs. per cwt.: how much neat weight? and what is the value thereof, at $7.25 per cwt.?

		lb.			*1qr.*	¼	$7.25
		126					68
		68					58.00
		1008					435.0
		756					493.00
8*lb.*	¼	8568	gross.		4*lb.*	¼	1.81.25
4*lb.*	½	612			2*lb.*	½	25,89
		306					12.94
		918	tare.				$495.20.08 Answer.
		7650	lb. neat=68*cwt.* 1*qr.* 6*lb.*				

2. Sold 36 barrels of sugar, each 2 cwt. 3 qrs. 24 lbs. gross, and tare 18 lbs. per cwt.: what is the neat weight, and what does it amount to, at $7.24 per cwt.?

3. In 21 casks of currants, each 2 cwt. 3 qrs. 18 lbs. gross, tare per cwt. 13 lbs.: how much neat, and what is the amount, at $8.16 per cwt.?

4. In 33 chests of tea, each 2 qrs. 24 lbs. gross, tare per cwt. 7 lbs.: how much neat weight, and what is the value thereof, at 59 cents per lb.?

5. What is the neat weight of 3 tierces of rice, each weighing 14 cwt. 2 qrs. 14 lbs. gross, tare 10 lbs. per cwt., and the value thereof, at $5.36 per cwt.?

§ 118. CASE 4. When both tare and tret are allowed.

RULE. Find the tare as before, subtract it from the gross, and the remainder or suttle, divided by 26, gives the tret, which subtract from the suttle; the remainder will be the neat weight.

EXAMPLES.

1. In 56 cwt. 0 qr. 26 lbs. of sugar, gross; tare 110 lbs.; tret 4 lbs. per 104 lbs.: how many lbs. neat, and what is the value thereof, at $7.16 per cwt.?

cwt.	*qr.*	*lb.*				
56	0	26	14*lb.*	$\frac{1}{8}$	$7.16	
4					53	
224					2148	
28					3580	
6298	gross.				37948	
110	tare.				89½	
26|6188	suttle.				$380.37½	Answer.
238	tret.					
5950	lbs. neat weight=53*cwt.* 0*qr.* 14*lb.*					

2. In 4 bags of coffee, each 2 cwt. 0 qr. 26 lbs. gross; tare 9½ lbs. per bag; tret 4 lbs. per 104 lbs.: what is the weight, and what is its value, at $11.13 per cwt.?

3. How much neat weight in 3 hhds. of tobacco, each 13 cwt. 3 qrs. 23 lbs.; tare 3 qrs. 23 lbs. per hhd.; tret 4 lbs. per 104 lbs.; and what is the value thereof, at 15 cents per pound?

4. In 17 chests of tea, each weighing 78 lbs. gross; tare in the whole 52 lbs.; tret 4 lbs. per 104 lbs.; how much neat weight, and what is the value thereof, at 57½ cents per pound?

5. What is the neat weight of 14 tierces of rice, each containing 5 cwt. 1 qr. 12 lbs. gross; tare 14 lb. per cwt.; and tret 4 lbs. per 104 lbs.; and what is the value of the same, at $4.94 per cwt.?

Q. What is tare and tret?
Q. What is gross weight?
Q. For what purpose is tare allowed?
Q. For what is tret allowed?
Q. What is the rule for finding the tare?
Q. What is suttle weight?
Q. How may tret be found?
Q. What is neat weight?

CUSTOM-HOUSE BUSINESS.

§ **119.** 1. Customs are duties charged upon commodities which are exported from, or imported into any country.

2. The custom-house is the house or office where merchandise is entered for exportation or importation, where the duties, drawbacks, &c., payable or receivable on such commodities, are paid or received, and where ships are cleared out, &c.

3. The duties on merchandise imported into the United States are either specific or *ad valorem* duties.

4. When duty is a certain sum paid on a ton, hundred weight, pound, square yard, foot, gallon, &c., it is denominated a *specific duty*.

5. An *ad valorem* duty is a certain per cent. or sum paid on the actual cost of the goods in the country from which they are imported.

Draft is an allowance for probable waste, made from the weight of each box, bag, &c., of goods, before the tare is computed.

EXAMPLES.

1. What is the duty on 2 ceroons of indigo, weighing gross as follows: 1st, 3*qr.* 19*lb.*; 2d, 3*qr.* 15*lb.*; draft, 1*lb.* per ceroon; tare, 10 per cent.; duty, 5 cents per pound?

3*qr.* 19*lb.* + 3*qr.* 15*lb.* = 202*lb.* gross weight.

202*lb.* — 2*lb.* draft and 20*lb.* tare = 180*lb.* nett. × 5 = $9,00.

2. What is the duty on 3*lb.* 3*oz.* sewing silk, at $2 per pound?

3. What is the duty on 76*lb.* 12*oz.* sheet copper, at 2 cents per pound?

4. What is the duty on an invoice of raw hides, the value of which is $148.50, at the rate of 5 per cent. ad valorem?

5. What is the duty on an invoice of 2 pieces of woollen cloth, the value of which is £29 6*s.* 0¾*d.* sterling, at 40 per cent. ad valorem?

For valuation of foreign moneys of account, in computation at custom-houses in the United States, see § 195, page 243.

6. What is the duty on an invoice of sheet tin, the value of which is £61 13*s.* 11*d.* sterling, at 2½ per cent. ad valorem?

7. What is the duty on an invoice of Manilla hemp, the gross weight of which is 58 *T.* 12*cwt.* 3*qr.* 20*lb.*; tare, 3 *T* 9*cwt.* 1*qr.* 10*lb.*; draft, 10*cwt.* 1*qr.* 16*lb.*, at $25 per ton?

INTEREST.

§ 120. Interest is the profit obtained by lending a sum of money for a certain time, at a fixed rate.

INTEREST IS EITHER SIMPLE OR COMPOUND.

SIMPLE INTEREST.

§ 121. Simple interest is that which is computed on the principal only.

The principal is the money lent.

The rate per cent. is the sum per cent. agreed to be paid for the use of the principal per annum. In most of the states the rate of interest is limited by law to 6 per cent., that is, six dollars for a hundred dollars. In New York, 7 per cent.

The amount is the principal and interest added together.

When no particular per cent. is mentioned, 6 per cent. is to be understood.

GENERAL RULE.

Multiply the principal by the rate per cent., and divide the product by 100, the quotient will be the interest for one year, which, multiplied by the years given, will be the interest for the required time; if there be months and days, take aliquot parts of the interest for a year, add the interest to the principal, and their sum will be the amount. Or,

Let the per cent. be considered the decimal of a hundred dollars, and multiply the principal by it. Thus, .06 is the decimal for 6 per cent., and .07 is the decimal for 7 per cent.

If the given rate be any other than 6 per cent., add or subtract such part as the given rate exceeds or falls short of 6 per cent.

EXAMPLES.

1. What is the interest of $842 for 2*yr.* 9*m.* 15*d.*?

		$842		
		.06		
6*m.*	$\frac{1}{2}$	50.52	Interest for 1 year.	
		2		
		101.04	"	" 2 years.
3*m.*	$\frac{1}{2}$	25.26	"	" 6 months.
15*d.*	$\frac{1}{6}$	12.63	"	" 3 months.
		2.10.5	"	" 15 days.
		$141.03.5	Answer.	

2. What is the amount of $340 for 3½ years, at 7 per cent.?

		Or,	
$340			$340
.07			.06
23.80			20.40
3½			3½
71.40			61.20
11.90			10.20
83.30	interest for 3½ years.	⅙	71.40 int. at 6 per ct.
340.	principal.		11.90 " " 1 "
$423.30	amount.		83.30 " " 7 "
			340.
			423.30 amount.

3.	What is the interest of $241 for 1 year?	14.	What is the interest of $174.64 for 7 yr. 2 m.?
4.	$234 for 1 year?	15.	$475 for 1 yr. 10 m. 25 d.?
5.	$1350 for 2 years?	16.	$1248.16 for 3 years?
6.	$891.24 for 3 years?	17.	$1968.01 for 4½ years?
7.	$349.25 for 4 years?	18.	$425.29 for 5½ years?
8.	$1181.04 for 5 years?	19.	$40.81¼ for 6 years?
9.	$1886.99 for 6 years?	20.	$606.25 for 2 years?
10.	$901.21 for 7 years?	21.	$794.31 for 2¼ years?
11.	$98.99 for 8 years?	22.	$863.87½ for 3 yrs. 9 m.?
12.	$101.20 for 9 years?	23.	$1087.75 for 9½ years?
13.	$836 for 1 yr. 7 m. 17 d.?	24.	$1196.33⅓ for 6⅔ years?

25. What is the amount of $268.44 for 3 yr. 5 m. 26 d.?
26. $934.50 for 1 year, 7 months, 15 days?
27. $6050 for 2 years, 3 months, 12 days?
28. $474.54 for 3 years, 7 months, 4 days?
29. $981.87 for 6 years, 1 month, 29 days?
30. $8122.14 for 11 years, 11 months, 23 days?
31. What is the interest of $241.18 for 10 years, at 4½ per cent.?
32. What is the interest of $709.39 for 11 years, at 5 per cent.?
33. What is the interest of $511.57 for 12 years, at 7 per cent.?
34. What is the interest of $910.50 for 3 years, 9 months and 26 days, at 7 per cent.?
35. What is the amount of $850 for 12 years, 2 months and 20 days, at 5 per cent.?

36. What is the amount of $300.25 for 2 years, 6 months and 15 days, at 4 per cent.?

37. What is the amount of 237*l.* 16*s.* 8*d.* for 4 years, 6 months and 6 days?

§ **122.** CASE 1. To find the interest for months at 6 per cent.

RULE. Multiply the principal by half the number of months, and proceed as directed in the general rule; or let half the number of months be considered as a decimal.

It is evident that if the rate per cent. were 12, it would be one per cent. a month. Therefore, if it be 6 per cent. it will be half per cent. a month; that is, half the months will be the per cent.

EXAMPLES.

1. What is the interest of $468.25 for 6 months?

468.25		Or	468.25
3=half the mo.			.03=half the months.
100)1404.75			$14.0475 Ans.
$14.0475 Ans.			

2. What is the amount of $1684 for 19 months?

1684×.095=159.98=Interest for 19 months.
1684. =Principal.
$1843.98 Answer.

3. What is the interest of $482 for 2 months?	12. What is the amount of $1479.78 for 23 months?
4. $582.40 for 3 months?	13. $1001.18 for 25 months?
5. $942.24 for 7 months?	14. $69.01 for 27 months?
6. $1264.10 for 11 months?	15. $324.49 for 29 months?
7. $861.28 for 14 months?	16. $1987.41 for 30 months?
8. $988.66 for 15 months?	17. $784.16 for 17 months?
9. $1080.74 for 18 months?	18. $2479.99 for 9 months?
10. $12.80 for 19 months?	19. $1300.42 for 13 months?
11. $80.89 for 21 months?	20. $1708.61 for 10 months?

§ **123.** CASE 2. To find the interest when the time is in months and days, at 6 per cent.

RULE. Multiply by half the number of months and one-sixth of the days; and if the principal be dollars, point off

three figures from the right hand for cents and mills, and those at the left will be the dollars. But if there be dollars and cents in the principal, separate five figures from the right hand, and those at the left will be the dollars.

It is evident that by taking one-sixth of the days and annexing them to half of the months, they become decimals of a month, for as days are thirtieths of a month, they will be sixtieths of half months; consequently if one-sixth of the days be taken, they become tenths or decimals of a month.

When any other per cent. is wanted, proceed as above directed, and then multiply by the given rate per cent. and divide by 6; the quotient will be the interest.

EXAMPLES.

1. What is the interest of $524.82 for 4 months and 27 days?

$\frac{1}{2}$ of 4 months = .02 = the interest on $1 for 4 months.
$\frac{1}{6}$ of 27 days = .004$\frac{1}{2}$ = " " " 27 days.
.024$\frac{1}{2}$ = " for the given time.
524.82 × .024$\frac{1}{2}$ = $12.85,809 Answer.

2.	What is the interest of $263.48 for 2 m. 21 d.?	10.	Find the amount of $1325 for 14 months 8 days?
3.	$318 for 10 m. 16 days?	11.	$789.99 for 7 m. 5 days?
4.	$418 for 19 m. 17 days?	12.	$675 for 8 m. 9 days?
5.	$792.30 for 13 m. 9 d.?	13.	$1000 for 4 m. 13 days?
6.	$465.17 for 16 m. 13 d.?	14.	$3000 for 8 m. 1 day?
7.	$40.71 for 3 m. 18 days?	15.	$951 for 7 m. 26 days?
8.	$74.26 for 5 m. 12 days?	16.	$651 for 17 m. 14 days?
9.	$1680.60 for 11 m. 15 d.?	17.	$999.99 for 19 m. 29 d.?

§ 124. CASE 3. To find the interest for days.

RULE. Multiply the principal by the number of days, and divide by 6083, the quotient will be the interest at 6 per cent.; but if the rate be 5 per cent. divide by 7300.

The reason of the rule will appear from the following:

Thus, for 6 per cent. (100 prin. × 365 days) ÷ 6 = 6083 divisor.

For 5 per cent. (100 prin. × 365 days) ÷ 5 = 7300 divisor.

The interest of any sum at 6 per cent. will amount to the principal in 16$\frac{2}{3}$ years = 6083 days; and at 5 per cent. in 20 years = 7300 days.

EXAMPLES.

1. What is the amount of $1642.25, for 54 days?

Thus $\frac{1642.25 \times 54}{6083} = \$14.577$ interest.

$\$ \quad ct.\ m.$

1642.25 principal.

$\$1656.827$ amount.

2. What is the interest of $7086 for 39 days?	9. What is the interest of $1385.25 for 118 days?
3. $175.12 for 72 days?	10. $1544.08 for 96 days?
4. $2500 for 115 days?	11. $86.88 for 58 days?
5. $1675 for 200 days?	12. $999.99 for 165 days?
6. $184.06 for 45 days?	13. $360 for 315 days?
7. $6083 for 61 days?	14. $6001.04 for 224 days?
8. $3849 for 110 days?	15. $69985.18 for 69 days?

16. What is the amount of $4561.11 for 65 days, at 5 per cent.?

17. What is the amount of $8244.82 for 86 days, at 5 per cent.?

§ 125. CASE 4. To compute the interest between different dates.

RULE. Write down the given dates and subtract one from the other, reckoning 30 days to the month, and 12 months to the year; the remainder will be the time for which the interest is required; then proceed as before.

EXAMPLES.

1. What is the interest of $360, from July 10, 1843, to January 4, 1845?

	yrs.	*m.*	*d.*
	1845	1	4
	1843	7	10
Time	1	5	24

		360
		.06
4*m.*	$\frac{1}{3}$	21.60
1 "	$\frac{1}{4}$	7.20
15*d.*	$\frac{1}{2}$	1.80
$7\frac{1}{2}d.$	$\frac{1}{2}$	.90
$1\frac{1}{2}d.$	$\frac{1}{5}$	.45
		9
		$32.04 Answer.

Or $360 \times .089 = \$32.04$ Interest as before.

2. What is the interest of $1573.50 from Dec. 16, 1843, to May 18, 1844?
3. $420.30, from May 21, 1843, to June 15, 1845?
4. $180, from Nov. 15, 1836, to Dec. 1, 1844?
5. $1681.20, from March 31, 1842, to July 21, 1845?
6. $850.25, from Oct. 11, 1843, to Jan. 11, 1845?
7. $71.50, from Dec. 25, 1842, to Jan. 19, 1845?
8. $5956, from Nov. 10, 1840, to July 13, 1845?
9. $188.64, from April 25, 1841, to Jan. 30, 1845?
10. $672.18, from July 4, 1829, to May 24, 1845?
11. $36.14, from Aug. 30, 1842, to Feb. 6, 1845, at 4½ per ct.
12. $901.62, from Jan. 8, 1843, to April 4, 1845, at 5 per ct.
13. $344.44, from Feb. 3, 1841, to Dec. 31, 1844, at 7 per ct.
14. $101.01, from Sept. 10, 1842, to Mar. 4, 1845, at 8 pr ct.
15. $1104.54, from Apr. 7, 1826, to June 24, 1845, at 2 pr ct.
16. What is the amount of $144.89, from August 25, 1832, to March 5, 1845?
17. $1108.16, from Feb. 8, 1839, to June 23, 1845?
18. $2011.98, from March 19, 1841, to April 4, 1845?
19. $601.29, from May 6, 1840, to Dec. 1, 1844?
20. $1806.18, from June 16, 1826, to Oct. 21, 1841?

§ **126.** CASE 5. When the principal, rate per cent., and amount are given, to find the time.

RULE. As the interest of the principal for one year, is to the whole interest, so is one year to the time required. Or, divide the interest by the product of the rate and principal, and the quotient will be the time.

EXAMPLES.

1. In what time will $525 amount to $708.75, at 5 per ct.?

$525	Amount $708.75
.05	Principal 525.
$26.25 interest for 1 year.	Interest $183.75

As $26.25 : $183.75 : : 1 y. : 7 yrs.

Again, $183.75 \div (.05 \times 525) = 183.75 \div 26.25 = 7$ Ans.

2. In what time will $550.75 amount to $682.93, at 6 per cent.?
3. $450, to $540, at 4 per cent.?
4. $500, to $620, at 6 per cent.?
5. $543, to $705.90, at 6 per cent.?
6. $932, to $1071.80, at 5 per cent.?

7. In what time will $684 amount to $868.68, at 6 per ct.
8. $701, to $806.15, at 6 per cent.?
9. $275, to $342.37½, at 3½ per cent.?
10. $810.50, to $1005.02, at 4 per cent.?
11. In what time will a given principal double itself at 6 per cent.?
12. In what time will a given principal double itself at 5 per cent.?

§ **127.** CASE 6. When the amount, time, and rate per cent. are given, to find the principal.

RULE. As the amount of $100 at the rate, and for the given time, is to the amount given, so is $100 to the principal required. Or, to the rate, multiplied by the time, add unity; by which sum divide the amount, and the quotient will be the principal.

EXAMPLES.

1. What principal being put to interest, will amount to $708.75 in 7 years, at 5 per cent.?

$5 \times 7 = 35$

100

135 = amount of $100 for the rate and time.

Then as $135 : $708.75 :: $100 : $525. Answer.

Or, $.05 \times 7$ (rate and time) $+ 1 = 1.35$

Then $708.75 ÷ 1.35 = $525. Answer.

2. What principal being put to interest, will amount to $682.93 in 4 years, at 6 pr ct.?
3. To $540, in 5 years, at 4 pr ct.?
4. To $620, in 4 years, at 6 pr ct.?
5. To $705.90, in 5 years, at 6 pr ct.?
6. To $868.68, in 4½ years, at 6 pr ct.?
7. To $1071.80, in 3 years, at 5 pr ct.?
8. To $806.15, in 2½ years, at 6 per cent.?
9. To $342.37½, in 7 years, at 3½ pr ct.?
10. To $1005.02, in 6 years, at 4 pr ct.?

§ **128.** CASE 7. When the principal, amount, and time are given, to find the rate per cent.

RULE. As the principal is to $100, so is the interest of the whole to the interest of $100 for the same time. Divide that interest by the time, and the quotient will be the rate per cent.

Or, divide the interest by the product of the time and principal, and the quotient will be the rate per cent.

EXAMPLES.

1. At what rate per cent. will $525 amount to $708.75 in 7 years?

$708.75
525
$183.75 Interest.

As $525 : $100 : : $183.75 · $35
and 35÷7=5 per cent.

Or, 183.75÷(525×7)=183.75÷3675=.05=5 pr ct. Ans.

2. At what rate per cent. will $550.75 amount to $682.93 in 4 years?

3. $450, to $540 in 5 years.?
4. $500, to $620 in 4 years.?
5. $543, to $705.90 in 5 years?
6. $684, to $868.68 in 4½ years?
7. $932, to $1071.80 in 3 years?
8. $701, to $806.15 in 2½ years?
9. $275, to $342.37½ in 7 years?
10. $810.50, to $1005.02 in 6 years?

PARTIAL PAYMENTS.

§ **129.** When notes, bonds, or obligations, receive partial payments or endorsements, we must use the following rule, as laid down in the cases of Penrose *vs.* Hart, 1 Dallas' Reports, p. 378; Commonwealth *vs.* Miller, 8 Sergeant and Rawle's Reports, p. 458; Smith *vs.* Shaw, 2 Washington Circuit Court Reports, p. 167, and by Chancellor Kent, in Johnson's Chancery Reports, vol. i. p. 17.

RULE. The rule for casting interest, when partial payments have been made, is to apply the payment, in the first place, to the discharge of the interest then due.

If the payment exceed the interest, the surplus goes towards discharging the principal, and the subsequent interest is to be computed on the balance of principal remaining due.

If the payment be less than the interest, the surplus of interest must not be taken to augment the principal; but interest continues on the former principal until the period

when the payments, taken together, exceed the interest due, and then the surplus is to be applied towards discharging the principal; and interest is to be computed on the balance as aforesaid.

EXAMPLES.

Philadelphia, November 1, 1841.

1. For value received, I promise to pay to Peter Haines, or order, the sum of eighteen hundred and sixty dollars, on demand, with lawful interest. CHARLES BOOTH.

The following endorsements were made on this note:

1842, Oct. 6, there was endorsed . .	$183.21
1843, March 4, " " " . . .	266.99
1844, Dec. 10, " " " . . .	323.31
1845, April 25, " " " . . .	1210.04
1845, May 30, " " " . . .	110.00

What was the balance due July 20th, 1845?

The amount of the note, ...	$1860.00
Interest to Oct. 6, 1842, 11 months 5 days,	103.85
	1963.85
First payment, same date, ...	183.21
Balance due, ...	1780.64
Interest to March 4, 1843, 4 months 29 days,	44.22
	1824.86
Second payment, same date, . . .	266.99
Balance due, ...	1557.87
Interest to Dec. 10, 1844, 1 year 9 months 6 days,	165.13
	1723.00
Third payment, ...	323.31
Balance due, ...	1399.69
Interest to April 25, 1845, 4 months 15 days,	31.49
	1431.18
Fourth payment, ...	1210.04
Balance due, ...	221.14
Interest to May 30, 1845, 1 month 5 days, .	1.29
	222.43
Fifth payment, ...	110.00
Balance due, ...	112.43
Interest to July 20, 1845, 1 month 20 days, .	.93
Amount due July 20, 1845, ...	$113.36

Philadelphia, May 1, 1841.

2. For value received, I promise to pay to Jacob Smith, or order, six hundred and ninety-nine dollars, ninety-nine cents and six mills. THOMAS JONES.

Endorsements were made on this note as follows:

Dec. 25, 1841, there was paid, . . .	$99.996
June 30, 1842, " " . . .	9.996
Aug. 22, 1843, " " . . .	30.000
June 4, 1844, " " . . .	199.998

How much was due April 5, 1845?

The amount of the note is, . . .		$699.996
Interest to Dec. 25, 1841, 7 m. 24 days,		27.300
		727.296
First payment same date, ...		99.996
Balance due, ...		627.300
Interest to June 30, 1842, 6 m. 5 days,		19.342
Second payment same date, (less than interest,)	$9.996	
Interest to Aug. 22, 1843, 1 y. 1 m. 22 d.		43.074
Third payment same date, (less than interest,)	30.000	
Interest to June 4, 1844, 9 m. 12 days,		29.483
		719.199
Fourth payment same date, (exceeding interest,)	199.998	239.994
Balance due, ...		479.205
Interest to April 5, 1845, 10 m. 1 day,		24.040
Amount due April 5, 1845, . . .		$503.245

3. Suppose B has a bond against C, for $2000, dated January 1st, 1843, with no definite time fixed for payment; but pays $100 at the end of 3, 6, 9, 12 and 18 months respectively, with lawful interest, and at the end of two years they make a final settlement: what was then due?

4. Suppose G has a bond against H, for $2333.33.2, dated May 1, 1840, with lawful interest, upon which the following payments have been made:

December 25, 1840, . . .	$333.33.2
July 10, 1841, ...	33.33.2
September 1, 1842, . . .	100.00.0
June 14, 1843, ...	666.66.6
April 15, 1844, ...	1240.00.0

What will be due upon it August 3, 1845?

5. Suppose W holds a note against X, for $2951.50, dated January 1, 1842, with lawful interest, upon which the following payments have been made:

September 11, 1842,	$693.00
January 1, 1843,	590.00
June 5, 1843,	350.50
September 24, 1843,	384.88
July 4, 1844,	490.00

What will be due January 1, 1845?

6. On a bond for $1200, dated March 1, 1839, with lawful interest, there were endorsed the following sums:

May 1, 1840, received	$400
June 16, 1841,	160
September 17, 1842,	24
December 19, 1842,	30
March 1, 1843,	200
October 16, 1844,	300

What will be due August 31, 1845?

7. On a note dated June 16, 1838, for $450, with interest, were endorsed the following payments:

Received July 1, 1839,	$75
" September 16, 1840,	45
" December 10, 1842,	5
" June 1, 1843,	10
" August 16, 1843,	100
" March 1, 1845,	150

What will be due September 1, 1846?

COMPOUND INTEREST.

§ **130.** Compound interest is that which arises both from the principal and interest; that is, the interest due at each payment is added to the principal to bear interest for the next payment.

RULE. Find the amount of the principal for the time of the first payment, by simple interest. Call this amount the principal for the second payment, and find its amount as before, and so on for the number of payments required.

Subtract the first principal from the last amount, and it will give the compound interest required.

EXAMPLES.

1. Find the compound interest of $760.75 for 2 years, payable half-yearly, at 4 per cent.

$760.75 the given principal.
15.21.5 first half year's interest.
775.96.5 second half year's principal.
15.51.9 second half year's interest.
791.48.4 third half year's principal.
15.82.3 third half year's interest.
807.31.3 fourth half year's principal.
16.14.7 fourth half year's interest.
823.46.0 fourth half year's amount.
760.75. first principal subtracted.
$ 62.71. compound interest.

2. What will be the compound interest of $250 for 3 years?
3. What will be the compound interest of $1500 for 3 years?
4. What will $1200 amount to in 4 years, at 4½ per cent.?
5. What will $629 amount to in 7 years?
6. What is the amount and interest of $354.80 for 3½ years, at 5 per cent.?
7. What is the compound interest of $600 for 4 years?
8. What is the compound interest of $280.50 for 1½ years, at 5 per cent., payable half-yearly?
9. What is the amount of $100 for 2 years at 3 per cent., payable quarterly?
10. What is the amount of $50 for 2½ years, at 3½ per cent., payable quarterly?

Q. What is interest?
Q. What is the principal, rate, and amount.
Q. What is the rate per cent. established in most of the states?
Q. How many things are to be considered in interest?
Q. How is interest computed?
Q. How many kinds of interest are there?
Q. What is simple interest?
Q. What is the rule for calculating interest for years, months, and days?
Q. What is the rule for calculating interest for months, at 6 per cent.?

Q. What is the rule for calculating interest for months and days?

Q. What is the rule for calculating interest for days?

Q. How do you calculate interest between different dates?

Q. How do you find the time when the principal, amount, and rate are given?

Q. How do you find the principal, when the amount, time, and rate are given?

Q. How do you find the rate, when the principal, amount, and time are given?

Q. When partial payments are made, how is interest calculated?

Q. What is the rule for calculating compound interest?

COMMISSION.

§ **131.** Commission is an allowance of so much per cent. from merchants to their factors or agents, for the buying or selling of goods.

RULE. Multiply the sum by the rate per cent., and the product divided by 100 will give the commission.

EXAMPLES.

1. What does the commission come to on $1628.50, at $2\frac{1}{2}$ per cent.?

$1628.50 \times .025 = \$40.71.25$. Answer.

2. What is the commission on $1650, at $\frac{3}{8}$ per cent.?

3. What is the commission on $1974, at 5 per cent.?

4. What is the commission on $1088.25, at $2\frac{1}{2}$ per cent.?

5. What is the commission on $2049.10, at $\frac{5}{6}$ per cent.?

6. A factor receives $1976 to lay out; after having deducted his commission, at 4 per cent., how much will remain to be laid out?

7. Suppose I allow my correspondent $1\frac{3}{8}$ per cent. for his commission, what will it amount to for disbursing on my account $758.90?

BROKERAGE.

§ **132.** Brokerage is a small allowance per cent. to a person called a broker, for assisting merchants or factors in buying or selling goods.

RULE. The same as in commission.

EXAMPLES.

1. What is the brokerage on $1728, at $\frac{3}{4}$ per cent.?

17.28 × .0075 = $12.96. Answer.

2. What is the brokerage on $1298.53, at $\frac{3}{8}$ per cent.?

3. What is the brokerage on $2864.46, at $1\frac{1}{2}$ per cent.?

4. If I allow a broker $\frac{2}{5}$ per cent., what will his brokerage come to on $964.70?

5. A broker sold goods to the amount of $525.60: what will his brokerage amount to, at $2\frac{3}{8}$ per cent.?

6. A broker sells goods to the amount of $1050: what is his demand at $1\frac{3}{4}$ per cent.?

7. What is the claim of a broker, at $\frac{1}{4}$ per cent. on $1760.60?

INSURANCE

§ **133.** Insurance is an allowance per cent. paid by the proprietors of goods, &c., to certain persons or officers, who engage to make good the loss of ships, houses, goods, &c., which may happen by storms, fire, or other accidents: the security the insurer receives, as well as the premium he pays, is also called Insurance. Policy is the name of the writ, or instrument, by which the contract is effected between the parties.

RULE. Multiply the sum to be insured by the rate, and the product divided by 100, is the per centage to be paid. Or, instead of multiplying, take parts for the rate.

EXAMPLES.

1. What is the insurance of $850, at $12\frac{3}{4}$ per cent.?

		$850
		12
$\frac{1}{2}$	$\frac{1}{2}$	10200
$\frac{1}{4}$	$\frac{1}{2}$	425
		212|5
1|00)		108|37.5

$108.37.5 Answer.

Or, 10 | $\frac{1}{10}$ | 850

2 | $\frac{1}{5}$ | 85

$\frac{1}{2}$ | $\frac{1}{4}$ | 17

$\frac{1}{4}$ | $\frac{1}{2}$ | 4.25

2.12.5

$108.37.5 Answer.

2. What is the insurance of $1000, at $6\frac{3}{8}$ per cent.?

3. What is the insurance of $1784.60, at $1\frac{5}{8}$ per cent.?

4. What is the insurance of a ship's cargo, valued at $24630, at $18\frac{1}{4}$ per cent.?

5. What sum must a policy contain to cover $759, when the premium is 8 per cent.?

As 8 per cent. is already taken out, the sum covered must be $\frac{92}{100}$ of the policy.

6. If a policy be taken out for $2400, at 30 per cent., what is the sum covered?

It is clear, that if 30 per cent. be taken from any sum, 70 per cent. will remain.

Q. What is commission, brokerage and insurance?

Q. What is the rule for calculating each of them?

STOCKS.

§ **134.** Stock is a general name used for funds, established by government; and also for banks, or individuals in their corporate capacity.

Rule. If the sum given is above par (*i. e.* above 100) multiply the sum to be purchased by the excess above 100; divide the product by 100, and add the quotient to the given sum.

If the sum given is under par, multiply it by the price; and that product divided by 100, will give the answer.

Or, instead of multiplying, take parts for the whole price.

EXAMPLES.

1. What is the purchase of $736.50 bank stock, at $111\frac{3}{4}$ per cent.?

$\frac{1}{2}$ | $\frac{1}{2}$	736.50	
	11	
	810150	
$\frac{1}{4}$ | $\frac{1}{2}$	36825	
	184125	
100)	8653.875	
	86.53875	
	736.50	
	$823.03875	Answer.

Or

10 | $\frac{1}{10}$	73650	
1 | $\frac{1}{10}$	7365	
$\frac{1}{2}$ | $\frac{1}{2}$	7365	
$\frac{1}{4}$ | $\frac{1}{2}$	36825	
	184125	
	$823.03875	Answer.

Or $736.50 \times 1.11\frac{3}{4} = \$823.03875$.

2. Bought $782.82½ bank stock, at 91⅛ per cent., what did it cost me? 782.825×.91125=$713.34.9.

3. What must be paid for $1000 stock, at 95⅓ per cent.?

4. Sold 18 shares, $40 each, of bank stock, at 9½ per cent. advance: to what did they amount?

5. What is the value of $4200 of the 5 per cent. stock, at 81 per cent.?

6. What is the purchase of $1058.60 bank stock, at 116¼ per cent.?

7. What must be paid for the purchase of $10000 stock, at 105¾ per cent., and the broker's commission at ⅛ per cent.?

DISCOUNT.

§ **135.** Discount is an allowance made for the payment of money before it becomes due, and is as much as that money, if put to interest, would gain in the same time, and at the same rate.*

Thus $100 present money will pay a debt of $106 payable a year hence, the discount being made at 6 per cent.

The present worth, then, is the sum to be paid after the discount is deducted. The discount on any sum is less than its interest. Thus, the discount on 106, due a year hence is $6, the interest of the same is $6.36.

§ **136.** CASE 1. When the present worth is required.

RULE. As the amount of $100 for the given rate, and time, is to $100, so is the sum given, to the present worth.

Or, divide the sum on which the discount is to be made by the amount of $1 for the given time, and rate per cent.

EXAMPLES.

1. What is the present worth of $360.50 for 11 months at 6 per cent.?

6*m.*	½	$6.00	As $105.50 : $100 : : $360.50 : $341.70 Ans.
4	⅓	300	Or, 360.50÷1.0550=341.70 Ans.
1	¼	200	
		50	
		5.50	
		100.	
		$105.50	Amount of $100.

* Discount for prompt payment of bills, and for loans of money or promissory notes, is usually calculated in the same manner as interest.

2. What is the present worth of $651.50, due 18 months hence, at 6 per cent.?

3. Of $301, due 16 months hence, at 5 per cent.?

4. Of $615.75, due 7 months hence, at 4½ per cent.?

5. Of $1350, due in 6 months, at 6 per cent.?

6. Of $150, due in 63 days, at 6 per cent.?

7. Of $848.50, due in 3½ months, at 6 per cent.?

8. Of $1674.40, due 9 mo. and 9 days hence, at 6 per ct.?

9. Of $3334.40, due in 11 mos. and 6 days, at 4½ per ct.?

10. Of $1935, due 18 months hence, at 5 per cent.?

11. Of $240, one half payable in 4 months, and the other half 8 months, discount at 5 per cent.?

12. Bought goods to the amount of $1012, to pay as follows: $432 in 12 months, and $580 in 2 years: what is the present worth, allowing discount at 8 per cent.?

13. What is the present worth of $5150, due in 4½ months, at 8 per cent., and allowing 1 per cent. for prompt payment?

14. D is to pay $5927 on the 29th of April, 1846, and $5989, on the 29th of July following: it is required to know how much money will discharge both sums on the 29th of January, 1846, discount at 8 per cent.?

15. What is the difference between the interest of $1200 for 12 years, at 5 per cent., and the discount of the same sum for the same time, and at the same rate?

§ **137.** CASE 2. When the discount is required.

RULE. As the amount of $100, for the given rate and time, is to the given sum, so is the interest of $100 for the given time, to the discount of the given sum.

EXAMPLES.

1. What is the discount of $375, for 1 year and 9 months, at 4½ per cent.?

m.		$
6	1/2	4.50=1 year.
3	1/2	2.25
		1.125
		7.875
		100.
		$107.875

Then, $ $ $ $ *cts.m.*
as 107.875 : 375 : : 7.875 : 27.37.5

2. What is the discount of $270.24, for 1 year and 8 months, at 7 per cent.?

3. Of $325.25, due 18 months hence, at 6 per cent.?
4. Of $250, payable in 3 years, at 6 per cent.?
5. Of $401.25, due 1 year hence, at 7 per cent.?
6. Of $450, due 6 months hence, at 6 per cent.?
7. Of $63.60, due 1 year hence, at 6 per cent.?
8. Of $384.80, due 3 years and 5 months hence?
9. A note for $500, to be paid Dec. 20, 1846: what is the discount June 11, 1845?
10. A note for $1431, dated August 19, 1844, which becomes due January 13, 1845: what is the discount September 27, 1844?
11. A note becomes due Nov. 29, 1845, for $2744.25: what is the discount Jan. 6, 1845?
12. Sold goods for $748.80, one half to be paid at 3 months, and the other half at 6 months: what must be the discount for the present payment, at 5 per cent.?
13. How much is the discount of $853, at 2 per cent.?
14. How much is the discount of $985.75, at 4 per cent.?
15. What is the discount on $2575, due in $4\frac{1}{2}$ months hence?

Q. What is discount?
Q. What is the rule for finding the present worth of a given sum?
Q. How do you find the discount?

BANKING.

§ **138.** A bank is an incorporated institution, created for the purpose of loaning money and dealing in exchange.

Its capital is limited by law, and owned in shares by individuals called stockholders.

Banks are by law allowed to make notes, which are called bank bills, which circulate as money, because they are obliged to redeem them with specie.

When the banks loan money, it is customary to take the interest in advance; that is, to deduct it from the face or amount of the note, at the time the money is lent. The note is then said to be discounted.

The sum discounted is called the amount; the interest deducted the discount; and what remains the proceeds or present worth.

A note to be discounted, or bankable, must be made payable at some future time, and to the order of some person who endorses it.

The drawer is not obliged to pay his note till three days after the expiration of the time mentioned in it, these being called days of grace.

Banks take interest for four days more than the number intervening between the day when a note is discounted, and the day on which the time specified in it expires, taking $\frac{1}{72}$ of the rate per cent. per annum, for each 30 days of the time so estimated. Thus, on a note dated August 31, payable in 6 months, falling due February 28, days of grace expiring March 3, and which is discounted on the day of date, at the rate of 6 per cent. per annum, interest would be taken for 185 days $= 6\frac{1}{6}$ months $= 3\frac{1}{12}$ per cent.

§ **139.** CASE 1. To find the banking discount, and present worth of any sum of money.

RULE. Calculate the interest on the given sum for four days more than the time specified, then deduct the interest from the amount; the remainder will be the present worth.

EXAMPLES.

1. What is the bank discount on $450, payable 180 days hence, at 6 per cent per annum?

days.		$450			Or, $1.			
		.06			days.		.06	
180	$\frac{1}{2}$	27.00		days.	180	$\frac{1}{2}$	.06	days.
4	$\frac{1}{45}$	13.50	Int. for	180	4	$\frac{1}{45}$	.03 Int. on $1 for	180
		30	"	4			.0006$\frac{2}{3}$ "	4
		$13.80	discount.				.0306$\frac{2}{3}$ "	184

Therefore $450 × .0306$\frac{2}{3}$ = $13.80 discount.

2. What is the bank discount on $600, payable 90 days hence, at 6 per cent.; and what is the present worth?

3. If I have a note of $9000, payable in 45 days, discounted at 6 per cent., what sum should I receive?

4. What is the bank discount on $750, payable 99 days hence, at 7 per cent.?

5. On $1800, payable 90 days hence, at 5 per cent.?

6. On $4200, payable 276 days hence, at 6 per cent.?

7. The present worth of a note for $3600 payable in 120 days, discounted at 6 per cent.?

8. Of a note for $1520, payable in 150 days, discounted at 6 per cent.?

9. Of a note for $5184, payable in 90 days, discounted at 6 per cent.?

10. Of a note for $3930, payable in 180 days, discounted at 6 per cent.?

11. What is the bank discount, at 6 per cent., on $675.68, payable in 30 days?

12. What sum should be paid at bank, for a note for $4000, payable in 60 days, discounted at 6 per cent.?

§ **140.** CASE 2. To find what sum or amount must be named in a note, in order to obtain a particular loan at bank.

RULE. Calculate the banking discount on $1, for the given rate and time; subtract this discount from $1, and divide the present worth by the remainder, and the quotient will be the amount.

EXAMPLES.

1. What must be the face or amount of a bankable note, so that when discounted for 210 days, at 6 per cent per annum, its present worth shall be $400?

days.		$1
		.06
210	$\frac{7}{12}$	.06
4	$\frac{2}{105}$	.035
		.0006$\frac{2}{3}$
		.0356$\frac{2}{3}$ discount on $1.

Then:

$1 - .0356\frac{2}{3} = .9643\frac{1}{3}$

Therefore:

$400 \div .9643\frac{1}{3} = \$414.79$

2. What must be the face of a bankable note, so that when discounted for 60 days, at 6 per cent., it shall give a present worth of $7421.25?

3. Suppose your note is discounted at bank for 180 days, at 6 per cent., and $967.50 passed to your credit; what must have been the face of the note?

4. What must be the face of a bankable note, so that when discounted for 60 days, at 7 per cent., the borrower shall receive $150?

5. What must be the face of a bankable note, so that when discounted for 300 days, at 5 per cent., the present worth may be $6000?

6. What must be the face of a bankable note, so that when discounted for 150 days, at 6 per cent., it shall give a present worth of $1481.24?

EQUATION OF PAYMENTS.

§ 141. Equation of payments is a rule for finding the average or equated time of several sums had at different dates or on different terms of credit, or both; in order that the whole amount due may be paid in one sum.

Note. In sums to be equated no account is taken of a fraction of a dollar, if less than a half: if as great as a half, it is counted as a dollar. The same rule is applied to a fraction of a day in the equated time.

CASE 1. When the sums due have been obtained at the same date, but have different terms of credit.

RULE. Multiply each sum by its time of credit; add the products together, and divide their sum by the sum of the payments: the quotient will be the average time of credit.

EXAMPLES.

1. A owes B $560, of which $200 is to be paid at 3 months, $200 at 5 months, $100 at 6 months, and the rest at 9 months: required the equated time for the whole payment.

$200 × 3 = 600
$200 × 5 = 1000
$100 × 6 = 600
$60 × 9 = 540

560 | 2740 | 2740 ÷ 560 = 4m. 26d. Ans.

2. A merchant has owing to him $300, to be paid as follows: $50 at 2 months, $100 at 5 months, and the rest at 8 months; but it is agreed to make one payment of the whole: when should that be?

3. A debt of $3600 is to be paid as follows: $\frac{1}{4}$ at 3 months, $\frac{1}{3}$ at 4 months, $\frac{1}{5}$ at 5 months, $\frac{1}{8}$ at 6 months, and the rest at 7 months: what is the equated time for the whole?

CASE 2. When the sums due have been obtained at different dates, but on similar terms of credit.

RULE. Multiply each sum by the number of days from its date to that of the last sum. Divide the sum of the products by the whole debt. The quotient will be the number of days prior to the date of the last sum, at which the average or equated date of purchase should be fixed.

EXAMPLES.

A purchased of B, on 6 months credit, as follows: May 4, $200, June 6, $600, August 20, $300. What is the equated time of purchase?

$200 × 108 = 21600
$600 × 75 = 45000
$300

1100 | 66600 | 66600 ÷ 1100 = $60\frac{6}{11}$.

61 days prior to August 20 = June 20. Ans.

C purchased of D, on 8 months credit, as follows: March 12, $500, May 3, $900, July 6, $300. What is the equated time of purchase?
Ans. April 29.

E purchased of F, on 9 months credit, as follows: March 20, $1200, May 18, $560, August 27, $1680, September 23, $750. Required the equated time of purchase. Ans. July 4.

CASE 3. When the sums due have been obtained at different times, and on different terms of credit.

RULE. Multiply each sum by the number of days from the date when due to the latest date of payment. Divide the sum of the several products by the whole debt. The quotient will be the number of days prior to the latest date of payment, at which the average or equated date should be fixed.

EXAMPLES.

N purchased of O the following amounts, at the dates and credits respectively named. March 2, $400, at 8 months; April 25, $200, at 4 months; May 10, $900, at 6 months; June 7, $300, for cash. When will the whole amount be due?

Date of Purchase.	*Amount.*	*Credit.*	*When due.*	
March 2	$400	8 months	Nov. 2	$300 × 156 = 46800
April 25	$200	4 months	Aug. 25	$200 × 77 = 15400
May 10	$900	6 months	Nov. 10	$400 × 8 = 3200
June 7	$300	. . . .	June 7	$900
				1800 65400

65400 ÷ 1800 = 36¼.

36 days prior to Nov. 10 = Oct. 5. Ans. (Payment to be made Oct. 8; 3 days grace being allowed on payments, in the United States.)

R bought goods of S; the amounts, dates and credits of his several purchases being as follows: March 1, $840, at 8 months; April 9, $1206, at 6 months; August 26, $1525, for cash; and October 3, $767, at 7 months. Required the date when the whole amount will be due? Ans. November 3.

T owes V for bills of goods of the following amounts, purchased at the dates and on the credits respectively named: April 3, $1256, at 8 months; April 30, $625.75, at 7 months; May 21, $520.37, for cash; August 25, $816, at 6 months; September 19, $1286.62, at 8 months; and October 12, $906.50, at 6 months. When will the whole amount be due? Ans. January 27.

Equated time is also frequently ascertained thus: Find the interest on each item of indebtedness for the number of days, (or months and fractions of months) by which it would be multiplied in calculating by the preceding rules. Add together their several items of interest, and find in how many days the interest calculated on the whole amount of indebtedness would produce a similar sum. The number of days, thus ascertained, will be the equated time of *credit* in Case 1; and in Cases 2 and 3 will be the number of days prior to the date of *purchase* and of *payment* respectively, at which the equated date should be fixed.

BARTER.

§ **142.** Barter is the exchange of one commodity for another, without loss to either party.

§ **143.** CASE 1. When the quantity of one commodity is given with its value, to find what quantity of the other, at the rate proposed, may be had for the same money.

RULE. Find the value of that commodity whose quantity is given; then divide the value of the quantity exchanged by the price of an unit returned.

EXAMPLES.

1. How many pounds of sugar, at 12 cents per lb., must be delivered in barter for 198 lbs. of rice, at 8 cents per lb.?

Thus $\frac{198\times8}{12}=132$ lbs of sugar.

2. What quantity of chocolate, at 45 cents per lb., must be given in barter for 265 lbs. of tea, at 70 cents per lb.?

Thus $\frac{265\times70}{45}=412\frac{2}{9}$ lbs. of chocolate.

3. How much cloth, at $3 per yard, must be given in barter for 84 reams of paper, at $6.50 per ream?

4. How much cheese, at $2.37½ per cwt., must be given in barter for 20 cwt. of sugar, at $5.58⅓ per cwt.?

5. How much tobacco, $6.92½ per cwt., is equal in value to 5 cwt. 3 qrs. 14 lbs. of snuff, at 22½ cents per lb.?

6. How many dozen of wine, at $7.25 per dozen, must be received in exchange for 3 pipes of brandy, at $181.25 per pipe?

§ **144.** CASE 2. When part of the value is returned in cash, and the remainder in goods.

RULE. Deduct the cash from the value of the given commodity, and then proceed for the remaining commodity as before.

1. How many yards of velvet, at $1.45 per yard, must I give, with $79.30 in cash, for 50 yards of cloth, at $2.77½ per yard?

Thus $2.77½×50=138.75
Deduct the cash, 79.30
$59.45

Then $59.45÷1.45=41 yards. Answer.

2. A sold to be B 30 cwt. of sugar, at $6.60 per cwt., for which B returned him $46.75 in cash, and the rest in linen, at 62½ cents per yard: how many yards did A receive?

3. A and B bartered; A has 84 gallons of brandy, at 92½ cents per gallons, for which B gives him $30 in money and the rest in figs, at 3¾ cents per lb.: what quantity of figs must A receive?

4. A has a quantity of pepper, weighing neat 1800 lbs, at 19 cents per lb., for which O gives him $180 in cash, and the rest in rice, at 9½ cents per lb.: how many lbs. of rice must A receive?

5. Received in barter 1200 yards of muslin, at 8½ cents per yard, and returned 84 lbs. of tea, at 45 cents per lb. and the rest in sugar, at 8 cents per lb.: the quantity of sugar is required?

§ **145.** CASE 3. The rate of one commodity being given, to find how the other should be rated.

RULE. Divide the value of the one commodity by the quantity of the other.

EXAMPLES.

1. Bartered two pieces of linen containing 64 yards, at 75 cents, for 12 yards of cloth: what was the cloth per yard?

Thus $(64\times75)\div12=\$4$ per yard.

2. A delivered 884 gallons of brandy, at $1.25 per gallon, to B, for 450 yards of cloth: what was the cloth per yard?

3. O sends P 260 yards of muslin, and receives in return 4 cwt. 3 qrs. of hops, at $4.12½ per cwt.: the price of the muslin per yard is required?

4. Received of T 12 cwt. 2 qrs. of sugar, at $6.40 per cwt., and returned him as an equivalent 40 gallons of wine: what was the wine per gallon?

§ **146.** CASE 4. When the ready money price of one commodity has been raised in barter, to find how to raise the other in proportion.

RULE. As the ready money price of the one commodity is to that of the other, so is the bartering price of the one to the bartering price of the other.

EXAMPLES.

1. A has cloth at $4.80 per yard ready money, but in barter advances to $5.40; B has ginger at $2.40 per cwt. ready money: how much must B rate his ginger per cwt. to be equivalent to A's?

As $4.80 : $5.40 : : $2.40 : $2.70 per cwt. Answer.

2. A has nutmegs worth $1 per pound, ready money, but in barter will have $1.06 per pound; D has tobacco worth 10 cents per pound ready money: how must D rate his tobacco, that his profits may be equivalent with A's?

3. A merchant with whom I bartered tea for sugar, raised his sugar from 12 cents to 13½ cents per lb.: what ought I

to charge him for tea which I sold, ready money, for 66 cents per lb.?

4. W has linen worth 30 cents per yard, ready money, but in barter will have 36 cents; S has broadcloth worth $6 per yard, ready money: at what price ought the cloth to be rated in barter?

MISCELLANEOUS EXERCISES.

1. A has linen cloth worth 20 cents an ell, ready money, but in barter he will have 25 cents; B has broadcloth worth $2 per yard, ready money: at what price ought the broadcloth to be rated in barter?

2. A has 320 dozen of candles, at $1.20 per dozen, for which B agrees to pay him $160 in cash, and the rest in cotton, at 20 cents per pound: how much cotton must B give A?

3. A merchant has 1000 yards of canvass at 9½ cents per yard, which he barters for serge, at 10¼ cents per yard: how many yards must he receive?

4. K has 75 sheep at $1.74 each, for which L is to give him $42.24 and the rest in Indian corn, at 42 cents per bushel: how much corn must L give K?

5. A and B bartered; A had 5 cwt. of sugar, at 6 cents per lb., which he gave to B for a quantity of cinnamon, at $1.28 per lb.: how much cinnamon did B give A?

6. C has candles at $1.44 per dozen, ready money, but in barter he will have $1.56 per dozen; D has cotton at 18 cents per lb. ready money: what price must the cotton be in barter, and how much must be bartered for 100 dozen of candles?

7. A has linen at 10 cents an ell, ready money, but in barter 12 cents; B has 3610 lbs. of sugar at 7½ cents, ready money, and will have of A $84 in cash and the rest in linen: at what rate is the sugar in barter, and how much linen must A give B?

8. A has 7½ cwt. of rice at 8 cents per lb., for which B gave him 12½ cwt. of cheese: what was the cheese rated at per lb.?

9. A and B barter; A has 12½ cwt. of hops at $7.50 per cwt., but in barter insists on $8; B has wine worth 66 cents a gallon, which he raises in proportion to A's demand; on the balance A receives but a hogshead of wine: what had he in ready money?

10. A has 240 bushels of rye, which cost him 90 cents per bushel; this he barters with B at 95 cents for wheat, that stands B in 99 cents per bushel: how many bushels of wheat is he to receive in barter, and at what price is it to be rated, that their gains may be equal?

LOSS AND GAIN.

§ **147.** Loss and gain is a rule which enables merchants and traders to estimate their profit or loss, in buying and selling goods; also to raise or lessen the price of them, so as to gain or lose so much per cent.

This rule has several variations, but the questions are mostly performed either by the rule of three or practice.

§ **148.** CASE 1. To find the whole gain or loss on any quantity of goods.

EXAMPLES.

1. Bought 19 cwt. 3 qr. 14 lb. of sugar, at $5.85 per cwt., and sold it out at $6.30 per cwt.: what was the profit on the whole?

$6.30 — 5.85 = .45 gain per cwt.

lb. cwt. qr. lb. $ $

Then as 112 : 19 3 14 : : .45 : 8.94375. Ans.

2. Bought 28 lbs. of tea for $5.70, and sold it for $6.80: what was the gain per cwt.?

3. Paid $56 for 1 ton of steel, which I retailed at 6½ cents per lb.: what was the profit or loss by the sale of 5 tons?

4. Purchased 1000 yards of cloth, at $4.60 per yard, and sold the whole at $5.90 per yard: what was the whole gain?

§ **149.** CASE 2. To find the selling price of goods at a certain gain.

EXAMPLES.

1. At what price must I sell sugar per cwt. which cost $7.50 per cwt. to gain 12 per cent?

$ $ $ $

As 100 : 112 : : 7.50 : 8.40. Answer.

2. Bought soap at $7 per cwt.: at how much per lb. must I retail it to gain 10 per cent?

3. Purchased cotton stockings at 18 cents per pair: how must I sell them per pair to gain 20 per cent.?

4. Bought sugar at $7.85 per cwt.: how must I retail it per cwt. to gain 12 per cent.?

§ 150. CASE 3. To sell at a certain loss.

EXAMPLES.

1. Bought 120 yards of cloth for $360, which I find damaged: how must I sell it per yard so as to lose $30 by the whole?

Cost 360
To lose 30
330

yd. yd. $ $
Then as 120 : 1 : : 330 : 2.75. Ans.

2. I gave $38 for 2 cwt. 2 qrs. of tobacco, but becoming damaged, at what rate must I sell it per lb. to lose $10 by the whole?

3. Sold bacon at 7½ cents per lb. which I bought at 9¾ cents per lb.: what shall I lose by the sale of 7 cwt. 0 qrs. 24 lbs.?

4. A quantity of tea cost me 68 cents per lb., but proving damaged, how must I sell it per lb. to lose 10 per cent?

§ 151. CASE 4. To find the gain or loss per cent.

EXAMPLES.

1. If 7 cwt. 2 qrs. 15 lbs. of sugar cost $77.97½, what must it be sold at per lb. to gain 15 per cent.?

$ $ $ $
As 100 : 115 : : 77.97½ : 89.67125
7 cwt. 2 qrs. 15 lbs.=855 lbs.
And 89.67125÷855=.10,48 per lb. Answer.

2. If flannel cost 77½ cents per yard, how should it be sold per yard to clear 15 per cent.?

$ $ *cts. cts.*
As 100 : 115 : : 77½ : 89⅛ per yard. Answer.

3. How must I sell sugar per cwt. which cost me $8, to lost 20 per cent.?

100—20=80, Then as $ $ $ $
100 : 80 : : 8 : 6.40. Ans.

4. If I sell cheese at 6¼ cents per lb. which cost me 7½ cents per lb., what do I lose per cent.?

5. Bought coffee at 11½ cents per lb. and sold it at 14 cents per lb.: required the gain per cent.?

6. Bought flour at $5.60 per barrel: how should it be sold to gain 12½ per cent.?

7. If I buy cloth at $8.40 per yard, and sell it at $9.66, what is the gain per cent.?

8. A bankrupt's estate pays 60 cents in the dollar: what is the loss per cent.?

§ **152.** CASE 5. To find the prime cost of goods.

EXAMPLES.

1. If 350 yards of cloth be sold for $421.20, at 20 per cent. profit, what did it cost per yard?

$ $ $ $
As 120 : 100 : : 421.20 : 351 first cost.
yd. *yd.* $ $
Then as 350 : 1 : : 351 : 1.00⅞ per yard.

2. Sold 5 cwt. 2 qrs. of sugar for $38.50, gaining thereby 16⅔ per cent.: what was the cost of one cwt.?

3. In selling a hogshead of wine for $126, there is a gain of 25 per cent.: what was the prime cost?

4. Sold 126 yards of flannel for $95.60, and gained 20 per cent. by the bargain: required the prime cost per yard?

MISCELLANEOUS EXERCISES.

1. Bought knives for 20 cents each, and sold them at 17 cents each: how much is the loss by the sale of 120 dozen?

2. A draper bought 100 yards of cloth for $149: how must he sell it per yard to gain $51 in the whole?

3. Bought 60 reams of paper, at $2 per ream: what is the loss in the whole at 4 per cent.?

4. Bought a chest of tea, weighing 490 lbs., for $326, and sold it for $370.10: what was the profit on each lb.?

5. A draper bought 100 yards of cloth for $56: how must he sell it per yard to gain $19 in the whole?

6. Bought 60 reams of paper, at 75 cents per ream: what is lost in the whole quantity at 4 per cent.?

7. Paid $184 for 1 ton of steel: what is the profit or loss on the sale of 14 tons retailed at 7 cents per lb.?

8. A draper bought 100 yards of cloth for $134.40: how must he sell it per yard to gain 15 per cent.?

9. If 28 pieces of stuff be purchased at $9.60 per piece, and 10 of them sold at $14.40; and 8 at $12 per piece: at what rate must the rest be disposed of to gain 10 per cent. by the whole?

10. Sold 12 yards of silk for $15.20, by which was gained 8 per cent.: what was the prime cost of a yard?

11. Bought 12 pieces of cassimere for $16.50 per piece; paid $2.87 per piece for dying: for how much must I sell them each to gain 20 per cent.?

12. Bought 96 gallons of liquor for $10, but by accident 16 gallons of it were lost: how must I sell the remainder per gallon so as neither to gain or lose?

13. A merchant received from Lisbon 180 casks of raisins, which stand him here, in $1.60 each; and by selling them at $2.80 per cwt. he gains 25 per cent.: required the weight of each cask?

FELLOWSHIP.

§ **153.** Fellowship or partnership, is a rule by which merchants, &c. trading in company with a joint stock, ascertain their proper shares of the gain or loss, in proportion to their stock. By this rule a bankrupt's estate may be divided among his creditors; legacies are also adjusted by it, when there is a deficiency of assets or effects. Fellowship is either with or without time.

SINGLE FELLOWSHIP,

§ **154.** Is when the calculations are made in proportion to the shares only, without regard to time.

RULE. As the whole stock is to each man's share in the stock, so is the whole gain or loss to his share of the gain or loss.

EXAMPLES.

1. Three persons traded; A put in $75, B $155, and C $85; their gain was $65: what is each man's share thereof?

A	75	$	$	$	$ cts.
B	155	As 315	: 75	: : 65	: 15.47$\frac{13}{21}$ A's share
C	85				
	$315				

			$ cts.	
As 315	: 155	: : 65	: 31.98$\frac{26}{63}$	B's share.
As 315	: 85	: : 65	: 17.53$\frac{61}{63}$	C's share.

A's. B's. C's. $

Proof, $15.47\frac{13}{21}+31.98\frac{26}{63}+17.53\frac{61}{63}=65=$total gain.

2. A merchant at his decease owed to A $126.60, to B $241.50, to C $350.75, to D $470.50, and to E $550.65; but he left property to the amount of only $580: how much may each creditor receive?

$	$	$	$
A 126.60	as 1740	: 126.60	: : 580
B 241.50	580	42.20	A's, by cancelling.
C 350.75	as 1740	: 241.50	: : 580
D 470.50	580	80.50	B's.
E 550.65	as 1740	: 580.740	: : 580
1740.00	580	116.91⅔	C's.
	as 1740	: 740.580	: : 580
	580	156.83⅓	D's.
	as 1740	: 580.580	: : 580
	580	183.55	E's.

A's. B's. C's D's. E's.

Proof, 42.20+80.50+116.91⅔+156.83⅓+183.55=$580.

3. Two persons trade together; A puts into stock $240, and B $480, and they gain $300: what is each one's share of the profit?

4. Four merchants, D, E, F, G, made a stock; D put in $500, E 1000, F 1500, G 2000; by trading they gained in two years $2500: required each person's gain?

5. Four persons traded, and gained $1900, which was to be so divided that their shares might be to each other, as 1, 2, 3, and 4, respectively: what had each to receive?

6. A bankrupt is indebted to S $384.60, to T $786.75, and U $850.65, and his estate is worth but $1348: if the whole were divided how much would each creditor receive?

7. A ship worth $16000, being entirely lost, of which ⅛ belonged to W, ¼ to X, ¼ to Y, and ½ to Z: what loss will each sustain, supposing $4000 of her to have been insured?

8. Three merchants trading, lost goods to the value of $960; supposing A's stock was $1440, B's $5760, and C's $2400, what sum must each man sustain of the loss?

9. Suppose a merchant is indebted to M $336, P $1920, L $675, but upon his death his estate is found to be worth only $1966.56: how must it be divided among his creditors?

10. If the money and effects of a bankrupt amount to $3361.74, and he is indebted to C $1782.24, to D $1540.76, and to E $2371.17, how must it be divided among them?

11. O, P and Q, rent an estate, containing 720 acres, at $480 per annum., of which O holds 180, P 240, and Q 300 acres: what must each man pay, in proportion to the land he holds?

12. Three graziers, A, B and C, rent an estate, containing 585 acres 2 roods 32 perches, at $960 per annum, of which A pays $288, B $312, and C $360; they have agreed that the estate shall be divided in proportion to the rents: what is each man's share?

13. Four merchants, L, M, N, and O, freight a ship with 360 tuns of wine; L loaded 95 tuns, M 100, N 110, and O the rest; in a storm the seamen threw 72 tuns overboard: how much must each sustain of the loss?

14. Four persons join in the purchase of a house and lot for $2000; A paid $\frac{1}{3}$, B $\frac{1}{4}$, C $\frac{1}{5}$, and D the remainder; but the house being afterwards burnt down, and only $1500 insured: required what each lost?

15. A and B venturing equal sums of money, cleared by trade $1100; by agreement A was to have 8 per cent. on account of the time he spent in the execution of the project, and B was to have only 5 per cent.: what was A allowed for his trouble?

16. A person ignorant of numbers, left $3000 among his four children, and ordered that A should have $\frac{1}{3}$, B $\frac{1}{4}$, C $\frac{1}{5}$, and D $\frac{1}{6}$: what will be the just share of each, according to the intention of the donor?

DOUBLE FELLOWSHIP,

§ **155.** Is when the different shares are employed for different terms of time. When labor for a given time represents the capital invested this rule is applicable; but the profits arising from mercantile operations are rarely, if ever, apportioned by it.

RULE. 1st. Multiply each man's stock and time together.

2d. Add together the several products thence arising.

3d. Then say, as the sum of these products is to each man's particular product, so is the whole gain or loss to his share of the gain or loss.

EXAMPLES.

1. Three men jointly contract to construct a railroad: A employs 113 men for 6 months, B 80 men for 9 months, and

C 60 men for 10 months; and they gain $1320: what is each man's share of the gain?

113× 6=678	1998 : 678 : : $1320 :	447.92\frac{88}{111}$	A's.
80× 9=720	1998 : 720 : : $1320 :	475.67\frac{63}{111}$	B's.
60×10=600	1998 : 600 : : $1320 :	396.39\frac{7}{111}$	C's.
1998	Proof	$1320.00	

2. Three men engage, in company, to clear a raft of drift wood from a river: D employs 23 men for 9 weeks, E 25 men for 10 weeks, and F 30 men for 12 weeks. They gain $5000: what is each partner's portion of this sum?

23× 9=207	817 : 207 : : $5000 :	1266.82\frac{806}{817}$	D's.
25×10=250	817 : 250 : : $5000 :	1529.98\frac{634}{817}$	E's.
30×12=360	817 : 360 : : $5000 :	2203.18\frac{194}{817}$	F's.
817	Proof	$5000.00	

3. Two cellar diggers, A and B, together gain $250; A having employed 5 men for 3 months, and B 7 men for 5 months: how much of the sum gained should each have?

4. Three miners, Q, R and S, together gain $500; Q having employed 4 men for 6 months, R 4 men for 9 months, and S 10 men for 2 months: how much does each gain?

5. L, M and N hold a piece of grazing ground in common, for which they agree to pay $90 rent: L puts in 35 oxen for 30 days, M 25 oxen for 35 days, and N 40 oxen for 25 days: what portion of the rent should each pay?

6. Two persons, in company, contract to build a bridge: A employs 360 men for 18 days, and B 240 men for 27 days: supposing they gain $2100 by the job, what sum should each have?

7. Three men jointly contract to dig a race way: A employs 9 men for 30 days, B 5 men for 51 days, and C 12 men for 9 days; they lose $1200 by the operation: what is each partner's share of the loss?

8. Three men jointly contract to build a breakwater: T employs 30 men for 18 weeks, U 35 men for 21 weeks, and V 40 men for 24 weeks; they gain $1800: how much of the sum gained should each have?

9. A, B and C jointly engaged to dig the ditch of a fort, which was completed in 12 months: A at first employed 20 men, at the end of 4 months 30 men additional, and 4 months later discharged 6 men; B at first employed 50 men, at the

end of 5 months 12 men additional, and 5 months later added 20 men more to his force; C at first employed 60 men, at the end of 3 months discharged 40 of them, and 4 months later discharged 10 more of his men; they gained by the job only $319.20: what is each partner's share of it?

10. Three men jointly contract to make a turnpike road: A employs 28 men for $3\frac{1}{2}$ months, B 35 men for $3\frac{1}{4}$ months, and C 40 men for $2\frac{3}{4}$ months; they lose by the job $1050: how much should each contribute to make good this deficiency?

TAXES.

§ **156.** Taxes are sums required to be paid by the citizens of any country for the support of the government under which they live. The most common way in which the taxes are collected is from each individual in proportion to the amount of the property he possesses. In some states, however, every white male citizen, above the age of twenty-one years, is required to pay a certain tax, commonly known as a *poll-tax*, each person so taxed being called a *poll*. When a tax is to be laid upon a town or a section of country, the first operation to be performed is to make a complete inventory of all the property in that town or district. Then, if there is a poll-tax, the remainder which results from subtracting the product of the number of the polls, and the tax on each poll, from the whole tax to be raised, will be the tax to be raised on property both real and personal. Next find the tax on one dollar, by dividing the whole tax to be raised by the amount of taxable property. The tax on one dollar, when multiplied by the inventory of each individual, will give the tax on his property.

EXAMPLES.

1. A certain village is to be taxed $220.52. The value of the property on which the tax is to be levied is estimated at $10000. The poll-tax is 54 cents, and there are 38 polls. Required the amount of the taxes of a person whose property is estimated as worth $1200, and who pays 3 polls?

$0.54 × 38 = $20.52 amount of the poll taxes.

$220.52 − $20.52 = $200.00 amount to be levied on the property.

$200 ÷ $10000 = .02, or 2 cents on 1 dollar.

Then as the property is worth $1200, the tax on the property will be $1200×0.02=24.00, to which add his three polls=$1.62, and you will have $25.62, the whole amount of his taxes.

2. What will A's tax be if he own real estate to the amount of $12060, and personal property to the amount of $7866, if the city in which he resides, the inventory of which is $2903850, raise a tax of $10169.55, there being 2430 polls, which are taxed 60 cents each, and for one for which he pays?

2430×60=1458.00 amount of the poll taxes. Therefore $10169.55—$1458=$8711.55, the sum to be levied on the property. Hence 8711.55÷2903850=.003 mills, the tax on 1 dollar.

Then $12060×.003=$36.18, the tax on real estate, and $7866×.003=$23.598, the tax on personal estate, and 1 poll at 60 cents.

Therefore $36.18+$23.598+$0.60=$60.378, the whole amount of his taxes.

3. A tax of $1500 is to be raised on a certain town, the inventory of which is $30000; the number of polls 125, taxed each 75 cents: what is the amount of B's tax, whose property is valued at $1250, and who pays 1 poll.

INVOLUTION.

§ **157.** Involution is the raising of powers from any given number as a root.

A power is a quantity produced by multiplying any given number, called the root, a certain number of times continually by itself. Thus,

2= 2, the root, 1st power of 2.
2×2= 4, the 2d power, or square of 2.
2×2×2= 8, the 3d power, or cube of 2.
2×2×2×2=16, the 4th power of 2, &c.

And in this manner may be calculated the following table of the first nine powers of the first nine numbers.

A TABLE OF THE FIRST NINE POWERS OF NUMBERS.

Roots.	Squares.	Cubes.	4th power.	5th power.	6th power.	7th power.	8th power.	9th power.
1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512
3	9	27	81	243	729	2187	6561	19683
4	16	64	256	1024	4096	16384	65536	262144
5	25	125	625	3125	15625	78125	390625	1953125
6	36	216	1296	7776	46656	279936	1679616	10077696
7	49	343	2401	16807	117649	823543	5764801	40353607
8	64	512	4096	32768	262144	2097152	16777216	134217728
9	81	729	6561	59049	531441	4782969	43046721	387420489

The number denoting the power is called the index, or the exponent of that power; and is one more than the number of multiplications used in producing the same.

So 1 is the index or exponent of the first power or root, 2 of the second power or square, 3 of the third power or cube, 4 of the fourth power, &c.

Powers that are to be raised, are denoted by placing the index above the root or first power—thus,

2^2=4 the 2d power of 2.
2^3=8 the 3d power of 2.
2^4=16 the 4th power of 2.
341^5= the 5th power of 341, &c.

When two or more powers are multiplied together their product is that power whose index is the sum of the exponents of the factors, or powers multiplied, thus,

$2\times2=4$ the square of 2; $4\times4=16$ the fourth power of 2, and $16\times16=256$ the 8th power of 2.

Rule. Multiply the given number by itself, as many times less one, as there are units in the index of the required power.

To find any power of a fraction, bring each of the terms to the power required.

EXAMPLES.

1. What is the 6th power of 5?

$$5\times5\times5\times5\times5\times5=15625.$$

2. What is the 2d power of 38?
3. The cube of 29?
4. The 5th power of 45?
5. The 6th power of 23?
6. The 7th power of 3?
7. The 4th power 3.4?
8. The 3d power of .032?
9. What is the square of $\frac{24}{41}$?
10. The cube of $\frac{13}{14}$?
11. The 4th power of $\frac{5}{6}$?
12. The square of 32^2?
13. The cube of 8^3?
14. The square of 4^4?
15. The cube of 2^5?
16. The cube of 2^9?

Q. What is involution?
Q. What is the power, root, and exponent of a number?
Q. How do you find the power of a number?
Q. How do you raise a fraction to any power?

EVOLUTION.

§ 158. Evolution, or the extraction of roots, is the reverse of involution.

The root of any power is such a number as being multiplied into itself a certain number of times, will produce that power. Thus 2 is the square root of 4, because $2^2=2\times2=4$; and 5 is the cube root of 125, because $5^3=5\times5\times5=125$.

The root of a number is denoted by prefixing, with the degree of the root written over or within it, the sign $\surd$, thus 5th root of 13, is written $\sqrt[5]{13}$. But in writing the square root the degree of the root is omitted; thus the square root of 5 is expressed $\surd5$, and sometimes the root is written as a power having for its index the reciprocal of the number denoting the root. Thus 5th root of 13 would be written $13^{\frac{1}{5}}$.

Any power of a given number or root, may be found exactly, by multiplying the number continually into itself. But there are many numbers of which a proposed root can never be precisely determined. Yet by means of decimals we may approximate towards the root to any degree of exactness.

Those roots which only approximate, are called surd roots, surds, or irrational numbers; as $\surd3$, $\sqrt[3]{7}$, $\sqrt[4]{10}$, &c.,

while those which can be found exactly are called rational; as $\sqrt{16}=4$, $\sqrt[3]{27}=3$, $\sqrt[4]{256}=4$.

THE EXTRACTION OF THE SQUARE ROOT.

§ 159. The square of the sum of two numbers, equals the square of the numbers added to twice their product.

Thus
$$\begin{array}{r} 30+4 \\ 30+4 \\ \hline 900+120 \\ \quad 120+16 \\ \hline 900+240+16 \end{array}$$

Or $900+(30\times4\times2)+16=1156$, the square of 34.

Hence we derive the following

RULE. Divide the given number into periods of two figures each, by setting a point over the place of units, another over the place of hundreds, and so on over every second figure, both to the left hand in integers, and to the right in decimals.

Find by trial the greatest square in the first period on the left hand, and set its root on the right hand of the given number, after the manner of a quotient figure in division.

Subtract the square thus found from the said period, and to the remainder annex the next following period for a dividend. Double the root found, for a divisor, and find how often it is contained in the dividend, except the right hand figure, and set the result both in the quotient and divisor.

Multiply the whole augmented divisor by this last quotient figure, subtract the product from the said dividend, and bring down to it the next period for a new dividend.

Repeat the same process, viz. find another new divisor, by doubling all the figures now found in the root, from which and the last dividend, find the next figure of the root as before, and so on through all the periods to the last.

To extract the square root of a vulgar fraction, reduce it to its lowest terms, then extract the root of the numerator for a new numerator, and the root of the denominator for a new denominator, when both terms are complete squares; otherwise reduce it to a decimal, and extract the root by the general rule.

Proof. Multiply the root by itself and add the remainder.

If, when the given number is pointed off, as the rule requires, the left hand period should be deficient, it must nevertheless stand as the first period.

If there be decimals in the given number, it must be pointed both ways from the place of units. If, when there are integers, the first period in the decimals be deficient, it may be completed by annexing so many ciphers as the rule requires. And the root must be made to consist of so many whole numbers and decimals as there are periods belonging to each; and when the periods belonging to the given numbers are exhausted, the operation may be continued at pleasure by annexing ciphers.

EXAMPLES.

1. What is the square root of 77284?

$$\begin{array}{r} \dot{7}\dot{7}\dot{2}\dot{8}\dot{4}(278 \\ 4 \\ \hline 47)372 \\ 329 \\ \hline 548)4384 \\ 4384 \\ \hline \end{array}$$

ELUCIDATION.—In order to extract the square root of a number it is necessary, first, to ascertain how many figures the root sought will consist of; which may be done by placing a dot over every alternate figure, commencing with the units; the number of these dots will be the number of figures in the root; for since the square root of 100 is 10, the square root of every whole number less than 100 must consist of but one figure; and since the square root of 10000 is 100, it follows that the square root of every number between 100 and 10000 must consist of but two figures; likewise, since the square root of 1000000 is 1000, the square root of every number between 10000 and 1000000 must consist of but three figures, &c. in each of these cases the number of dots will be 1, 2, 3, &c. respectively. Here in this example the points indicate that the root consists of but three figures, and the greatest square contained in 7 is 4, therefore the first figure of the root is 2, whose square taken from 7 leaves 3, to which the two next figures, 72 being connected, gives 372 for the first dividend. Put 4, the double of the root figure, in the divisor's place, and it is contained in 37, 9 times; but

upon trial 9 is found to be too great, the next figure of the root is therefore 7, which number being placed in the divisor completes it; and the product of this divisor, and last root figure taken from the dividend, leaves 43, to which the remaining two figures are connected, and the same operation repeated. To exhibit, however, more clearly, let the figures of the number 77284 be represented according to their values in the arithmetical scale; thus the value of the first figure 7 is 70000, that of the second 7000, of the third 200, of the fourth 80, and of the last 4. Now, as it is necessary that the first term should be a square, and as in this case it is not, it will be proper to substitute for 70000, 40000+30000, 40000 being the greatest square contained in it; the operation will then be as follows:

40000+30000+7284 (200+70+8
40000 or
400+70 | 30000+7000 278
or or
470 | 37000
32900
400+140+8 | 4100+200+80+4
or or
548 | 4384
4384

2. What is the square root of $\frac{2116}{3364}$?
$\frac{2116}{3364}=\frac{9}{16}$, and the square root of $\frac{9}{16}=\frac{3}{4}$ Answer.
3. What is the square root of $76\frac{1}{7}\frac{2}{9}$?

76.823529(8.764+
64
167)1282
1169
1746)11335
10476
17524)85929
70096
15833

4. What is the square root of 1764?
5. What is the square root of 2401?
6. What is the square root of 7569?
7 What is the square root of 10201?

8. $\surd 20449$?	28. $\surd 1100401$?	48. $\surd 6319$?
9. $\surd 37249$?	29. $\surd 2614689$?	49. $\surd 7221$?
10. $\surd 41616$?	30. $\surd 3798601$?	50. $\surd .4$?
11. $\surd 59536$?	31. $\surd 6482116$?	51. $\surd .056$?
12. $\surd 77284$?	32. $\surd 10419984$?	52. $\surd .00789$?
13. $\surd 96721$?	33. $\surd 13972644$?	53. $\surd .009$?
14. $\surd 121801$?	34. $\surd 16386304$?	54. $\surd 368863$?
15. $\surd 150544$?	35. $\surd 20421361$?	55. $\surd 3.1721812$?
16. $\surd 160801$?	36. $\surd 34058896$?	56. $\surd .00032754$?
17. $\surd 190096$?	37. $\surd 36529936$?	57. $\surd \frac{9}{16}$?
18. $\surd 201601$?	38. $\surd 42680089$?	58. $\surd \frac{49}{64}$?
19. $\surd 268324$?	39. $\surd 60093504$?	59. $\surd \frac{324}{576}$?
20. $\surd 299209$?	40. $\surd 72726784$?	60. $\surd \frac{7}{2}$?
21. $\surd 381924$?	41. $\surd 7475.3316$?	61. $\surd \frac{14}{9}$?
22. $\surd 419904$?	42. $\surd 81.108036$?	62. $\surd 11\frac{1}{16}$?
23. $\surd 514089$?	43. $\surd .88981489$?	63. $\surd 7\frac{13}{36}$?
24. $\surd 692224$?	44. $\surd 930067.36$?	64. $\surd \frac{5}{12}$?
25. $\surd 790321$?	45. $\surd 9686.4964$?	65. $\surd \frac{1}{17}$?
26. $\surd 881721$?	46. $\surd 99.980001$?	66. $\surd 10\frac{3}{10}$?
27. $\surd 996004$?	47. $\surd 3717$?	67. $\surd 6\frac{1}{5}$?

68. $\surd 8639$?	71. $\surd 780811249$?
69. $\surd 236144689$?	72. $\surd 285970396644$?
70. $\surd 1607448649$?	73. $\surd 48303584206084$?

EXTRACTION OF THE CUBE ROOT.

§ 160. The cubes or third powers of two numbers increased by 300 times the square of the first number multiplied by the second, and also increased by 30 times the first multiplied by the square of the second, will give the cube of the sum of those numbers.

Thus $\left.\begin{matrix}20+6\\20+6\end{matrix}\right\}$ multiplied.

$400+120$

$\quad 120+36$

Multiplied $\left\{\begin{matrix}400+240+36=\text{2d power}\\20+6\end{matrix}\right.$

$8000+4800+720$

$\quad 2400+1440+216$

$8000+7200+2160+216=\text{3d power.}$

Or $8000+(300\times4\times6)+(30\times2\times36)+216$

Hence we derive the following:

RULE. 1. Divide the given numbers into periods of three places, beginning at the place of units, and pointing to the left in integers, and to the right in decimals.

2. Find the nearest root of the first period, which set in the quotient, subtract its cube from the period, and bring down the next period to the remainder for a new dividend.

3. Find a divisor by multiplying the square of the part of the root found by 300, divide the dividend by it, and place the quotient figure for the second figure in the root.

4. Multiply the part of the root formerly found by the last figure placed in the root, and this product by 30, which place under the divisor, under which write the square of the figure last placed in the root.

5. Multiply the sum of these three by the figure last placed in the root, and subtract the product from the dividend; to the remainder annex the next period for a new dividend, with which proceed as before.

If the given decimal has not as many places as are required, supply them by annexing ciphers.

To extract the cube root of a fraction reduce it to its lowest terms, and extract the cube roots of each term separately; or, reduce the fraction to a decimal, and then extract the root.

The reason for dividing the given number into periods of three figures each, is, that the cube of any single figure never exceeds three places, thus $9^3=729$.

EXAMPLES.

1. Extract the cube root of 41421736.

$$
\begin{array}{rl}
 & 41\dot{4}21\dot{7}36(346 \\
3^3=27 & \\
\hline
 & 14421 \\
3^2\times300\ =2700 & \\
3\ \times4\times30=\ 360 & \\
4^2\ \ =\ 16 & \\
\hline
\text{First divisor } 3076 & 12304 \\
\hline
 & 2117736 \\
34^2\times300\ =\ 346800 & \\
34\ \times6\times30=\ 6120 & \\
6^2\ \ =\ 36 & \\
\hline
\text{Second divisor } 352956 & 2117736 \\
\hline
\end{array}
$$

ELUCIDATION.—The nearest root of 41, the first period, is 3, which we set in the quotient as the first figure of the root sought, and subtract 27, its cube, which leaves a remainder of 14; to this bring down 421, the next period, which makes 14421. The square of 3 multiplied by 300, is 2700, the divisor, and the dividend divided thereby gives 4; then 4 times 3, multiplied by 30, is 360, and the square of 4 is 16. The sum of these three numbers is 3076, and being multiplied by 4 gives 12304, which being subtracted from 14421 leaves a remainder of 211; to this bring down 736, the next period, which makes 211736. The square of 34 multiplied by 300 is 346800 the divisor, and the dividend divided thereby gives 6; then 34 times 6 multiplied by 30 is 6120, and the square of 6 is 36, and the sum of these three numbers is 352956, which being multiplied by 6 gives 2117736, the same as the dividend. The root, therefore, is exactly 346.

2. $\sqrt[3]{132651}$.
3. $\sqrt[3]{658503}$.
4. $\sqrt[3]{970299}$.
5. $\sqrt[3]{2460375}$.
6. $\sqrt[3]{3307949}$.
7. $\sqrt[3]{5359375}$.
8. $\sqrt[3]{24137569}$.
9. $\sqrt[3]{63521199}$.
10. $\sqrt[3]{123505992}$.
11. $\sqrt[3]{212776173}$.
12. $\sqrt[3]{451.217663}$.
13. $\sqrt[3]{.700227072}$.
14. $\sqrt[3]{890277.128}$.
15. $\sqrt[3]{.7854}$.
16. $\sqrt[3]{20874107.909304}$.
17. $\sqrt[3]{1551328.215978515625}$.
18. $\sqrt[3]{7.15410916753}$.
19. $\sqrt[3]{\frac{54010152}{110592000}}$.
20. $\sqrt[3]{405\frac{28}{125}}$.
21. $\sqrt[3]{13\frac{3}{8}}$.

ANOTHER METHOD.

§ 161. RULE. By trials take the nearest cube to the given number, whether it be greater or less, and call it the assumed cube.

Then say, as the sum of the given number and double the assumed cube is to the sum of the assumed cube and double the given number, so is the root of the assumed cube to the root required nearly. Again, by using, in like manner, the cube of the root last found as a new assumed cube, another root will be obtained still nearer.

EXAMPLES.

1. What is the cube root of 21035.8?

Here we soon find that the root lies between 20 and 30, and then between 27 and 28. Taking therefore 27, its cube is 19683, which is the assumed cube.

Then 19683 21035.8
2 2
39366 42071.6
21035.8 19683.
As 60401.8 : 61754.6 : : 27 : 27.6047 the root nearly.

Again, assuming 27.6 and working as before, the root will be found 27.60491.

2. $\sqrt[3]{2.439}$. | 4. $\sqrt[3]{5.842}$. | 6. $\sqrt[3]{.7759}$. | 8. $\sqrt[3]{7121.10216198}$.
3. $\sqrt[3]{2.363}$. | 5. $\sqrt[3]{.6199}$. | 7. $\sqrt[3]{10.00}$. | 9. $\sqrt[3]{61218.00121}$.

Q. What is evolution?
Q. What is the rule for extracting the square root?
Q. How do you find the square root of a vulgar fraction?
Q. How do you point off when there are decimals in the given number?
Q. Why do you separate the figures of the dividend into periods?
Q. How is the operation proved?
Q. What is the rule for extracting the cube root?
Q. What other method of finding the cube root is given?

ARITHMETICAL SYMBOLS.

§ **162.** Certain signs prefixed to numbers indicate the operations that are to be performed.

+ This sign denotes addition; and is read, plus or more.
— This sign denotes subtraction; and is read, minus or less.
× The sign of multiplication; and is read, multiplied by.
÷ The sign of division; and is read, divided by.
$\frac{6\,7}{1\,7}$ or $\frac{3\,4\,2}{9\,9}$ Numbers placed like a fraction also denote division; the upper number being to be divided by the lower.
= The sign of equality; and is read, equal to.

A number placed above another number, a little to the right, is called an exponent; as 3^2, 5^3, and denotes that the quantity is to be used as a factor as many times as there are units in the exponent, as $3^3=3\times3\times3=27$.

: is to; : : so is; : to; the signs of proportion.
$\sqrt{}$ or $\sqrt[3]{}$ Signs of the square root.
$\sqrt[3]{}$ and $\sqrt[4]{}$ Signs of the cube and biquadrate root

A vinculum is a line drawn over quantities, and it is employed to signify that the total result of these is to be considered but one quantity; which said resulting quantity is to be operated upon by some affixed number or expression. The parenthesis ( ) or bracket [ ] implies the same.

EXEMPLIFICATION. The arithmetical expression $3+7+4+1$ is read, three plus seven plus four plus one. The expression 17—8 is read, seventeen minus eight. The expression 9×7 is read, nine multiplied by seven. The expression $48\div8$ is read, forty-eight divided by eight. A quantity that is to be divided is more conveniently expressed by placing it over the quantity by which it is to be divided; thus, instead of $69\div9$, it may be written $\frac{69}{9}$; and $141\div8$ is implied by $\frac{141}{8}$; and $349\div112$ is the same as $\frac{349}{112}$.

The expression $7+3—1=7+2$ denotes that the aggregate of the numbers $(7+3—1)$ on one side of the sign $(=)$, is of the same value as the sum of those $(7+2)$ on the other side of the sign; and therefore, seven plus three minus one, are read, equal to seven plus two.

What is the total amount of the expression of $6\times5+2—4$? It is read, six times five plus two minus four; now $6\times5+2—4=30+2—4=32—4=28$.

What is the value of $14—8+4\times3$? It is read, fourteen minus eight plus four times three; now $14—8+4\times3=14—8+12=26—8=18$.

What is the value of $4\times6\times5+8\times3—7\times2$? It is read, four times six times five plus eight times three minus seven times two; now $4\times6\times5+8\times3—7\times2=120+24—14=144—14=130$.

What is the value of $63\div7+3\times8—24\div6$? It is read, sixty-three divided by seven plus three times eight minus twenty-four divided by six: now $63\div7+3\times8—24\div6=9+24—4=33—4=29$.

What is the value of $12\times8\div6+14$? It is read, twelve times eight divided by six plus fourteen; now, $12\times8\div6+14=96\div6+14=16+14=30$.

What is the value of $5\times32\div4\times8$? It is read, five times thirty-two divided by four times eight; now $5\times32\div4\times8=160\div32=5$.

What is the value of $6\times30\times12\div4\times90—2$? It is read, six times thirty times twelve divided by four times ninety minus two; now $6\times30\times12\div4\times90—2=6\times360\div360—2=2160\div360—2=6—2=4$.

What is the value of $96\times32\div8\times24\times2+14-18$? It is read, ninety-six times thirty-two divided by eight times twenty-four times two plus fourteen minus eighteen; now $96\times32\div8\times24\times2+14-18=3072\div384+14-18=8+14-18=22-18=4$.

The expression $\overline{3+6}\times7$ becomes $9\times7=63$. The expression $\overline{4-3+12}\times6$ becomes $13\times6=78$. The expression $(11-6)\times30$ becomes $5\times30=150$. The expression $(12-2+3-4)\times(10-2)$ becomes $9\times8=72$. The expression $(14+2)\div(29-25)$ becomes $16\div4=4$. The expression $(6\times5-6)\times(3\times4+2\times3)$ becomes $(30-6)\times(12+6)=24\times18=432$. The expression $(2\times6\times3-9+4\times2)\div7$ becomes $(36-9+8)\div7=35\div7=5$. The expression $(14-2\times3+8)\times6\div32$ becomes $(14-6+8)\times6\div32=16\times6\div32=96\div32=3$.

The expression $(26-6)\times4\div(7\times3-5)$ becomes $20\times4\div16=80\div16=5$. The expression $(8-2)\times(11\times3-4\times6)\div2\times9$ becomes $6\times9\div2\times9=54\div18=3$. The expression $(80\div2-3\times8+5)\times(11+6\div3-2\times4)$ becomes $(40-24+5)\times(11+2-8)=21\times5=105$. The expression $(4\times6-11\times8+12\times14)\times(42\div6+9)\times(2\times3\times7-21\times13\div39)$ becomes $(24-88+168)\times(7+9)\times(42-7)=104\times16\times35=58240$.

The expression $[(\frac{3}{70}+\frac{1}{28})\div\frac{1}{21}-\frac{3}{8}]\times\frac{1}{3}-\frac{1}{20}$ becomes $(\frac{11}{140}\div\frac{1}{21}-\frac{3}{8})\times\frac{1}{3}-\frac{1}{20}=(\frac{22}{140}-\frac{3}{8})\times\frac{1}{3}-\frac{1}{20}=(\frac{5}{40}\times\frac{1}{3}-\frac{1}{20}=\frac{17}{40}-\frac{1}{20}=\frac{3}{8}$.

The expression $2^2+7^3-4^2+3^4$ becomes $4+343-16+81=428-16=412$. The expression $\frac{(\frac{2}{3})^2+(\frac{1}{2})^3}{(\frac{1}{2}+\frac{1}{4})^2}$, becomes $\frac{41}{72}\div\frac{9}{16}=\frac{82}{81}$.

The expression $\frac{(1.2^3-\frac{3.6^2}{10})\div4}{(3.6-1.2)^2\div6}$ becomes

$$\frac{(1.728-1.296)\div4}{(2.4)^2\div6}=\frac{.432\div4}{5.76\div6}=\frac{.108}{.96}=.1125.$$

The expression $(\sqrt{64}-\sqrt[3]{125})\times3$ becomes $(8-5)\times3=3\times3=9$.

The expression $\sqrt{72}+\sqrt{128}$ becomes $\sqrt{(36\times2)}+\sqrt{(64\times2)}=6\sqrt{2}+8\sqrt{2}=14\sqrt{2}$.

The expression $\sqrt[3]{40}+\sqrt[3]{135}-\sqrt[3]{\frac{5}{27}}$ becomes $\sqrt[3]{(8\times5)}+\sqrt[3]{(27\times5)}-\sqrt[3]{(\frac{1}{27}\times5)}=2\sqrt[3]{5}+3\sqrt[3]{5}-\frac{1}{3}\sqrt[3]{5}=5\sqrt[3]{5}-\frac{1}{3}\sqrt[3]{5}=4\frac{2}{3}\sqrt[3]{5}$.

EXAMPLES.

1. What is the value of $3+8+11+17$?
2. What is the value of $29-18$?
3. What is the value of 31×7?
4. What is the value of $112\div14$?
5. What is the value of $9+5-4$?
6. What is the value of $5\times7+4-3$?
7. What is the value of $16-9+5\times2$?
8. What is the value of $3\times7\times9+4\times6-8\times12$?
9. What is the value of $56\div8+4\times5-12\div3$?
10. What is the value of $8\times5\div10+12$?
11. What is the value of $10\times16\div4\times8$?
12. What is the value of $3\times15\times6\div2\times45-1$?
13. What is the value of $48\times16\div4\times12\times2+7-9$?
14. What is the value of $\overline{5+7}\times8$?
15. What is the value of $\overline{6-3+11}\times6$?
16. What is the value of $(14-4+5-6)\times(12-2)$?
17. What is the value of $(12+3)\div(29-26)$?
18. What is the value of $(5\times4-6)\times(2\times7+3\times2)$?
19. What is the value of $(3\times7\times2-5+6\times3)\div14$?
20. What is the value of $(15-3\times2+9)\times7\div21$?
21. What is the value of $(28-4)\times5\div(9\times2-3)$?
22. What is the value of $(7-4)\times(12\times3-2\times7)\div3\times11$?
23. What is the value of $(60\div2-4\times9+7)\times(12+8\div4-3\times2)$?
24. What is the value of $(7\times3-12\times9+10\times13)\times(49\div7+11)\times(3\times4\times5-18\times4\div36)$?
25. What is the value of $\frac{5}{8\,4}+\frac{3}{3\,0\,8}-\frac{1}{7\,8}$?
26. What is the value of $(\frac{3}{5}+\frac{1}{5})\times\frac{4}{5}$?
27. What is the value of $(\frac{3}{4}+\frac{1}{2}-\frac{5}{6})\div(\frac{4}{5}-\frac{2}{3})$?
28. What is the value of $(\frac{3}{3\frac{5}{7}}+\frac{3}{14})\div\frac{3}{8}-\frac{1}{8}$?
29. What is the value of $(\frac{15}{16}+.906)\div.92$?
30. What is the value of $(\frac{2}{4}-.475+\frac{3}{5})\times3.3765$?
31. What is the value of $(7\frac{1}{3}+.997-5.504)\div6.75$?
32. What is the value of $(11\frac{3}{5}+14.775)\div(4.375-\frac{3}{4})$?
33. What is the value of $2.1\div(6.39-1.5)+3.75$?
34. What is the value of $\dfrac{13.11-5.25}{14\frac{1}{4}-7\frac{1}{2}}$?
35. What is the value of $(\frac{3.6}{7.25}+3.942)\div6$?
36. What is the value of $\dfrac{12+.64\div4-10}{(12+.64)\div4-3}$?

37. What is the value of $\frac{5\frac{1}{2}+\cdot\frac{7\frac{1}{4}}{.5}-.725}{(4+3.45)\div 2\frac{1}{2}}$?

38. What is the value of $\frac{12\frac{1}{2}+2\frac{1}{4}}{45}\div\frac{9\frac{1}{3}-4\frac{1}{2}}{.75}$?

39. What is the value of $\frac{9\frac{4}{5}-\frac{1}{2}}{5\frac{3}{4}}\times\frac{4\frac{1}{5}-3\frac{1}{2}}{2\frac{1}{2}}$?

40. What is the value of $[(\frac{3}{35}+\frac{1}{14})\div\frac{2}{21}-\frac{3}{4}]\times\frac{2}{3}-\frac{1}{10}$?

41. What is the value of

$$\frac{6+.68\div 2-1.2}{(6+.68)\div 2-1.2}\div\frac{6+.68\div(2-1.2)}{(6+.68)\div(2-1.2)}?$$

ARITHMETICAL PROGRESSION,

§ **163.** Is when a series of numbers increases or decreases regularly, by the continual addition or subtraction of some equal number, called the common difference; so 1, 2, 3, 4, 5, 6, &c. are in arithmetical progression, by the continual addition of one; and 11, 9, 7, 5, 3, 1, by the continual decreasing or subtracting of two.

The numbers which form the series are called the terms of the progression, of which there are five.

The first term and the last term, (called the extremes,) the number of terms, the common difference and the sum of all the terms. Any three of these being given, the other two may be readily found.

§ **164.** CASE 1. The first term, the last term, and the number of terms, being given, to find the sum of all the terms.

RULE. Multiply the sum of the extremes by the number of terms, and half the product will be the answer.

EXAMPLES.

1. The first term of an arithmetical progression is 2, the last term 53, and the number of terms 18: required the sum of the series?

Thus $2+53=55$, and $\frac{55\times 18}{2}=495$ Answer.

Or $\frac{(2+53)\times 18}{2}=495$ Answer.

2. The first term is 1, the last term 21, and the number of terms 11: required the sum of the series?

3. How many strokes do the clocks of Venice strike in 24 hours, from 1 to 24?

4. If 100 stones are placed in a right line, 3 yards asunder, and the first 3 yards from a basket, what length of ground will a man go over who gathers them up one by one, returning with each to the basket?

5. The first term of an arithmetical progression is 5, the last is 74, and the number of terms 24: what is the sum of the series?

6. What debt may be discharged in a year, by weekly payments, in arithmetical progression, of which the first payment is 15 cents, and the last $5.25?

7. A mercer bought 30 yards of silk, and gave for the first yard 15 cents, and for the last $3.05, increasing in arithmetical progression: what did the whole cost him?

§ **165.** CASE 2. The first term, the last term, and the number of terms, being given to find the common difference.

RULE. From the last term subtract the first, the remainder divided by the number of terms, less one, gives the common difference.

EXAMPLES.

1. The extremes are 2 and 53, and the number of terms is 18: required the common difference?

$$\text{Thus } \frac{53-2}{18-1}=\frac{51}{17}=3$$

2. The extremes are 1 and 24, and the number of terms 24: required the common difference?

3. The first term is 6, the last term 600, and the number of terms 100: what is the common difference?

4. The first term of an arithmetical progression is 5, the last is 74, and the number of terms 24: what is the common difference?

5. The first and last terms of an arithmetical series are 3 and 105, and the number of terms 52: what is the common difference?

6. Bought 30 yards of cloth, and gave for the first yard 15 cents, and for the last $3.05: what is the common difference of the price of each yard?

7. If the extremes be 3 and 19, and the number of terms 9, it is required to find the common difference, and the sum of the whole series?

8. A man is to travel from Washington to a certain place in 12 days, and to go but 3 miles the first day, increasing every day by an equal excess, so that the last day's journey may be 58 miles: required the daily increase, and the distance of the place from Washington?

§ 166. CASE 3. Given the first term, the last term, and the common difference, to find the number of terms.

RULE. Divide the difference of the extremes by the common difference, and the quotient increased by 1 is the number of terms required.

EXAMPLES.

1. The extremes are 2 and 53, and the common difference 3: what is the number of terms?

Thus $\frac{53-2}{3}=17$, and $17+1=18$ Answer.

2. If the extremes be 5 and 74, and the common difference 3, what is the number of terms?

3. If the first and last terms of an arithmetical progression are 3 and 105, and the common difference 2, what is the number of terms?

4. Bought cloth, and gave for the first yard 15 cents, and for the last $3.05, the common difference of the price of each yard is 10 cents: required the number of yards?

5. A man going a journey travelled the first day 12 miles, the last day 64 miles, and each day increased his journey by 4 miles: how many days did he travel?

§ 167. CASE 4. The last term, the number of terms, and the common difference given, to find the first term.

RULE. Multiply the number of terms less one, by the common difference, the product subtracted from the last term, gives the first.

EXAMPLES.

1. A person in 13 days travelled to a certain place, every day's journey increasing the former by 5, and the last he travelled was 64 miles: what was the first?

$(13-1)\times 5=60$, and $64-60=4$ Answer.

2. A person had 9 sons, his family having increased 1 every 3 years; the eldest was 29 years of age: what was the age of the youngest?

3. If the last term be 74, the number of terms 24, and the common difference 3, what is the first term?

4. If the last term be 105, the number of terms 52, and the common difference 2, what is the first term?

5. Purchased 30 yards of cloth, and gave for the last yard $3.05, the common difference of the price of each yard was 10 cents: how much was given for the first yard?

§ **168.** CASE 5. The common difference, the number of terms, and the sum of all the terms, given to find the first term.

RULE. Divide the sum of all the terms by the number of terms, and from the quotient subtract half of the product of the common difference, multiplied by the number of terms less one for the answer.

EXAMPLES.

1. A man is to receive $540, at 12 several payments, each exceeding the former by 6: what will be the first payment?

$$540 \div 12 = 45$$
$$(12-1) \times 3 = 33$$
$$\overline{12} \text{ the first payment.}$$

2. A person had 9 sons, his family having had an increase of 1 every 3 years, and the amount of all their ages was 153: what was the age of the youngest?

3. If the common difference be 2, the number of terms 52, and the sum of all the terms 2808, what will be the first term?

§ **169.** CASE 6. The first term, the number of terms, and the common difference given to find the last term.

RULE. Multiply together the number of terms and the common difference; from that product subtract the common difference; to that remainder add the first term, and it will give the last.

EXAMPLES.

1. A person received a sum at 12 several payments, the first was $12, and each succeeding payment exceeded the former by 6: what was the last payment?

Thus $(12\times6)-6=66$
Add the first term 12
Answer 78 the last term.

2. The first term of an arithmetical series is 5, the number of terms is 24, and the common difference is 3: what is the last term?

3. What is the last number of an arithmetical progression beginning with 3, and continuing by the increase of 2 to 30 places?

GEOMETRICAL PROGRESSION.

§ 170. Any series of numbers, the terms of which gradually increase or decrease by a constant multiplication or division, is said to be in geometrical progression. Thus 1, 2, 4, 8, 16, 32, 64, &c., and 81, 27, 9, 3, 1, &c., are series in geometrical progression, the one increasing by a constant multiplication by 2, and the other decreasing by a constant division by 3.

The number by which the series is constantly increased or diminished, is called the ratio.

Any three of the five following terms being given, the rest may be readily determined.

1. The first term, } commonly called the extremes.
2. The last term, }
3. The number of terms.
4. The ratio.
5. The sum of all the terms.

In order to abridge the tedious operation of finding the last term of a long series of numbers by continual multiplication, we use a series of numbers in arithmetical proportion called indices; the first term of which is unity, and the common difference one. When any number of indices are used, as many terms of the geometrical series should be written under them. Thus

1. 2. 3. 4. 5. 6. 7. 8. indices.
2. 4. 8. 16. 32. 64. 128. 256. numbers in geometrical proportion.

But if the first term in geometrical proportion be different from the ratio, the indices must begin with a cipher, thus:

0. 1. 2. 3. 4. 5. 6. 7. indices.
1. 2. 4. 8. 16. 32. 64. 128. numbers in geometrical proportion.

When the indices begin with a cipher, the sum of the indices made choice of must always be one less than the number of terms given in the question, for one in the indices stands over the second term, and two in the indices over the third, &c.

Add any two of the indices together, and that sum will agree with the product of their respective terms; thus,

In either table of indices, $2+5=7$ and $2+4=6$.

So in geometrical proportion, $4\times32=128$ and $4\times16=64$.

In any geometrical progression of three terms the square of the mean term is equal to the product of the extremes. Thus, in 2, 6, 18, it will be $2\times18=6^2=36$, and the same of any series of three terms.

The middle term is called a geometrical mean between the two extremes, and is always equal to the square root of their product.

In any geometrical series of four terms the product of the two means is equal to that of the two extremes. Thus, if $3:6::12:24$, $3\times24=6\times12$.

If any number of terms be continued in geometrical progression, the product of the two extremes will be equal to the product of any two means, equally distant from the extremes; where, as in 2, 4, 8, 16, 32, 64, $2\times64=4\times32=8\times16$, each product being 128.

If the number of terms be odd, then the square of the mean will be equal to any two terms equally distant, as in 1, 3, 9, 27, 81, 243, 729, where $1\times729=3\times243=9\times81=$ the square of the mean 27, the product of each being 729.

§ **171.** CASE 1. Given the first term, the last term, and the ratio, to find the sum of the series.

RULE. Multiply the last term by the ratio, and from the product subtract the first term; the remainder divided by the ratio, less one, will give the sum of the series.

EXAMPLES.

1. The first term of a series in geometrical progression is 1, the last term is 2187, and the ratio 3: what is the sum of the series?

Thus $\frac{(2187\times3)-1}{3-1}=\frac{6561-1}{2}=3280$. Answer.

2. The extremes of a geometrical progression are 1 and 65536, and the ratio 4: what is the sum of the series?

3. The extremes of a geometrical series are 1024, and 59049, and the ratio is $1\frac{1}{2}$: what is the sum of the series?

4. The first term of a series in geometrical progression is 2, the last term is 13122, and the ratio 3: what is the sum of the series?

5. Sold 12 bushels of wheat, and received for the first bushel one cent, and for the last 1048576 cents: the ratio of each bushel is 4: what were the 12 bushels sold for?

6. The extremes of a geometrical series are 20 and 10000, and the ratio is 2: what is the sum of the series?

7. A thresher worked at a farmer's 24 days during the winter months, and received for his first day's work 2 grains of wheat, for the second 4, for the third 8, &c., doubling them each day, and for the last day's work 16777216 grains: the sum of the series is required?

§ **172.** CASE 2. Given the first term, the number of terms, and the ratio, to find the last term.

1. When the first term is equal to the ratio.

RULE. 1. Find a few of the leading terms, over which place their indices.

2. Find what figures of these indices, when added together, will give the index of the terms wanted.

3. Multiply the numbers standing under such indices into each other, and the last product will be the term required.

EXAMPLES.

1. A man agrees for 20 horses, to pay only the price of the last, reckoning 3 cents for the first, 9 cents for the second, &c., trebling the price to the last, the ratio being 3: what must he give?

1. 2. 3. 4. 5. 6. 7. indices.

3. 9. 27. 81. 243. 729. 2187. terms in geometrical proportion.

Take any of the indices, which added together make 20 and multiply by the terms underneath.

Thus $7+7+6=20$

Multiply 2187= 7

by 2187= 7

4782969=14

and by 729= 6

Answer $34867844.01=20

2. What is the last term of a geometrical series, having 12 terms, of which the first term is 2, and the ratio 2?

3. A draper sells 21 yards of cloth, the first yard for 3 cents, the second for 9 cents, the third for 27 cents, &c. in a triple proportion geometrical: required the price of the last yard.

2. When the first term is not equal to the ratio.

RULE. 1. Write down a few of the leading terms, as before, and place over them their indices, beginning with a cipher.

2. Find what figures of the indices, when added together, less one, will give the index of the term wanted.

3. Multiply the numbers standing under such indices into each other, observing to divide every product by the first term.

EXAMPLES.

1. The first term of a geometrical series is 5, the ratio 3, and the number of terms 15: what is the last term?

0	1	2	3	4	5
5	15	45	135	405	1215

1215= 5
1215= 5

5)1476225
295245
405= 4

5)119574225=14
23914845 Answer.

2. A man agrees to purchase 22 yards of cloth, at 3 cents for the first yard, 6 cents for the second, 12 cents for the third, &c., the ratio being 2, geometrical proportion: what was the charge for the last yard?

3. A sum of money is to be divided among 9 persons; the first is to have $30, the second $60, the third $120, &c.: what will the last receive?

4. A moneyed man, ignorant of numbers, left 12 sons, and bequeathed his estate thus: to his executor $50, to his youngest son double that sum, and each son was to exceed the next younger by as much more: what was the oldest son's portion?

§ **173.** CASE 3. The first term, the number of terms, and the ratio given, to find the sum of all the terms.

RULE. Find the last term, as before, then subtract the first from it, and divide the remainder by the ratio less one, to the quotient of which add the last term, and it will give the sum required.

EXAMPLES.

1. A servant agreed to serve his master 12 months, for 1 cent the first month, 3 cents the second, 9 cents the third, &c., the ratio being 3: what did his wages amount to?

0 . 1 . 2 . 3 . 4 . 5 . 6 indices.
1 . 3 . 9 . 27 . 81 . 243 . 729 terms in geom. progr.

729= 6
243= 5
177147=11

Then $\frac{177147-1}{\text{ratio } 3-1}=88573$,

And 88573+the last term 177147=$2657.20 Ans.

2. A man bought a horse, and by agreement was to give 1 cent for the first nail, 2 for the second, 4 for the third, &c., the ratio being 2; the number of nails was 32: what was the price of the horse?

3. One new-year's day a gentleman married, and received of his father-in-law 1 dollar, with a promise that it should be doubled on the first day of every month for one year: what was the lady's portion?

4. A person agrees to purchase 15 yards of lace, for three pins the first yard, 9 pins the next, &c. in treble proportion; reckoning 100 pins for 1 cent, what is the amount?

5 A goldsmith, when his customer refused to give $200 per lb. for gold, offered to sell it him at a cent for the first ounce, four for the second, &c., in quadruple proportion geometrical, to which he agreed: what did the lb. of gold cost him?

§ **174.** CASE 4. The first term, the last term, and the sum of the series being given to find the ratio.

RULE. From the sum of the series subtract the first term, and divide the remainder by the difference between the sum of the series and the last term, and the quotient will be the ratio required.

EXAMPLES.

1. If the extremes of a series be 2 and 13122, and the sum of the series be 19682, what is the ratio?

$$\frac{19682-2}{19682-13122}=\frac{19680}{6560}=3 \text{ the ratio. Ans.}$$

2. Sold wheat, and received for the first bushel 1 cent, and for the last 1048576 cents, and for the whole 1398101 cents: required the ratio?

3. A person bought 10 acres of land, and gave for the first acre 3 cents, for the last 59049 cents, and for the whole 88572 cents: what was the ratio?

4. A gentleman received as his wife's portion, in one year $4095, by monthly payments in geometrical progression; his first receipt was $1, and his last $2048: what was the ratio?

PERMUTATION.

§ **175.** To find in how many different ways any given number of things may be arranged, we have the following rule of permutation.

RULE. Multiply continually together all the terms of the natural series of numbers, from 1 up to the given number, and the product will be the answer required.

EXAMPLES.

1. How many different ways may six different notes in music be varied?

$$1\times2\times3\times4\times5\times6=720 \text{ Answer.}$$

2. For how many days can seven persons be placed in a different position round a table at dinner?

$$1\times2\times3\times4\times5\times6\times7=5040.$$

3. How many changes can be made of the letters in the word Charleston?

4. In those forming the word Philadelphia?

5. How many permutations may be made of the letters of the alphabet?

6. How many changes may be rung on 12 bells, and how long would they be in ringing them once over, supposing 26 changes to be rung in one minute, and the year to consist of 365¼ days?

COMBINATION.

§ **176.** By combination we learn how often a less number of things can be taken out of a greater and combined together, without reference to their order or position.

To find how many changes may be made out of any given number of different things, by taking any given number of them at a time, we have the following

RULE. Form a series of numbers, beginnning at the number of things given, and decreasing by 1 to the number of quantities to be taken at a time. The required answer will be the continued product of all the terms of this series.

EXAMPLES.

1. How many changes may be rung with 3 bells out of 9?

$$9\times8\times7\times6\times5\times4=60480 \text{ Answer.}$$

2. A pedler wished to purchase several pieces ot check from a merchant, but objected to the price. He finally bought 8 pieces, agreeing to give the merchant two cents for each different choice of 8 in 16: what did the 8 pieces cost him?

3. How many different cubes may be made with 27 out of 34 cubical blocks?

ALLIGATION.

§ **177.** Alligation is a rule that teaches either to find the value of any compound, or how to mix things of different values so as to ascertain the quantities. The whole may be comprised in four cases, viz. medial, alternate, partial and total.

§ **178.** CASE 1. Alligation medial. When the rates and quantities of the several ingredients are given, to find the value of the mixture.

RULE. Multiply each quantity of the mixture by its rate, then divide the sum of the products by the sum of the quantities, and the quotient will give the rate of the mixture required?

EXAMPLES.

1. If 19 bushels of rye, at 30 cents per bushel; 40 of corn, at 20 cents; and 12 of oats, at 15 cents, be mixed together, what is a bushel of this mixture worth?

$$\begin{array}{rl} 19 \times 30 = & 570 \\ 40 \times 20 = & 800 \\ 12 \times 15 = & 180 \\ \hline 71 & 71)\overline{1550}(21\frac{59}{71} \text{ cents. Answer.} \end{array}$$

2. If 4 ounces of silver, worth 60 cents the ounce, be melted with 8 ounces, at 48 cents, what is 1 ounce of the mixture worth?

3. A wine merchant mixes 12 gallons of wine, at 58 cents the gallon, with 24 gallons, at 66 cents, and 16 at 75 cents: what is a gallon of this mixture worth?

4. A goldsmith melted together 8 ounces of gold 22 carats fine, 1 lb. 8 ozs., 21 carats fine, and 10 ozs., 18 carats fine: what is the quality of the composition?

5. A refiner melted 5 lbs. of silver bullion 8 ozs. fine, with 10 lbs. of 7 ozs., and 15 lbs., 6 ozs. fine: of what fineness is 1 lb. of this mass?

6. A composition is made of 18 lbs. of tea, at 66 cents per lb., with 20 lbs. at 69 cents, 24 lbs. at 75 cents, and 16 lbs. at 78 cents per lb.: what is the worth of 3 lbs. of this mixture?

7. A vintner compounds a pipe of wine, of 36 galls. at $1.44 per gall., 40 galls. at $1.56 per gall., and 50 galls. at $1.68 per gall.: what will a gallon of this mixture be worth?

§ **179.** CASE 2. Alligation alternate. When the rates of several things are given, to find what quantity of each must be taken to make a mixture of a certain mean value.

RULE. 1. Place the rates of the ingredients under each other, and place the mean rate on the left hand of them.

2. Link the several rates together, so that one greater than the mean rate may be joined to one that is less.

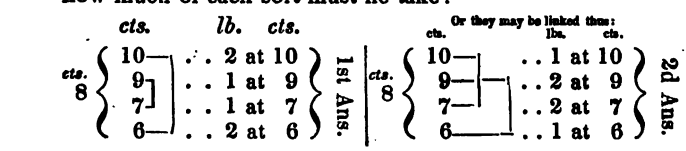
3. Take the difference between each price and the mean rate, and place it opposite to the rate to which it is linked.

4. If only one difference stand against any rate, that difference will be the answer; but if more than one, their sum will be the answer.

Proof. By alligation medial.

EXAMPLES.

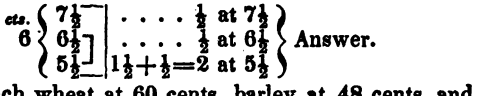
1. A grocer would mix sugar, at 10 cents, 9 cents, 7 cents, and 6 cents per lb., to make a mixture worth 8 cents per lb.: how much of each sort must he take?



2. A tobacconist would mix tobacco at 42 cents, 36 cents, 24 cents, and 18 cents per lb.: what quantity of each must he take to make a mixture worth 30 cents per lb.?

3. A maltster has several sorts of malt, at 48 cents, 60 cents, 72 cents, and 78 cents per bushel: how much of each sort must be taken to make a mixture worth 66 cents per bushel?

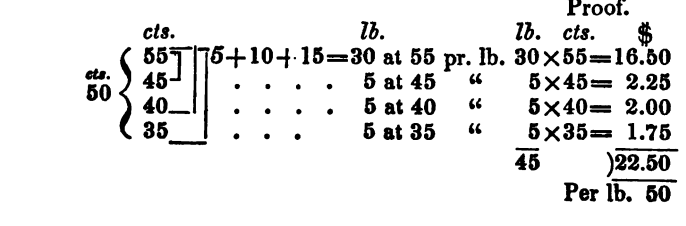
4. What quantity of raisins at $7\frac{1}{2}$, $6\frac{1}{2}$, and $5\frac{1}{2}$ cents per pound, must be mixed together to sell at 6 cents per pound?



5. How much wheat at 60 cents, barley at 48 cents, and oats at 36 cents, will make a mixture worth 42 cents per bushel?

6. A brewer had ale at 16 cents, 12 cents, and 8 cents per gallon: how much of each sort must he take to sell at 10 cents per gallon?

7. A tea dealer has several sorts of tea, at 55 cents, 45 cents, 40 cents, and 35 cents per pound: how much of each sort must be used that the whole quantity may be afforded at 50 cents per pound?



8. How many ounces of gold, of 22, 18, 17 and 16, carats fine, must be mixed, so that the composition may be 20 carats fine?

9. How much sugar, at 7 cents, 8 cents, 9 cents, and 16 cents per pound, must be mixed together to make a mixture that may be sold at 10 cents per pound?

§ **180.** CASE 3. Alligation partial is when one of its ingredients is limited to a certain quantity.

RULE. 1st. Take the difference between each price and the mean rates as before.

2d. State, as the difference of that commodity whose quantity is given is to the rest of the differences severally, so is the quantity given to the several quantities required.

EXAMPLES.

1. A farmer would mix 54 bushels of wheat at 90 cents per bushel, with rye at 54 cents, and barley at 63 cents per bushel, to make a mixture worth 72 cents per bushel.

				bus.	Proof.
72	90 ⌐ 18	18+9=27	As 27 : 18 :: 54	54 at 90	=48.60
	54 ⌋	... 18	54	36 at 54	=19.44
	63 ⌊	... 18	72	36 at 63	=22.68
			90	126	126)90.72
			27)972(36 bush.		.72

2. A distiller would mix 30 gallons of French brandy at $2.88 per gallon, with English at $1.44, and spirits at 96 cents per gallon: what quantity of each must be taken to be afforded at $1.92 per gallon?

3. A grocer mixes 24 lb. of fine tea at 90 cents per lb. with others at 65 cents and 60 cents per lb. to make a mixture worth 75 cents per lb.: what quantity of each does he take?

4. How much wine at $1.26, $1.50, and $2.16 per gallon, must be mixed with 18 gallons at $1.98 per gallon, to make a composition worth $1.80 per gallon?

180	198 ⌐ 54	As 54 : 18 :: 18 :	6	gallons.
	126 ⌋ 18	54 : 36 :: 18 :	12	"
	150 ⌐ 36	54 : 30 :: 18 :	10	"
	216 ⌋ 30			

5. A grocer would mix 56 lbs. of tea at 36 cents per lb.

with others at 45 cents, 51 cents, and 54 cents, to make a composition worth 48 cents per lb.: how much of each must he take?

6. A mealman mixes 60 bushels of flour at $1.26, with others at $1.08 and 96 cents, to make a mixture worth $1.14 per bushel: what quantity of each does he take?

§ **181.** Case 4. Alligation total is when the whole of the ingredients are limited to a certain quantity.

Rule. 1st. Take the difference between each price and the mean rate, as before.

2d. State, as the sum of the differences is to each particular difference, so is the quantity given to the quantity required.

EXAMPLES.

1. A grocer has sugar at 12 cents, 10 cents, and 8 cents per lb., and he would make a composition of 54 lbs., to sell for 9 cents per lb.: how much of each must he take?

9	12	. . . 1	As 6 : 1 : : 54 : 9 at 12 cts.=$1.08
	10	. . . 1	6 : 1 : : 54 : 9 at 10 cts.= .90
	8	3+1=4	6 : 4 : : 54 : 36 at 8 cts.= 2.88
		6	And 54 lbs at 9 cts.=$4.86

2. A druggist who has drugs at 96 cents, 60 cents, and 48 cents per lb., would make a composition of 112 lb. worth 72 cents per lb.: what quantity of each must he take?

3. A goldsmith has several sorts of gold, some of 24 carats fine, some 22, and some 18 carats fine, with which he would make make a compound of 30 ounces of 20 carats fine: how much of each sort must he take?

4. A person has rice at 12 cents, 7 cents, 6 cents, and 5 cents per lb., with which he makes a composition of a quarter of a cwt., worth 8 cents per lb.: what quantity of each does he take?

				lbs. oz.
8	12	1+2+3=6	As 18 : 6 : : 28 : 9	$5\frac{1}{3}$ at 12 cents.
	7	 4	18 : 4 : : 28 : 6	$3\frac{4}{9}$ " 7 "
	6	 4	18 : 4 : : 28 : 6	$3\frac{5}{9}$ " 6 "
	5	 4	18 : 4 : : 28 : 6	$3\frac{5}{9}$ " 5 "
		18		28 lbs.

5. A wine merchant has four sorts of wine, viz. Canary at 70 cents per gallon, Malaga at 65 cents, Rhenish at 55

cents, and Oporto at 50 cents per gallon; and he is desirous of making a composition of a pipe (126 gallons) to sell for 60 cents per gallon: the quantity of each is required?

6. I have teas at 48 cents, 60 cents, 84 cents, and $1.08 per lb., and I would make a mixture of ½ a cwt. (56 lbs.) to sell at 72 cents per lb.; what quantity of each will be required?

POSITION.

§ **182.** Position is the method of performing such questions as cannot be resolved by the common direct rules, and is of two kinds, called single and double.

SINGLE POSITION.

§ **183.** Single position teaches to resolve those questions whose results are proportioned to their suppositions.

RULE. 1. Take any number, and perform the same operations as are directed to be performed in the question.

2. Then say, as the result of the operation is to the result in the question, so is the supposed number to the number required.

Any supposed number will produce the true answer; but for convenience in working, those numbers are to be preferred, from which all the parts can be taken without remainders. Some, however, recommend the number one to be made the constant supposition?

Only those questions belong to single position whose parts are certain proportions of the suppositions, or of some power or root of their suppositions.

EXAMPLES.

1. A schoolmaster being asked how many scholars he had, said, if I had as many more, half as many, and a quarter as many, I should have 330: how many had he?

Suppose	140
As many more	140
½ as many	70
¼ as many	35
	385

Then as 385 : 330 : : 140 : 120 Ans.

Proof.

120+120+60+30=330.

2 A person, after spending ⅓ and ¼ of his money, had $60 left: what had he at first?

3. A certain sum of money is to be divided between 4 persons, in such a manner, that the first shall have $\frac{1}{3}$ of it, the second $\frac{1}{4}$, the third $\frac{1}{6}$, and the fourth the remainder, which is $80: what was the sum?

4. What number is that which being increased by $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{3}{4}$ of itself, the sum shall be 155?

5. A person, after spending $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{7}$ of his money, had $216\frac{3}{4}$ left; how much had he at first?

6. A, B and C purchased a house for $3300; A paid a certain sum, B paid twice as much as A, and C four times as much as B: how much did each man pay?

7. Lent a sum of money to receive 6 per cent. per annum simple interest, and at the end of ten years received for principal and interest $1000: what was the sum lent?

8. Borrowed a sum of money, for which I paid $5 per cent. per annum, simple interest; in 7 years it amounted to $810: what was the sum borrowed?

9. Three unequal vents will severally empty a vessel of 120 gallons, in 1 hour, 2 hours, and 3 hours: if running together what time is necessary?

10. Of a certain sum given, A has $\frac{1}{3}$, B $\frac{1}{4}$, C $\frac{1}{6}$, and D the rest, which is $67.20: the sum is required?

11. What is the age of a person who says that if $\frac{3}{5}$ of the years I have lived be multiplied by 7, and $\frac{2}{3}$ of them be added to the product, the sum will be 292?

12. A saves $\frac{1}{5}$ of his income; but B, who has the same salary, by living twice as fast as A, sinks $120 a year: how much then have they per annum?

13. A person bought a carriage, horse, and harness, for $720; the horse came to twice the price of the harness, and the carriage to twice the price of the horse and harness: what did he give for each?

DOUBLE POSITION.

§ **184.** Double position teaches to resolve questions by making two suppositions in the operation.

RULE. 1. Take any two convenient numbers, and proceed with each according to the nature of the questions.

2. Find how much each result differs from the true result, and call this difference its error.

3. Multiply each of the errors by the contrary supposition.

4. If the errors are alike, that is, both greater or both less than the given number, take their difference for a divisor, and the difference of their products for a dividend. But if unlike, that is, one too much and the other too little, take their sum for a divisor, and the sum of their products for a dividend, the quotient will be the answer.

This rule is founded on the supposition that the first error is to the second, as the difference between the true and first supposed number is to the difference between the true and second supposed number; when that is not the case, the exact answer to the question cannot be found by this rule.

EXAMPLES.

1. A, B, and C would divide $1200 between them, so that B may have $100 more than A, and C $100 more than B: how much had each?

Suppose	Suppose again
A had 400	A had 350
B had 500	B had 450
And C 600	And C 550
1500 too much by 300.	1350 too much by 150.

Here the errors are of one kind.

Then 300—150=150=the difference of the errors.

Also 400×150=60000=the product of the first supposition and second error.

And 350×300=105000=the product of the second supposition and first error.

And 105000—60000=45000=their difference.

Whence 45000÷150=300=A's share. Then 300+100=400=B's share; and 400+100=500=C's share.

Proof. 300+400+500=1200.

2. A, B, and C are indebted to D in certain sums. A's and B's debts united amount to $5000, B's and C's to $7000, and A's and C's to $6000: what is each man's particular debt?

Suppose A's	1900
Then B's	3100
And C's	3900
A's and C's	5800 too little by 200.

Suppose again A's	2200
Then B's	2800
And C's	4200
A's and C's	6400 too much by 400.

Here are errors unlike.

Then 400+200=600=the sum of error.

Also 1900×400=760000 the product of the first supposition and second error.

And 2200×200=440000=the product of the second supposition and the first error.

And 760000+440000=1200000=their sum.

Whence 1200000÷600=2000=A's debt.

Proof. A's 2000+B's 3000=5000, B's 3000+C's 4000 =7000, C's 4000+A's 2000=6000.

3. Two persons, A and B, have both the same income; A saves $\frac{1}{3}$ of his yearly; but B, by spending $50 per annum more than A, at the end of 4 years finds himself $100 in debt: what is their income, and what do they spend?

4. Two persons, A and B, lay out equal sums of money in trade; A gains $126 and B loses $87, and A's money is now double of B's: what did each lay out?

5. Divide $500 between two persons, A and B, so that A may have $120 more than B: what is each person's share?

6. A, B, and C built a house which cost $3000, of which A paid a certain sum, B paid $400 more than A, and C paid $400 more than B: what sum did each pay?

7. A person dying bequeathed to three of his friends $6000, as follows: to the first a certain portion, to the second half as much more, wanting $100, and to the third double the sum, wanting $300: what sum did each receive?

8. A labourer hired for 40 days upon this condition, that he should receive 20 cents for every day he wrought, and forfeit 10 cents for every day he was idle; at settlement he received $5: how many days did he work and how many was he idle?

9. The head of a fish is 9 inches long, and its tail is as long as its head and half its body, and the length of the body equals the sum of the length of the head and tail: what is its whole length?

10. A person being asked the time of day, replied, the day is now 16 hours long, and the sun rises at 4 o'clock. Now, if you add $\frac{1}{2}$ of the hours that have passed since the sun rose to $\frac{3}{4}$ of those which must elapse before he sets, you will have the exact time of day: what was the time?

ANNUITIES.

§ 185. A certain sum of money to be paid at regular intervals, is called an annuity.

The value or present worth of an annuity is that sum which, when placed at compound interest, would pay the annuity. The amount of an annuity is the sum of the payments increased by their compound interests.

§ 186. CASE 1. To find the amount of an annuity at compound interest.

RULE. Find the sum of a geometrical series whose terms shall equal in number the number of the years, whose first term is $1.00, and whose ratio is the amount of $1.00 at the given rate per cent. Then the amount will be the product of the sum thus found, multiplied by the given annuity.

EXAMPLES.

1. What is the amount of an annuity of $40, to continue 5 years, at 5 per cent.

$1+1.05+1.05^2+1.05^3+1.05^4=5.52563125$

$5.52563125\times 40=\$221.025$. Answer.

2. What is the amount of an annuity of $500, in 5 years, at 6 per cent.?

3. Of $50, to continue 7 years, at 4 per cent.?

4. Of $800, to continue 3 years, at 6 per cent.?

§ 187. CASE 2. To find the present worth of an annuity.

RULE. Find the product of the given annuity and the present worth of an annuity of $1, for the given time.

EXAMPLES.

1. What is the present worth of an annuity of $500 for 5 years, at 6 per cent.?

Present worth of $1.00 for			1 year,	0.943396
"	"	"	2 years,	0.889996
"	"	"	3 "	0.839619
"	"	"	4 "	0.792094
"	"	"	5 "	0.747258
$4.212363\times 500=\$2106.181500$				$\$4.212363$

2. What is the present worth of an annuity of $400, to continue 5 years, at 5 per cent.?

3. Of an annuity of $1200, to continue 4 years, at 6 per ct.?

EXCHANGE.

§ **188.** Exchange is a term chiefly employed by merchants, to denote that species of commercial transactions by which individuals cancel debts due to persons residing at a distance, without the transmission of money. This payment is usually made by means of a bill of exchange.

A bill of exchange, then, may be defined to be an order directing a person at a distance to pay a certain sum to the person in whose favour the bill is drawn, or to his order. Three persons are usually concerned in the bill or contract. These are, the *drawer*, or person who draws the bill; the *drawee*, or person who is directed to pay or give value for the bill; and the *remitter*, or person in whose favour it is drawn. After the drawee has accepted the bill, or engaged himself to pay it when due, he is called the *acceptor*. The remitter may pass the bill to any other person, who may in turn give it to a third, and so on. These persons are termed the endorsers, from their writing their names on the back of the bill. The person in whose possession the bill is at any time, is called the *holder*.

Bills of exchange are either made payable at sight, at a certain specified time after sight, or after date, or at usance, the usual term allowed by custom or law where the bill is payable.

Days of grace are a few days generally allowed for payment beyond the time when the bill becomes due, the number of which varies in different countries. Thus, in Great Britain and Ireland, 3 days' grace are allowed for all bills, except those payable at sight, which must be paid as soon as presented. In Amsterdam, Rotterdam, Antwerp, Cadiz, Lisbon, Venice, Oporto, and Rio Janeiro, 6 days of grace are allowed. At Geneva 5, at Bremen 8, at Hamburg and Altona 12, at Paris, Dantzic, and Bordeaux 10, at Madrid, Bilboa, Gibraltar, and Barcelona 14, and at Genoa 30. In drawing foreign bills of exchange, it is usual to draw them in sets of three, lest the first should miscarry, or be lost. When bills are drawn in sets, each must contain a condition that it shall be payable only while the others remain unpaid; thus, the first only is payable, "second and third unpaid;" the second, "first and third being unpaid," and the third, "first and second unpaid." The following is the usual

FORM OF A BILL OF EXCHANGE.

Philadelphia, July 15th, 1845.

Exchange for 5000 francs.

Sixty days after sight of this my first of exchange, second and third of the same tenor and date unpaid, pay to Peter Mason, or order, five thousand francs, with or without further advice from me. PETER PARKER.

Messrs. DES CARTES & Co., Merchants, Paris.

A bill of exchange, though only a simple contract, is always presumed to have been originally given for a good and valuable consideration. It is assignable to a person not named in the bill or party to the contract, so as to vest in the assignee a right of action in his own name, which no release by the drawer to the acceptor, nor set off due from the former to the latter, can affect.

ACCEPTANCE OF A BILL.

§ **189.** An *acceptance* is an engagement to pay a bill according to a particular tenor. It may either be absolute or qualified.

An *absolute* acceptance is an engagement to pay a bill according to its request, which is done by the drawee writing the word "accepted" on the bill, and subscribing his name, or writing "accepted" only, or merely subscribing his name across the bill or at the bottom of it.

When a bill is accepted on certain conditions, as when goods conveyed to the drawee are sold, or when any other future event has occurred, it is called a *qualified* acceptance, and is not binding till the contingency named has happened. An acceptance may also be partial, as to pay $100 instead of $200, or to pay at a different time and place required by the bill. In such case, the holder should give notice of such partial or qualified acceptance to the other parties to the bill, if he mean to resort to them in default of payment.

When one of several partners accepts a bill drawn upon the firm, and concerning their joint trade, it binds all the others. But if the bill concern the acceptor only in a separate and distinct interest, and the holder of the bill, at the time he becomes so, is aware of that circumstance, his acceptance will not bind his partners. A clerk or servant may accept a bill for his master if authorized to do so, or if he usually transacts business of that kind for him, and his acceptance will bind his master; but if the bill be drawn nominally on

the servant, and he accepts it without stating that he does so on his master's account, the acceptance binds the servant only, and not the master.

When a bill is drawn for the account of a third person, and is accepted as such, and he fails in business without making provision for its payment, the acceptor must discharge the bill, and can have no action against the drawer.

The drawers and indorsers are discharged from liability, unless due notice is given of non-acceptance when presented, or non-payment when the bill becomes due.

INDORSEMENT OF BILLS.

§ **190.** An indorsement is the act by which the holder of a bill of exchange, or other negotiable instrument, transfers his right to another person termed the indorsee. It is usually done by merely writing the name on the back of the bill. Bills may be transferred by delivery only, or by indorsement and delivery. By the latter mode only, bills payable to order must be transferred, but when payable to bearer they may be transferred by either mode. When a person makes a transfer by delivery only, he ceases to be a party to the bill, but when he indorses the bill, he is chargeable as a new drawer.

A bill originally transferable may be restrained by restrictive words; for the payee having absolute property in the bill, may make a restrictive and limited indorsement, by making it payable to "A B, only," or "for his use." Such an indorsement prevents the person in whose favour it is made from making a transfer so as to give another person a right of procedure against the special indorser, or any other of the parties. Partial indorsements are invalid, except with regard to those who make them. In the case of the decease of the holder of a bill, his executors may indorse it, but in so doing they become personally liable, and not as executors.

PRESENTING BILLS.

§ **191.** The holder of a bill should be always careful to present it for payment when due, to the acceptor, or to his heirs or assigns should he be dead, bankrupt, or insolvent. If the bill has a qualified acceptance, the presentment should be at the place mentioned in that acceptance. Delay or neglect in this respect will exonerate the drawer and indorsers from their liability.

Bills payable at sight, or at usance, ought not to be presented for payment at the expiration of the time mentioned in the bill, but at the end of the days of grace.

Payment of a bill should be made only to the holder, and the bill must be produced and delivered up with a receipt written on the back. On the non-acceptance or non-payment of a bill, the holder should authorize a public notary to *protest* it, that is, to draw up a notice of the refusal to accept or pay the bill, and the declaration of the holder against sustaining loss thereby. This protest is admitted in foreign countries as legal proof of the fact. The damages occurred by non-acceptance and non-payment, consisting besides interest, of the exchange or re-exchange, commission, postage, and expenses of protest, and interest on those expenses, may all be recovered by the holder of a bill, of either the drawer, the acceptor, or the indorser of that bill. Nothing whatever will discharge them from their liability but the payment of the money.

§ **192.** DRAFTS are similar to bills of exchange, but are mostly restricted to one country, thereby forming what is commonly denominated inland exchange. A draft vests in the assignee the same right of action against the assignor as a bill of exchange.

THE FOLLOWING IS THE FORM.

$5000 *Philadelphia, February* 11, 1845.

Ninety days after date, pay to the order of James Y. Hoopes, five thousand dollars, value received, and charge the same to account of SMITH & BAKER.

To JACOB JONES, Merchant, New York.

EXAMPLES.

1. New York on Liverpool; £515 10*s.* sterling. Required its value in federal money, exchange being at 10 per cent. premium, that is, the pound sterling being worth 10 per cent. more than its *nominal* par of $4\frac{4}{9}$.

2. New Orleans on Liverpool; £1080 16*s.* 6*d.* sterling. Required its value in federal money, exchange being at $8\frac{1}{2}$ per cent. premium.

3. Liverpool on New York; $4520.50. Required its value in sterling money, exchange being at 4*s.* 2*d.* per dollar.

4. New Orleans on Havre; francs 10520.50. Required its value in federal money, exchange being at 5 francs 25 centimes per dollar.

5. Charleston on Havre; francs 2850.25. Required its value in federal money, exchange being at 5 francs $22\frac{1}{2}$ centimes per dollar.

6. Paris on New York; $860.50. Required its value in French money, exchange being at 5 francs 30 centimes per dollar.

7. Baltimore on Hamburg; 2540 marcs 8*s.* banco. Required its value in federal money, exchange being at 34½ cents per marc banco.

8. Boston on St. Petersburg; 1650 roubles 75 copecs. Required its value in federal money, exchange being at 77 cents per rouble.

9. Trieste on New York; $880. Required its value in Austrian money, exchange being at 48¼ cents per florin.

10. Boston on Genoa; 4750 lire 50 centesimi. Required its value in federal money, exchange being at 5 lire 20 cents per dollar.

11. New York on Leghorn; 1460 pezze 10 soldi 8 denari. Required its value in federal money, exchange being at 89 cents per pezza.

12. Cadiz on Boston; $974.25. Required its value in the money of Spain, exchange being at 10 rials of old plate 12 quintos per dollar.

13. New York on Lisbon; 2:800⊕500 rees. Required its value in federal money, exchange being at $1.12½ per milree.

14. Boston on St. Domingo; $950.75 currency. Required its value in federal money, exchange being at $2.80 currency per dollar federal money.

15. Rio Janeiro on London; £860 4*s.* 6*d.* sterling. Required its value in the currency of Brazil, exchange being at 2*s.* 1*d.* per milree currency.

16. Philadelphia on Buenos Ayres; $10510 currency. Required its value in federal money, exchange being at 16 dollars currency per dollar federal money.

17. Boston on London; Calcutta on ditto; } £500 stg. { Exch. at 10 pr. ct. prem. Exch. at 2*s.* 1*d.* pr. Co's rupee.

What is the cost of the Company's rupee, in federal money, by this circuitous remittance from Boston to Calcutta?

18. New York on London; Canton on ditto; } £800 stg. { Exch. at 9 pr. ct. prem. Exch. at 4*s.* 8*d.* pr. dollar.

What is the cost of the dollar of 72 candarines, in federal money, by this circuitous remittance from N. York to Canton?

19. Boston on London; Batavia on ditto; } £600 stg. { Exch. at 8½ pr. ct. prem. Exch. at 12 florins pr. £ stg.

What is the cost of the florin, in federal money, by this circuitous remittance from Boston to Batavia?

§ 193. TABLE OF MONEYS OF ACCOUNT.

Moneys of account are those denominations of money in which accounts are kept, and sales and purchases usually effected. They generally correspond in *name* with some of the coins of the realm, but not unfrequently differ in *value*. Thus, in our own country, the coins are pieces of gold, of the value of $10, $5, and $2½; of silver, of the value of $1, 50 cts. 25 cts. 10 cts. and 5 cts.; of copper, of the value of 1 ct. and ½ ct.: and accounts are kept in dollars and cents. At the present time, the dollar of account is equivalent to the specie dollar; but quite recently, over the greater part of the Union, its value was from 10 to 25 per cent. less than that of the specie dollar.

In the subjoined table, the moneys of account of the principal commercial nations of the world are given, with their intrinsic values estimated according to the U. States Mint valuation of the metal (gold or silver) of which the coin is made. If the money of account is fluctuating in value, or does not correspond with any of the coins of the realm, it is so stated, and an approximate value given.

The relative valuation of gold and silver varying in different countries, it follows that in computing the value of foreign money in the currency of the U. States, where such difference exists the result will differ as the gold or silver coins of the two countries are compared. Thus, the Louis d'or, or French 20 franc piece of gold, contains 89.55 grains of pure gold; 20 francs of silver contain 1386 grains of pure silver: the relative value of the gold and silver coinages of France is, therefore, 1386:89.55 or 15.4775:1.—The American eagle, or gold piece of ten dollars, contains 232.2 grs. of pure gold; ten silver dollars contain 3712.5 grs. of pure silver: the relative value of the gold and silver coinages of the U. States is, therefore, 3712.5:232.2 or 15.9884:1.—Gold as compared with silver is, therefore, worth, in the U. States, 3.3 per cent. more than in France: hence in making a remittance of specie from the U. States to France, silver would be preferable to gold. It is the business of the exchange broker and merchant to ascertain which of the precious metals affords the *best* medium of remittance from one country to another, and this constitutes the par of exchange.

UNITED KINGDOM OF GREAT BRITAIN AND IRELAND.

Accounts are kept in *pounds*, *shillings*, *pence*, and *farthings*. Prices of articles are also not unfrequently quoted in guineas.

4 farthings (qr) = 1 penny, (d) 12 pence = 1 shilling, (s)
20 shillings = 1 pound sterling, (£ stg.) 21 shillings = 1 guinea, (g)

The sovereign, or pound stg. in gold, is worth $4.86 U. S. currency. This is the *true* par of exchange; the *nominal* par is $4.$\frac{4}{5}$, which was established many years ago.

FRANCE.

Accounts are kept in *francs* and *centimes*. 100 centimes (c) = 1 franc (f)

The franc in silver is worth nearly 18.7 cts. U. S. currency.

BELGIUM.

Accounts are kept in *florins* or *guilders*, and *cents* or hundredths.

The florin in silver is worth a trifle over 40 cts. U. S. currency.

The coinage of Belgium since 1832 has been similar to that of France.

HOLLAND.

Moneys of account same as in Belgium.

Bremen.

Accounts are kept in *current rix-dollars*, *grotes*, and *schwares*.

5 schwares = 1 grote, 72 grotes = 1 rix dollar current = 71½ cts. U. S. currency.

The old rix-dollar (silver) = 81.45 cts., that since 1840 = 71.4 cts. U. S. currency.

Hamburg.

Accounts are kept in *marcs*, *sols* or *schillings*, and *pfenings*, *currency* or *lubs*, (meaning of "Lubeck.")

12 pfenings = 1 schilling, 16 schillings = 1 marc.

The marc *currency*, which is a silver coin, = 28½ cts., U. States currency. The marc *banco*, which is an imaginary money, based upon certificates of deposit of bullion and jewelry in the bank of Hamburg, fluctuates 2 or 3 per cent. above or below its par value, which is 23 per cent. more than the marc currency, or about 35 cts., U. States currency.

Accounts are also sometimes kept, particularly in exchanges, in *pounds*, *schillings*, and *pence*, *Flemish*.

12 grotes or pence = 1 schilling, 20 schillings = 1 pound.

1 pound Flemish, banco = 7½ marcs banco.

DENMARK.

Accounts are kept in *rix-dollars*, *marcs* and *skillings*.

16 skillings = 1 marc, 6 marcs = 1 rix-dollar.

The new rix-dollar, or *rigsbank* dollar (silver) is worth about 52½ cts. U. S. currency. The money used in commercial transactions is bank money, which is commonly at a heavy discount.

NORWAY.

Accounts are kept in *rix-dollars species* and *skillings*.

120 skillings = 1 rix-dollar species.

Value of the species dollar (silver) in U. S. currency $1.05. Norway has no gold coinage.

SWEDEN.

The currency of Sweden consists almost entirely of paper money, of which there are two descriptions, the one issued by the government bank, called *riksgald*, the other issued by the national bank, called *banco*, the latter being worth 50 per cent. more than the former.

Accounts are kept in *rix-dollars*, *skillings* and *rundstycks*, riksgald, or banco.

12 rundstycks = 1 skilling, 48 skillings = 1 rix-dollar.

The value of the specie dollar, (silver) or coin of the realm, in the currency of the U. S., is $1.06+: that of the paper money fluctuates, but may be said to be, of the rix-dollar banco 37½ cts; of the rix-dollar riksgald 25 cts.

RUSSIA.

Prior to the year 1840, accounts were kept in *paper* roubles, 3½ of which were equal to 1 silver rouble: from that period the *silver rouble* and the *copec*, of one hundredth part its value, have been adopted by law as the moneys of account.

Value of the silver rouble in the currency of the U. S., 75+cts.

PRUSSIA.

Accounts are kept in *thalers* or *dollars*, *silver groschen*, and *pfenings*.

12 pfenings = 1 silver grosch, 30 silver groschen = 1 dollar.

The value of the dollar, (silver) in the currency of the U. States, is 69+ cts.

AUSTRIA.

Accounts are kept in *florins* and *kreutzers*. 60 kr. = 1 florin.
The florin, (silver) is worth, in the currency of the U. States, 48¾ cts.

Venice.

Accounts are kept in *lire* divided into 100 *centesimi*. These are imaginary moneys of the value of the French franc and centime respectively.

SARDINIA.

Accounts are kept in *lire* and *centesimi* or hundredths; corresponding in value with the French franc and centime respectively.
The value of the gold and silver coinages, also corresponds with that of France, differing only in name.

TUSCANY.

Accounts are principally kept in *pezze* of 8 rials, or dollars, *soldi* di pezza and *denari* di pezza.
12 denari di pezza = 1 soldo di pezza, 20 soldi di pezza = 1 pezza of 8 rials.
But for transactions of small amount accounts are kept in *lire*, *soldi* and *denari*. 12 denari = 1 soldo, 20 soldi = 1 lira,
5¾ lire = 1 pezza (silver) = 90+ cts., currency of the U. States.

PAPAL STATES.

Accounts are kept in crowns or *scudi* and *bajocchi*.
100 bajocchi, or 10 paoli = 1 scudo (silver) = $1.00.6 currency of U. S.
Payments of greater amount than 5 scudi are made in paper money, which, not being redeemable in specie on demand, is uniformly at a discount.

NAPLES.

Accounts are kept in *ducati* and *grani*. 100 grani, or 10 carlini = 1 ducat.
Value of the ducat, (silver) in U. S. currency, a trifle under 79 cts.

SICILY.

Accounts are kept in *oncie*, *tari* and *grani*.
20 grani = 1 taro, 30 tari = 1 oncia.
1 oncia = 3 ducati, and 1 taro = 1 carlino, of Naples.
The oncia, (gold) = $2.48½ U. States currency.

SPAIN.

Accounts are kept at Cadiz, Malaga, and in most of the provinces of Spain, in *rials of old plate*, *quintos* and *maravedis*.
2 maravedis = 1 quinto 16 quintos = 1 rial of old plate.
10⅔ rials of old plate = 1 silver dollar, or $1. U. States currency.
At Barcelona accounts are kept in *libras*, *sueldos*, *dineros* and *mallas*.
2 mallas = 1 dinero, 12 dineros = 1 sueldo 20 sueldos = 1 libra.
The libra is likewise divided into *rials de plata Catalan*, of 3 sueldos each, and into *rials ardites*, of 2 sueldos each.
1 libra = $\frac{8}{15}$ silver dollar, or 53⅓ cts., U. States currency.

PORTUGAL.

Accounts are kept in *milrees* and *rees*. In notation the milrees are separated from the rees by a crossed cypher (⊕), and the milrees from the millions of rees, by a colon (:)—thus, 5:400⊕300 rees = 5,400 milrees and 300 rees. 1 milree or crown, (silver) = $1.12 U. States currency.

GREECE.

Accounts are kept in *drachmè* and *lepta*. 100 lepta = 1 drachmè.
Value of the drachmè, (silver) in U. States currency 16.6 cts.

TURKEY.

Accounts are kept in *piastres*, *paras* and *aspers*.

3 aspers = 1 para, 40 paras = 1 piastre.

Some foreign merchants resident in Turkey reckon 80, others 100 aspers to the piastre, in their accounts.

The value of the piastre, (silver) was depreciated from 60 cts. in 1764, to 3 cts., in 1832. Its present value, in the currency of the U. States, is about 3.8 cts. Its commercial value is about $\frac{1}{2}\frac{1}{3}$ of a dollar.

EGYPT.

Accounts are kept as in Turkey. The piastre (silver) = 4.8 cts., U. States currency.

CAPE OF GOOD HOPE.

Accounts are kept in *pounds*, *shillings*, *pence* and *farthings*, sterling; or in *rix-dollars*, *schillings* and *stivers*.

6 stivers = 1 schilling, 8 schillings = 1 rix-dollar = 1s. 6d stg.

MAURITIUS.

Accounts are kept in *dollars* and *cents* (or hundredths) currency. The current dollar is worth 4 shillings stg., when exchange is at par. The dollar of the U. States is valued at $1.08\frac{1}{3}$ cts., currency.

EAST INDIA COMPANY'S TERRITORIES.

Accounts are kept in *Company's rupees*, *annas* and *pice*.

12 pice = 1 anna, 16 annas = 1 Company's rupee.

Value of the Company's rupee, (silver) in U. States currency, $44\frac{1}{2}$ cts.

Notation. 1,00,000 (100 thousand) = 1 lac, 1,00,00,000 (10 millions) = 1 crore.

JAVA.

Accounts are kept in *florins* and *cents*, as in Holland.

Manilla.

Accounts are kept in *dollars* and *rials* 8 to the dollar.
The dollar (Spanish) = the dollar of the U. States.

Sydney, N. S. W.

Accounts are kept in sterling money.

Canton.

Accounts are kept in *taels*, *mace*, *candarines* and *cash*.

1 tael = 10 mace = 100 candarines = 1000 cash.

The cash is the only money coined by the Chinese government, and is made of a mixture of copper and lead. It is generally worth more than its nominal value, and when much in demand for change a tael will sometimes purchase only 750.

72 candarines are considered equal to 1 U. States dollar *of account*; but specie dollars are generally worth from 3 to 5 per cent. more than the dollar of account.

BRITISH PROVINCES OF NORTH AMERICA.

Accounts are kept in *pounds*, *shillings*, *pence* and *farthings*, *Halifax currency*. The pound = $4 U. States currency.

4 farthings = 1 penny, 12 pence = 1 shilling, 20 shillings = 1 pound.

BRITISH WEST INDIA COLONIES.

In these islands accounts are kept in *pounds*, *shillings*, *pence* and *farthings*, of the same relative value as in England; but the value of the pound *currency* as compared with the pound *sterling*, varies very much in the different islands, and is in all cases less. The following values are affixed to the currencies of the respective islands:—£100 stg. = £135 of Barbadoes = £140 of Jamaica = £175 of the Windward islands, except Barbadoes, = £200 of the Leeward islands. But exchange on England in these several islands, is generally at from 10 to 20 per cent. premium on the values above given. Thus it would require about £160 currency of Jamaica, to purchase a bill of exchange on London for £100 stg.

CUBA AND PORTO RICO.

Accounts are kept in Spanish *dollars* and *rials plate* of 8 to the dollar. The dollar corresponds in value with the dollar of the U. States.

ST. DOMINGO.

Accounts are kept in *dollars*, and *centimes* or hundredths.

The dollar, (silver) = about 33⅓ cts., U. States currency. In January 1845, $2.75 currency = $1. U. States currency.

MEXICO.

Accounts are kept in *dollars* and *rials*.

8 rials = 1 dollar (silver) = that of the United States.

VENEZUELA, NEW GRENADA, AND ECUADOR.

Accounts in these republics, formerly united under the name of Colombia, are kept in *dollars* and *rials* eight to the dollar.

The coinage of these States is very irregular in value; some dollars (silver) being worth rather more than the dollar of the U. States; some not more than ⅞ its value; but most being of less value than our coin.

BRAZIL.

Accounts are kept in *milrees* and *rees* currency. 1000 rees = 1 milree. In notation the milrees are separated from the rees by ||.

The piece of 1||200 rees (silver) = 99.4 cts. U. States currency.

The currency of Brazil is chiefly paper and copper, redeemable in silver only at a very great discount: the rate varying very much in different provinces, and at different times in the same province. At Rio Janeiro, in December 1844, Spanish dollars were worth 2||040 rees.

Monte Video.

Accounts are kept in *dollars currency*, *rials* and *centesimos*.

100 centesimos = 1 rial, 8 rials = 1 dollar currency.

The dollar of account or currency = 83⅓ cts., U. States currency.

There is no paper currency.

Buenos Ayres.

Accounts are kept in *dollars currency* and *rials* 8 to the dollar. The value of the dollar currency fluctuates very much. In December, 1843, it was worth 6¼ cts., U. States currency.

CHILI.

Accounts are kept in *dollars* and *rials* 8 to the dollar. The dollar, (silver) is worth about $1.01 U. States currency, but is generally estimated to be of equal value with it.

PERU.

Accounts are kept in *dollars* and *rials* 8 to the dollar. The dollar (silver) is about equal in value with that of Chili, and the same remarks apply to it. There is, moreover, a duty of 5 per cent. on dollars exported.

§ 194. *Foreign Coins made current in the United States, and their values affixed by the several Acts of Congress.*

GOLD COINS.

Law of June 28, 1834.

	FINENESS.	RATE PER DWT.	APPROXIMATE VALUES OF COINS AT RATES GIVEN.	
PORTUGAL, BRAZIL	22 carats. . .	94.8 cts. . .	Moidore of 1705	$6.59
			No coin of this fineness.	
SPAIN, MEXICO, COLOMBIA	20 carats and $3\frac{7}{16}$ grains	89.9 cts. . .	Doubloon of 1810—24	$15.50
			" of Republic	$15.55
			" of Bogota	$15.61

Law of March 3, 1843.

GT. BRITAIN	$\frac{915.5}{1000}$	94.6 cts. . .	Sovereign, of Victoria	$4.86
FRANCE	$\frac{899}{1000}$	92.9 cts. . .	Napoleon, or 20 francs	$3.85

SILVER COINS.

Law of June 25, 1834.

	FINENESS.	WEIGHT.	VALUE OF EACH.	
Dollar of CHILI, CENT. AM'A.	$\frac{215}{240}$	415 grs.	100 cts.	Dollars of Chili worth about 101 cts; those of Cent. Am'a. about 100 cts.

BRAZIL. Dollars re-stamped in Brazil, same as above.

Law of March 3, 1843.

Pillar dollar of SPAIN and dollar of MEXICO, PERU, BOLIVIA	$\frac{897}{1000}$	415 grs.	100 cts.	These dollars are generally worth rather more than the value affixed.
5 Franc piece of FRANCE	$\frac{900}{1000}$	384 grs.	93 cts.	The value affixed is very nearly the intrinsic worth.

§ 195. *Value of Foreign Moneys of Account, in computation at the Custom-Houses in the United States.*

By Law of May, 1846.

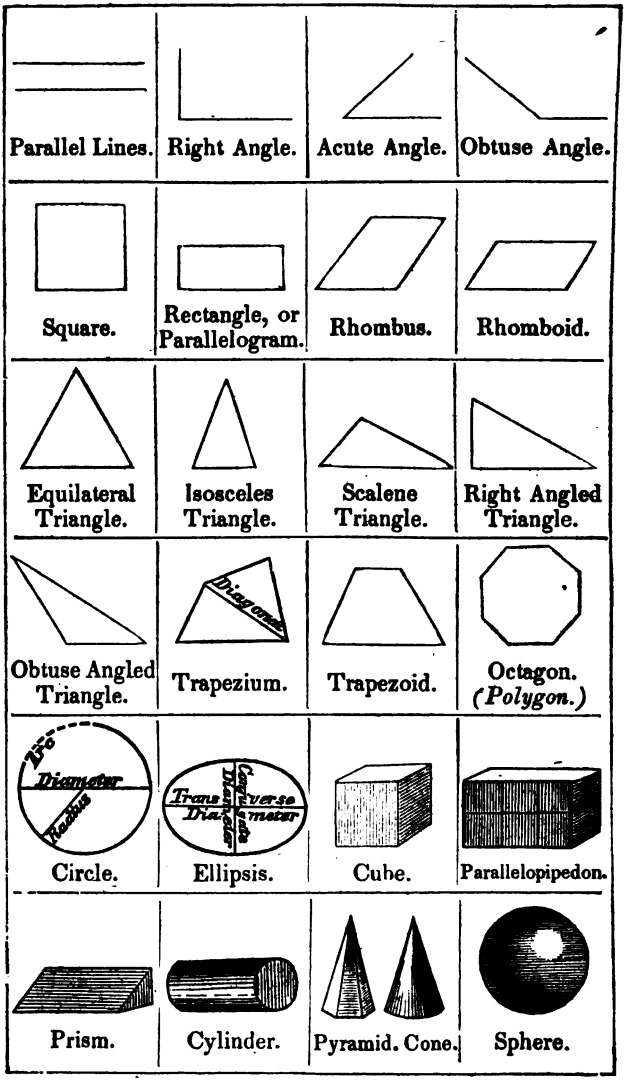
	$. c. m.
Franc of France and Belgium	18.6
Lira of Sardinia	
Thaler of Prussia and Northern States of Germany	69.0
Florin of Southern States of Germany	40.0
Florin of Austria and City of Augsburg	48.5
Lira of Tuscany and Lombardo Venetian Kingdom	16.0
Ounce of Sicily	2.40.0
Specie Dollar of Denmark	1.05.0
" " of Sweden and Norway	1.06.6

By Law of March 3, 1843.

	$. c. m.
Milree of Portugal	1.12.0
" of Madeira	1.00.0
" of Azores	.83½c.
Rix Dollar of Bremen	.78½c.
Thaler of Bremen, 72 Grotes	.71.0
Marc Banco of Hamburg	.35.0
Rouble of Russia	.75.0
Rupee of British India	.44.5

By Previous Laws.

Pound Stg. of Gt. Britain	4.84.0
Pound of Brit. Prov. of N. A.	4.00.0
Pound of Jamaica	3 00:0
Ducat of Naples	.80.0
Rial of Plate of Spain	.10.0


Parallel Lines.
Right Angle.
Acute Angle.
Obtuse Angle.
Square.
Rectangle, or Parallelogram.
Rhombus.
Rhomboid.
Equilateral Triangle.
Isosceles Triangle.
Scalene Triangle.
Right Angled Triangle.
Obtuse Angled Triangle.
Diagonal
Trapezium.
Trapezoid.
Octagon. (Polygon.)
Circumference
Diameter
Radius
Circle.
Transverse Diameter
Conjugate Diameter
Ellipsis.
Cube.
Parallelopipedon.
Prism.
Cylinder.
Pyramid. Cone.
Sphere.

MENSURATION,

§ **196.** Is the art of measuring superficies or solids.

DEFINITIONS.

1. A point is that which has position but not magnitude.

2. A line has length but not breadth, and its bounds or extremes are points.

3. A right line is the shortest line that can be drawn between any two points.

4. A superficies or surface is that which has length and breadth but not thickness.

5. A figure is space bounded by one or more lines.

6. The area or superficial content of any figure, is the space contained within the line or lines which bound it.

7. Parallel lines are those which are in the same plane, and which being produced both ways do not meet.

8. An angle is the inclination or opening of two lines, having different directions and meeting in a point.

9. One line is perpendicular to another when it makes the angles on both sides equal to each other.

10. A right angle is that which is formed by two lines that are perpendicular to each other, and contains 90°.

11. An acute angle is less than a right angle.

12. An obtuse angle is greater than a right angle.

13. A triangle is a figure bounded by three right lines, and has three angles.

14. An equilateral triangle has its three sides all equal.

15. An isosceles triangle has only two of its sides equal.

16. A scalene triangle has three sides all unequal.

17. A right-angled triangle has one right angle; the side opposite which is called the hypothenuse, and the other two sides the legs, or the base and perpendicular.

18. A figure of four sides is called a quadrangle, or a quadrilateral.

19. A square is a quadrilateral, whose sides are all equal, and its angles all right angles.

20. A rhombus is a quadrilateral, whose sides are all equal, but its angles not right angles.

21. A parallelogram is a quadrilateral, whose opposite sides are parallel.

22. A rectangle is a parallelogram, whose angles are all right angles.

23. A rhomboid is a parallelogram, whose angles are not right angles.

24. A trapezium is a quadrilateral, which has not its opposite sides parallel.

25. A trapezoid is a quadrilateral, with two sides parallel.

26. A diagonal is a line joining any two opposite angles of a quadrilateral.

27. Figures having more than four sides are called polygons.

28. Polygons having five sides, are called pentagons; those of six sides, hexagons; those of seven, heptagons; &c.

29. The base of any figure is that side on which it is supposed to stand, and the altitude is the perpendicular falling upon it from the opposite angle.

30. A circle is a plane figure bounded by a curve line which is equally distant from a point within, called the centre. The boundary line is called the circumference.

31. An arc is any portion of the circumference.

32. A semicircle is half, and a quadrant, quarter of a circle.

33. The radius of a circle is a line drawn from the centre to the circumference.

34. The diameter of a circle, is a right line drawn through the centre, terminated both ways by the circumference.

35. A solid is that which has length, breadth, and thickness, and its bounds or extremes are superficies.

36. A cube is a solid contained by six equal square sides.

37. A parallelopipedon is a solid contained by six quadrilateral planes, every opposite two of which are equal and parallel.

38. A prism is a solid whose ends are two equal parallel and similar plane figures, and whose sides are parallelograms.

39. A cylinder is a solid described by the revolution of a rectangle about one of its sides, which remains fixed.

40. A cone is a solid formed by the revolution of a right-angled triangle about one of its legs, which is fixed.

41. If a cone be cut by a plane passing obliquely through its two slant sides, the section will be an ellipsis.

42. The longest diameter of the ellipsis is called the tranverse, and the shortest the conjugate.

43. A pyramid is a solid whose sides are all triangles meeting in a point at the vertex, and the base any plane figure whatever.

44. A sphere or globe is a solid described by the revolution of a semicircle about its diameter, which remains fixed

45. The diameter of a sphere is a right line passing through the centre, and terminated on each side by the convex surface.

46. The areas of circles are to each other as the squares of their diameters, radii, or circumferences. The areas of similar figures are to each other, as the squares of their like sides.

47. The surfaces of all similar solids are to each other as the squares of their like dimensions; such as diameters, circumferences, like linear sides, &c. And their solidities, as the cubes of those dimensions.

PROBLEM I. To find the area of a square.

RULE. Multiply the side by itself.

EXAMPLES.

1. What is the area of a board whose side is 19 inches?

$19 \times 19 = 361 =$ area required.

2. How many acres in a square field, whose side is 50 rods?
3. What is the area of a square, whose side is 4 ft. 2 in.?

PROBLEM II. The area of a square being given to find the length of the side.

RULE. Extract the square root of the area.

EXAMPLES.

1. The area of a square is 729 yards: what is the side?

$\sqrt{729} = 27 =$ side required.

2. What is the side of a square floor containing 2025 sq. ft.?
3. What is the side of a square, whose area is 6*A*. 1*R*.?

PROBLEM III. The diagonal of a square being given, to find the area.

RULE. Divide the square of the diagonal by 2.

EXAMPLES.

1. The diagonal of a square is 8 chains: what is the area?

$(8 \times 8) \div 2 = 32$ chains $= 3$ a. 2 r.

2. The diagonal of a square is 16 feet, what is the area?
3. The diagonal of a square is 12 feet, what is the area?

PROBLEM IV. The area of a square given, to find the diagonal.

RULE. Extract the square root of double the area.

EXAMPLES.

1. The area of a square piece of land is 28.8 acres, what is the diagonal in chains?

$$\sqrt{(288\times 2)}=\sqrt{576}=24 \text{ chains}$$

2. The area of a square is 578 feet, what is the diagonal?
3. The area of a square is 128 yards, what is the diagonal?

PROBLEM V. The diagonal of a square given, to find the side.

RULE. Find the square root of half the square of the diagonal.

EXAMPLES.

1. The diagonal of a square is 12yds., what is the side?

$$\sqrt{((12\times 12)\div 2)}=\sqrt{72}=8.485.$$

2. The diagonal of a square is 36 feet, what is the side?
3. The diagonal of a square is 48 chains, what is the side?

PROBLEM VI. To cut off a given area from a square, parallel to either side.

RULE. Divide the given area by the length of the side, the quotient will be the length on the other side.

EXAMPLES.

1. What length must be cut from a square whose sides are 25 chains, to leave 30 acres at the end?

$$300 \text{ ch.}\div 25=12 \text{ chains, length required.}$$

2. The sides of a square are 17 feet, what must be the length of the other side to leave 153 square feet?
3. The side of a square being 42 rods, what must be the length of the other side, to leave 4 acres?

PROBLEM VII. The length and breadth of a rectangle being given, to find the area.

RULE. Multiply the length by the breadth.

EXAMPLES.

1. What is the area of a rectangle whose length is 14.5 feet, and breadth 11.6 feet?

$$14.5\times 11.6=168.2 \text{ feet.}$$

2. What is the area of a rectangle whose length is 117.8 yards, and breadth 204.7 yards?
3. What is the area of a rectangular field whose length is 87 rods, and breadth 56 rods?

PROBLEM VIII. The area and either side of a rectangle being given, to find the other side.

RULE. Divide the area by the given side.

EXAMPLES.

1. The area of rectangle is 456 yards, and the length 30 yards: what is the breadth?

$$456 \div 30 = 15.2 \text{ breadth.}$$

2. The area of a rectangular piece of land is 6*A*. 2*R*. 16*P*., and breadth $29\frac{1}{2}P$.: what is the length?

3. The area of a rectangle is 846 chains, and its length 41 chains: what is the breadth?

PROBLEM IX. To find the area of a rhombus or rhomboid.

RULE. Multiply the length by a perpendicular let fall from one of the angles to the opposite side.

EXAMPLES.

1. The length of a rhombus is $18\frac{1}{2}$ yards, and the perpendicular height $7\frac{3}{4}$ yards: required the area.

$$18.5 \times 7.75 = 143.375.$$

2. Required the area of a rhombus whose length is 14 feet 4 inches, and its height 12 feet 2 inches.

3. The length of a rhomboid is 36 chains, and the perpendicular height 19 chains: required the area.

PROBLEM X. To find the area of a triangle.

RULE. Multiply the base by the perpendicular height, and half the product will be the area.

EXAMPLES.

1. What is the area of a triangle whose base is 37 feet, and height 23 feet?

$$(37 \times 23) \div 2 = 425.5 \text{ feet area.}$$

2. Of a triangle, whose base is $12\frac{1}{4}$ chains, height $8\frac{1}{2}$ chains?

3. The base of a triangle is 72 perches, and the height 12 perches: how many acres does it contain?

PROBLEM XI. Given three sides of a triangle, to find the area.

RULE. From half the sum of the three sides subtract each side severally. Multiply the half sum and the three remainders continually together, and the square root of the product will be the area required.

EXAMPLES

1. What is the area of a triangle whose sides are 26, 28, and 30 feet?

$(26+28+30)\div 2=42$=half sum of the sides.

Also $42-26=16$=first difference, $42-28=14$=second difference, and $42-30=12$=third difference.

Whence $\sqrt{(42\times 16\times 14\times 12)}=336$ feet area.

2. How many acres are there in a triangular field whose sides are 49, 50.25 and 25.69 chains?

3. Required the area of a triangle whose sides are 3, 4 and 5 yards?

PROBLEM XII. The proportion of the three sides of a triangle being given, to find the length of the sides of a triangle corresponding with a given area.

RULE. Find the area of the triangle according to the given proportion; then as that area is to the area given, so is the square of either of the sides to the square of the similar side.

EXAMPLES.

1. A person wishes to enclose 6 acres, 1 rood, 12 perches, in a triangle similar to a small triangle whose sides are 6, 8 and 9 perches respectively: required the sides of the triangle?

The area of a triangle whose sides are 6, 8 and 9, is found to be equal 23.5252 by Prob. XI.

6 acres, 1 rood, 12 perches=1012 perches.

$23.5252 : 1012 :: 8^2 : 2753.1328$, and

$\sqrt{2753.1328}=52.47$=one of the sides.

Now $8 : 9 :: 52.47 : 59.029$ } the other two sides.
And $8 : 6 :: 52.47 : 39.353$ }

2. What, if they are in the proportion of 3, 4 and 5, is the length of the sides of a triangle containing 24 acres?

3. The area of a piece of land is 10 acres: what must be the length of the sides, in the proportion of 5, 11 and 13.

PROBLEM XIII. Any two sides of a right-angled triangle being given, to find the third side.

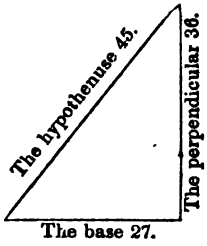
RULE. 1. When the base and perpendicular are given to find the hypothenuse. Extract the square root of the sum of the squares of the base and perpendicular.

2. When the hypothenuse and either leg are given to find the other. Extract the square root of the difference of the squares of the hypothenuse and the given leg.

EXAMPLES.

1. The base of a right-angled triangle is 27, and the perpendicular 36 yards: required the length of the hypothenuse?

$36 \times 36 = 1296$
$27 \times 27 = 729$
$\sqrt{2025} = 45$, the hypothenuse.



2. The base of a right-angled triangle is 24, the perpendicular 18 feet: what is the length of the hypothenuse?

3. The hypothenuse of a right-angled triangle is 40 chains, and the perpendicular 26 chains: what is the base?

4. The base of a right-angled triangle is 16 chains, and the perpendicular 12 chains: required the hypothenuse.

5. The hypothenuse of a right-angled triangle is 25 chains, and the base 20 chains: what is the perpendicular?

6. The wall of a fort standing on the brink of a river is 42.426 feet high, and the river is 23 yards wide: how long must a cord be to reach from the top of the fort across the river?

7. Two ships sail from the same port, one east 50 miles, the other south 84 miles: how far are they apart?

8. A line 27 yards long will reach from the top of a fort, on the opposite bank of a river, to the water edge on this side of the river: what is the height of the fort, the river being 24 yards across?

9. A ladder of 100 feet in length was placed against a building of 100 feet high, in such a manner that the top of it reached the top of the building within 6 inches: what was the distance of the foot of the ladder from the base of the edifice?

10. A ladder 30 feet long, placed near the middle of a street, reached the buildings at one side 24 feet from the ground; and the opposite side, without moving the foot, 18 feet: what was the breadth of the street?

PROBLEM XIV. To find the area of a trapezium, the length of the diagonal and perpendiculars being given.

RULE. Multiply the sum of the perpendiculars by the diagonal, and half the product will be the area.

EXAMPLES.

1. What is the area of a trapezium whose diagonal is 108 feet 6 inches, and the perpendiculars 56 feet 3 inches, and 60 feet 9 inches?

ft. in. ft. in. ft. in. ft. in. ft. in. ft. in.
(56 3+60 9)×108 6=12694 6, and 12694 6÷2=6347 3.

2. How many acres are there in a trapezium whose diagonal is 33 perches, and the sum of the perpendiculars 24 perches?

3. How many yards are there in a trapezium whose diagonal is 80.5 feet, and perpendiclars 22.4 and 28.3 feet?

PROBLEM XV. To find the area of a trapezoid, having the length of the two parallel sides, and the perpendicular distance between them given.

RULE. Add the two parallel sides together, and multiply the sum by the perpendiculars, and half the product will be the area.

EXAMPLES.

1. The perpendicular distance between the two parallel sides of the trapezoid is 4.5 yards, and the length of the parallel sides are 12.75 and 16.67 yards: what is the area?

(12.75+16.67)×4.5=132.39, and 132.39÷2=66.195.

2. Required the area of a trapezoid whose parallel sides are 20.5 and 12.25 chains, and perpendicular 10.75 chains?

3. What is the area of a trapezoid whose parallel sides are 51 feet and $37\frac{1}{2}$ feet, and the perpendicular distance $20\frac{5}{6}$ feet?

PROBLEM XVI. To find the area of a regular polygon.

RULE. Multiply half the perimeter of the figure by the perpendicular falling from its centre upon one of its sides, and the product will be the area.

EXAMPLES.

1. What is the area of a regular pentagon, whose side is 21 feet, and perpendicular 12 feet?

(21×5)÷2=52.5 half perimeter, and 52.5×12=630 area.

2. What is the area of a hexagon, whose side is $16\frac{1}{2}$ feet, and perpendicular $14\frac{1}{3}$ feet?

3. What is the area of an octagon, whose side is 14.4 yards, and perpendicular 16.3 feet?

PROBLEM XVII. To find the circumference of a circle, the diameter being given.

RULE. Multiply the diameter by 3.1416, and the product will be the circumference.

EXAMPLES.

1. What is the circumference of a circle whose diameter is 12 chains?

$3.1416 \times 12 = 37.6992$, the circumference.

2. What is the circumference of a wheel whose diameter is 5 feet 2 inches?

3. If the diameter of the earth be 7935 miles, what is its circumference?

PROBLEM XVIII. To find the diameter of a circle, the circumference being given.

RULE. Divide the circumference by 3.1416, or multiply the circumference by .31831, and the result will be the diameter.

EXAMPLES.

1. The circumference of a circle is 16 yards: what is its diameter?

$16 \div 3.1416$ (or $16 \times .31831$) $= 5.09296$, the diameter.

2. If the circumference of a carriage wheel be 16 feet 6 inches, what is its diameter?

3. What is the diameter of a circle whose circumference is 128 feet?

PROBLEM XIX. To find the area of a circle.

RULE. Multiply the square of the diameter by .7854, and the product will be the area. Or multiply the square of the circumference by .07958, and the product will be the area.

EXAMPLES.

1. How many square feet are contained in a circle whose diameter is 4 feet 3 inches?

$4.25 \times 4.25 = 18.0625$ square of the diameter.

$18.0625 \times .7854 = 14.1862875$ area.

2. The circumference of a circle is 24 chains: how many acres does it contain?

3. How many acres are contained in a circle whose diameter is $20\frac{3}{4}$ perches?

PROBLEM XX. The area of a circle given, to find the diameter or circumference.

RULE. Divide the area by .7854, and the square root of the quotient will be the diameter. Or divide the area by .07958, and the square root of the quotient will be the circumference.

EXAMPLES.

1. The area of a circle being 2 acres, 3 roods, and 12 perches, what is the circumference?

2 acres, 3 roods, 12 perches=452 perches. $\sqrt{452 \div .07958} = \sqrt{5679.69} = 75.3657$ perches, the circumference.

2. The area of a circle is 5 acres, 3 roods, and 26 perches: what is the diameter?

3. The area of a circular garden containing 2 acres, what is the length of a brick wall which will enclose it?

PROBLEM XXI. To find the area of an ellipse, the transverse and conjugate diameters being given.

RULE. Multiply the transverse and conjugate diameters together, and the product again by .7854, and the result will be the area.

EXAMPLES.

1. The transverse and conjugate diameters of an ellipse are 60 and 40 perches: how many acres does it contain?

$(60 \times 40 \times .7854) = 188.4.96 = 11$ acres, 3 roods, 4.96 per.

2. The transverse and conjugate diameters of an ellipse are 8 and 6 chains: how many acres does it contain?

3. What is the area of an ellipse whose diameters are 24 and 18 feet?

PROBLEM XXII. The transverse and conjugate diameters being given to find the circumference.

RULE. Multiply the square root of half the sum of the squares of the two diameters by 3.1416, and the product will be the circumference.

EXAMPLES.

1. The transverse and conjugate diameters of an ellipse are 12 and 10 yards: required the circumference.

$$\sqrt{((144+100) \div 2)} = \sqrt{122} = 11.045, \text{ and}$$
$$11.045 \times 3.1416 = 34.699.$$

2. The transverse and conjugate diameters of an ellipse are 30 and 20 feet: what is the circumference?

3. What is the circumference of an ellipse whose transverse and conjugate diameters are 80 and 60 chains?

PROBLEM XXIII. To find the side of a square equal in area to any given superficies.

RULE. The square root of any given superficies will be the side of the square required.

EXAMPLES.

1. If the area of a given triangle be 961 perches, what is the side of a square equal in area thereto?

$\sqrt{961}=31$ perches, the side required.

2. If the area of a given circle be 9876 perches, what is the side of a square equal in area?

3. If an elliptical fish-pond contain one acre (=4840 square yards), required the side of a square fish-pond of equal dimensions.

PROBLEM XXIV. To find the area of the surface of a cube.

RULE. Multiply the square of the length of one side by 6, and the product will be the area.

EXAMPLES.

1. The side of a cube is 15 inches: what is the area of its surface?

$15\times15\times6=1350$ inches$=9\frac{3}{8}$ feet.

2. The side of a cube is 21 inches: what is the area of its surface?

3. The side of a cube is 6 feet: what is the area of its surface?

PROBLEM XXV. The area of the surface of a cube being given, to find the length of the side.

RULE. Divide the area by 6, and extract the square root of the quotient.

EXAMPLES.

1. The area of a cube is 2400 square inches: what is the length of the side?

$\sqrt{(2400\div6)}=\sqrt{400}=20$ inches.

2. The area of a cube is 216 square feet: what is the length of its side?

3. What is the length of a side of a cube whose area contains 96 square feet?

PROBLEM XXVI. To find the solidity of a cube, the height of one of its sides being given.

RULE. Cube the given side.

EXAMPLES.

1. What is the solidity of a cube whose side is 4 feet?

$4\times4\times4=64$ solid feet.

2. How many solid feet in a cubic box, whose depth is 23 inches?

3. How many cubic inches are contained in a cubical piece of timber, whose length is 76 inches?

PROBLEM XXVII. To find the side of a cube, the solidity being given.

RULE. Extract the cube root of the solidity.

EXAMPLES.

1. What is the length of the side of a cubic box containing 36 solid feet?

$\sqrt[3]{36}=3.302$ the side required.

2. The height of a cubic box containing 60 bushels, each bushel containing 2150.4 solid inches?

3. The height of a cubic box containing 16 bushels?

PROBLEM XXVIII. To find the solidity of a parallelopipedon.

RULE. Multiply the length by the breadth, and that product by the depth or altitude, and it will give the solidity required.

EXAMPLES.

1. The length of a parallelopipedon is 4 feet, its breadth 3 feet, and thickness 2 feet: what is the solidity?

$4\times3\times2=24$ solid feet.

2. How many bushels are contained in a bin 5 feet 6 inches long, 4 feet 9 inches wide, and 3 feet 9 inches deep?

3. What is the solidity of a block of marble whose length is 12 feet, breadth $5\frac{1}{4}$ feet, and thickness $2\frac{1}{2}$ feet?

PROBLEM XXIX. To find the solidity of a prism.

RULE. Find the area of the base, and multiply it by the height, the product will be the solidity.

EXAMPLES.

1. What is the solidity of a triangular prism, whose length is 12 feet, and each of its equal sides $2\frac{1}{2}$ feet?

The area of the base found by problem XI.=2.7063.

And $2.7063 \times 12 = 32.4756$ solid feet.

2. What is the value of a triangular prism, whose height is 48 feet, and each side of the base 4 feet, at 16 cents per solid foot?

3. What is the solidity of a square prism, each of its sides being 7 feet, and height 24?

PROBLEM XXX. To find the surface of a cylinder.

RULE. Multiply the circumference of the end by the length, and the product will be the convex surface, to which add the area of each end, and the sum will be the whole surface of the cylinder.

EXAMPLES.

1. What is the convex surface of a cylinder, whose length is 12 feet, and the diameter of its end 3 feet?

$3.1416 \times 3 = 9.4248$ circum. and $9.4248 \times 12 = 113.0976$ convex surface.

2. What is the whole surface of a cylinder, whose length is 16 feet, and the circumference of its base 8 feet?

$8 \times 16 = 128$ convex surface.

$8 \times 8 \times .07958 \times 2 = 10.18624$ area of both ends.

Then $128. + 10.18624 = 138.18624$ the whole surface.

3. How many yards are there in the convex surface of a cylinder 13 feet long, and diameter 4 feet?

PROBLEM XXXI. To find the solidity of a cylinder.

RULE. Multiply the square of the diameter by .7854, or the square of the circumference by .07958, and that product by the length will give the solidity.

EXAMPLES.

1. How many solid inches are there in a grindstone, whose diameter is 20 inches, and thickness $4\frac{1}{2}$ inches?

$.7854 \times 400 \times 4\frac{1}{2} = 1413.72$ solid inches.

2. How many gallons of water, each containing 282 solid inches, can be put into a cylindrical vessel 18 inches in diameter, and 2 feet in height?

3. The circumference is 12 feet 4 inches, and length 18 feet: what is its solidity?

PROBLEM XXXII. To find the whole surface of a cone.

RULE. Multiply the circumference of the base by the slant height, or the length of the side of the cone, and half the product will be the area of the convex surface, to which add the area of the base.

EXAMPLES.

1. The slant height of a cone is 20 feet, and the diameter is 4 feet: what is the whole surface of the cone?

$(4\times3.1416\times20)\div2=125.664$ convex surface.

$.7854\times16=12.5664$ area of the base.

$125.664+12.5664=138.2304$ whole surface.

2. The slant height of a cone is 18 feet, and the diameter 5 feet: required the convex surface of the cone?

3. The circumference of a cone is 12 feet, and the slant height 10: what is the whole surface of the cone?

PROBLEM XXXIII. To find the solidity of a cone, the diameter of the base and perpendicular altitude given.

RULE. Multiply the square of the diameter of the base by .7854, or the square of the circumference of the base by .07958, and that product by $\frac{1}{3}$ of the perpendicular altitude, the product will be the solidity.

EXAMPLES.

1. The diameter of the base of a cone is 4 feet, 6 inches, and the perpendicular altitude is 10 feet: what is the solidity?

$4.5\times4.5\times.7854\times(10\div3)=53.0145$ solidity.

2. The circumference of the base is 12 feet, and the perpendicular altitude is 16 feet: what is the solidity?

3. The altitude of a cone is 8 feet, and the diameter of the base 6 feet: what is its solidity?

PROBLEM XXXIV. To find the solidity of a pyramid.

RULE. Multiply the area of the base by $\frac{1}{3}$ of the altitude, and the product will be the solidity.

EXAMPLES.

1. What is the solidity of a pyramid, the area of whose base is 418 feet, and the altitude is 18 feet?

$418\times(18\div3)=2508$ feet solidity.

2. What is the solidity of a triangular pyramid, the area of whose base is 270.6 yards, and the altitude is 13 yards?

3. What is the solidity of a hexagonal pyramid, the area of whose base is 4156.92 feet, and the altitude is 31½ feet?

PROBLEM XXXV. To find the convex surface of a sphere or globe.

RULE. Multiply the square of the diameter by 3.1416, the product will be the area of the convex surface.

EXAMPLES.

1. What is the convex surface of a sphere whose diameter is 6 feet?

$$3.1416 \times 6 \times 6 = 113.0976.$$

2. Of a globe whose diameter is 12½ inches?

3. If the diameter of the globe we inhabit be 7935 miles, what is the convex surface?

PROBLEM XXXVI. To find the solidity of a globe.

RULE. Multiply the cube of the diameter by .5236, and the product will be the solidity.

EXAMPLES.

1. What is the solidity of a globe whose diameter is 4 ft. 6 in.?

$$4.5 \times 4.5 \times 4.5 \times .5236 = 47.71305 \text{ feet solidity.}$$

2. What is the solidity of a globe whose diameter is 3½ ft.?

3. How many cubic miles are contained in the solidity of the earth, if its diameter be 7935 miles?

PROBLEM XXXVII. The solidity of a globe being given, to find the diameter.

RULE. Divide the solidity by .5236, and extract the cube root of the quotient.

EXAMPLES.

1. The solidity of a globe is 2100 solid inches: what is its diameter?

$\sqrt[3]{(2100 \div .5236)} = \sqrt[3]{4010.695} = 15.88$ inches diameter.

2. A globe is 12 solid feet: what is its diameter?

3. A globe is 6 solid feet: what is its circumference?

ARTIFICERS' WORK.

§ **197.** Artificers compute their work by different measures; thus, flooring, partitioning, weather-boarding, roofing, &c. are measured by the square of 100 feet.

Painting, plastering, paving, &c. by the square yard.
Glazing by the foot, and sometimes by the light.
Bricklaying by the foot or rod.
Masons' work by the foot or perch.

MASONS' WORK.

§ 198. All sorts of stone work are performed by masons.

Masons measure walls by the solid perch, which is 16½ feet in length, 1 foot in depth, and 1½ feet in breadth. All walls of less breadth than 1½ feet, are considered of that thickness; but if more than that, the excess is charged for proportionately. Walls having angles are measured on the outer side, and no deduction is made for doors, windows, &c. Columns, or blocks of marble or other stone, are measured by the solid foot; but chimney pieces, slabs, and other flat work, by the square foot.

Materials.—Building stone is sold by the solid perch, containing 24¾ solid feet=4 ft.×2¾ ft.×2¼ ft., as measured at the quarry.

In measuring walls to ascertain the quantity of building stone in them, the space occupied by doors, windows, &c., is deducted.

EXAMPLES.

1. Required the solid contents of a wall 48 feet 6 inches long, 11 feet 4 inches high, and 2 feet thick.

ft.	*in.*	*ft.*	*in.*	*ft.*	*ft.*	*in.*
48	6 ×	11	4 ×	2 =	1099	4

ft.	*in.*	*perches.*
And 1099	4 ÷ 24.75 =	44.41

2. What is a mantle slab worth, whose length is 10 feet 6 inches, and breadth 3 feet 2 inches, at 80 cents per foot?

3. The dimensions of a certain building are as follows: 58 feet by 26 on the outside, height of the building 22 feet, height of the gable at each end 12 feet, thickness of the wall 1½ feet, two doors 4½ by 8 feet each, 28 windows each 3½ by 6 feet: what will the mason-work amount to, at 50 cents a perch, and what must be paid for the stone, at 40 cents per perch?

(58+26)×2=168=the whole girth of the building, and 168×22=3696=the surface of the wall to the gable. Also 26×12=312=the surface of the two gables. Hence (3696 +312)÷16.5=242.91 perches of mason-work. And 242.91×50 cents=$121.45 the amount of the mason-work.

Again 26—1.5×2=23 the width of the building within the walls. Hence (58+23)×2 = 162 the whole perimeter, 162×22=3564 content of the wall to the gable; 26×12= 312 content of the two gables; and 3564+312=3876 the superficial content of the building, including doors and windows; (4½×8×2)=72 deduction to be made for doors; (3½×6×28)=588 deduction to be made for windows; hence 3876—(72+588)=3216, and 3216×1½=4824, the contents of the wall in solid feet; 4824÷24.75=194.909 perches=the quantity of stone. Hence 194.909×40 cents =$77.96 the cost of the stone.

BRICKLAYERS' WORK.

§ **199.** Bricklayers' work is often measured by the rod, which is applied only to the surface of the work, the price being regulated by the thickness. But the dimensions of the rod which is used often vary. The following are some which are most frequently employed. The rod square of 16½ feet, or 272¼ square feet; that of 18 feet, or 324 square feet; that which is 16½ feet long and 1 foot high, and also that which is 21 feet long and 3 feet high, and which contains 63 square feet. But brick-work is now most commonly estimated by the thousand, the number of bricks used being ascertained by counting the courses or rows of bricks laid. Sometimes the number of solid feet in the wall are taken, and an allowance of 20 bricks is made for the solid foot, each brick being 9 inches long, 4½ wide, and 2½ thick. Or when the wall is 9 inches thick the area of the side may be ascertained, and an allowance of 13 bricks to a foot will give the number very nearly. But when brick-work is done by the rod, it should be reduced to standard thickness of a brick and a half thick. For this purpose the following rule is given.

Rule. Multiply the total number of feet in the wall by the number of half bricks in the thickness of it, and divide the product by 3, which will give the standard measure. Then divide by 272¼, or by 324, as the case may be, and the quotient will be the number of rods required. The ¼ is generally omitted in dividing, in favour of the workmen.

EXAMPLES.

1. If a wall measure 125 feet in length, 12 feet high, and 2½ bricks thick, how many rods does it contain?

(125×12×5)÷3=2500, and 2500÷272=9$\frac{116}{136}$ rods.

2. How many bricks in a wall whose length is 24 feet 3 inches, and the height 8 feet, and the thickness one brick or 9 inches, reckoning 13 bricks to the square foot of that thickness.

24 ft. 3 in. $\times$ 8 ft. = 194, and 194 $\times$ 13 = 2522, the number of bricks.

3. How many bricks are there in a wall whose length is 36 feet, and in height 18 feet, and the thickness 2 bricks, or 1 foot 6 inches, reckoning 20 bricks to the solid foot?

36 ft. $\times$ 18 $\times$ 1.5 ft. = 972, and 972 $\times$ 20 = 19440, the number of bricks.

4. How many rods are contained in a wall whose length is 67 feet, and height 32 feet 6 inches, the wall being 3 bricks thick?

5. If each side of a wall of a building be 60 feet long on the outside, each end wall 35 feet 6 inches in the inside, and the height of the building 30 feet, and the gable at each end of the wall 10 feet, and the thickness 1 brick or 9 inches: how many bricks are contained in said wall, and what is the value thereof, at $6.50 per thousand?

CARPENTERS' AND JOINERS' WORK.

§ **200.** Carpenters' and joiners' work comprehends all the wood work of buildings, and is measured by the square of 100 feet. Framing is sometimes measured by the foot long measure, and sometimes by the square foot. Joiners' work is often measured by the yard or foot, the line being made to ply close to every part of the work it passes over. When doors, window cases, cornices, &c. are thus measured, each member is measured by itself. Circular work is usually measured twice, and oval or elliptical work three times. Sometimes doors and shutters are computed by the panel, and sometimes are measured one and a half times. Venitian blinds are usually measured by the foot.

It is customary to reckon the flat and half of any building within the walls to be the measure of the roof of that building, when the roof is a true pitch, or so that the length of the rafters is three-fourths of the breadth of the building.

EXAMPLES.

1. If a house measure within the walls 44 feet 6 inches long, and 18 feet 3 inches broad, and the roofing of true pitch: what will the roofing cost at $1.25 per square?

44 ft. 6 in. × 18 ft. 3 in. = 812.125, and
812.125 + 812.125 ÷ 2 = 812.125 + 406.0625 = 1218.1875 ft.
and 1218.1875 ÷ 100 = 12 sq. 18 ft.

12.18 × 1.25 = $15.22, the amount.

2. If a floor is 60 feet long, and 28 feet 9 inches broad, how many squares will it contain?

3. What is the expense of flooring a building 45 feet 6 inches long, 26 feet 9 inches wide, two stories high, at $1.45 per square?

4. A partition is 116 feet 4 inches long, and 12 feet 3 inches broad: how many squares will it contain?

5. If the front or breadth of a building be 30 feet 3 inches, and the length or depth 52 feet 6 inches, what will be the expense of roofing, at $1.37½ per square, the length of the rafters being ⅗ of the depth of the building?

PLASTERERS' WORK.

§ **201.** Plasterers' work is of two kinds, viz. plastering upon laths, called ceiling; and plastering upon walls, called rendering; and these different kinds must be separately measured, and their contents collected into one sum, proper deduction being made for doors and windows.

Plasterers' work is generally measured by the yard, except cornice, mouldings, &c. by running or linear measure.

EXAMPLES.

1. If a ceiling be 61 feet 8 inches long, and 28 feet 4 inches broad, how many square yards does it contain?
61 ft. 8 in. × 28 ft. 4 in. = 1747 ft. 2⅔ in. and 1747 ft. 2⅔ in. ÷ 9 = 194. 13 yards.

2. There is a room 25 feet 6 inches long, 16 feet wide, and 9 feet 3 inches high: what will the plastering of the ceiling and walls come to, at 25 cents per yard?

3. The length of a room is 28 feet 10 inches, its breadth 26 feet 4 inches, and height 11 feet to the under side of the cornice, whose girth is 8½ inches, and its projection 5 inches from the wall on the upper part next the ceiling: what will be the quantity of plastering, supposing there be no deductions but for two doors whose size is 7 feet by 4 feet each.

4. If a ceiling be 82 feet 3 inches long, and 33 feet 9 inches broad, how many square yards does it contain?

PAINTERS AND GLAZIERS' WORK.

§ **202.** Painters' work is measured in square yards, except windows, which are done by the piece. In mouldings, &c. it is customary to allow twice the usual measure.

Glaziers' work is done at so much per light, that is computed at a given price for putting in each pane of glass, according to its size.

EXAMPLES.

1. If a room be painted whose height is 12 feet 4 inches, and its compass 111 feet 3 inches, how many yards does it contain, and what will it cost, at 18 cents per yard?

111 ft. 3 in. $\times$ 12 ft. 4 in. = 1372 ft. 1 in., and 1372 ft. 1 in. $\div$ 9 = 152.45 yds. Hence 152.45 $\times$ 18 = $27.44 amount.

2. What should a painter charge for painting a room, the walls of which were 8 feet high, the room 18 feet by 14, ceiling included, at 42 cents per yard?

3. What should be charged for three rooms, the first 15 feet by 12, walls 7 feet high; the second 13 feet by 9, walls 6 feet 9 inches high; the third 20 feet by 15, walls 8½ feet high, at 35 cents per yard?

4. There is a house with 26 windows, 12 of which contain 22 lights each, and 14 contain 18 lights each: what will the glazing come to, at 6 cents per light?

PAVERS' WORK.

§ **203.** Pavers' work is done by the square yard.

RULE. Multiply the length by the breadth, and the product will be the content?

EXAMPLES.

1. The length of a rectangular yard is 45 feet, and the breadth 18 feet 9 inches: what will the paving come to, at 54 cents per square yard?

45 ft. $\times$ 18 ft. 9 in. = 843.75 ft., and 843.75 $\div$ 9 = 93.75 yds., and 93.75 $\times$ 54 cts. = $50.62½ amount.

2. How many yards are there in a side walk, the length of which is 396 feet, and breadth 12 feet 4 inches?

3. What will be the expense of paving a rectangular yard, the length of which is 72 feet, and breadth 52 feet, in which there is laid a foot path 6 feet 6 inches wide, running the whole length, with stones, at 42 cents per yard, the remainder being paved with pebbles, at 25 cents per yard?

KEY

TO THE

UNITED STATES

ARITHMETIC.

FOR THE USE OF TEACHERS.

By WILLIAM VOGDES, A. M.

PROFESSOR OF MATHEMATICS IN THE CENTRAL HIGH SCHOOL OF PHILADELPHIA.

PHILADELPHIA:
E. C. & J. BIDDLE, No. 6 SOUTH FIFTH STREET.
1847.

Adoption of the "United States Arithmetic" as a class book for the Public Schools of the City and County of Philadelphia.

CHAMBER OF THE CONTROLLERS OF PUBLIC SCHOOLS,
FIRST SCHOOL DISTRICT OF PENNSYLVANIA.

Philadelphia, April 9th, 1845.

At a meeting of the Controllers of Public Schools, held at their Chamber, on Tuesday, April 8th, 1845, the following resolution was adopted:—

Resolved, That the "UNITED STATES ARITHMETIC," by PROFESSOR VOGDES, be introduced, as a class book, into the Public Schools of the District.

From the minutes.

THOMAS B. FLORENCE,
Secretary.

Entered according to Act of Congress, in the year 1845,
By E. C. & J. BIDDLE,
In the Clerk's Office of the District Court of the United States,
of the Eastern District of Pennsylvania.

KEY

TO THE UNITED STATES ARITHMETIC.

Pages 16—23.] ADDITION.

§ 3.

4. 1 636.
5. 1 502.
6. 2 374.
7. 2 627.
8. 3 022.
9. 1 801.
10. 2 541.
11. 8 896.
12. 14 251.
13. 12 262.
14. 27 610.
15. 20 086.
16. 17 531.
17. 5 765.
18. 14 895.
19. 14 602.
20. 25 175.
21. 23 382.
22. 262 362.
23. 268 795.
24. 262 794.
25. 127 536.
26. 1 670.
27. 8 122.
28. 1 790.
29. 2 647.
30. 1 740.
31. 1 755.
32. 2 566.
33. 3 184.
34. 1 166.
35. 15 172.
36. 175 406.
37. 21 070.
38. 26 470.
39. 88 996.
40. 688 795.
41. 43 089.
42. 913 942.
43. 72 364.
44. 2 487.
45. 300.
46. 728 000.
47. 4 800 000.
48. 1 288.
49. 1 016.
50. 49 500 000.
51. 57 270.
52. 40 099.
53. 58 350 000.
54. 3 187 100.
55. 3 296 798.
56. 3 394 570.
57. 293 040.
58. 7 767 401.
59. 12 230.
60. 35 252 805.

SUBTRACTION.

§ 4.

4. 499.
5. 332.
6. 273.
7. 307.
8. 209.
9. 19.
10. 773.
11. 1 096.
12. 924.
13. 654.
14. 1 725.
15. 869.
16. 1 299.
17. 740.
18. 1 041.
19. 2 657.
20. 8 657.
21. 11 937.
22. 16 740.
23. 10 361.
24. 28 965.
25. 10 604.
26. 150 612.
27. 243 099.
28. 5 069 407.
29. 6 082 520.
30. 8 006 655.
31. 4 754 783.
32. 191 991.
33. 15 500 000.

34. 16 731 037.
35. 10 955 576.
36. 1 899 871.
37. 154 119.
38. 83 388 453.
39. 72 000 008.
40. 4 248 225.
41. 6 685 737.
42. $2 896.
43. 1 121 488 *increase.*
44. 92 *years.*
45. $17 897.
46. $889 *gain.*
47. 14 000 000 *popular'n*, 13 561 470 *bushels.*
48. 1 238 392 *increase.*
49. 88*hhds.*, 87 684*lbs.*
50. 1 737.
51. 289 *years.*
52. $24 899 *prime cost.*
53. 98 999.
54. 9 201 485.
55. $49 787.
56. 15 668.
57. $17 199.
58. 939 933.
59. $3 611 059.

ADDITION AND SUBTRACTION.

Pages 25, 26 and 27.

1. 2 443 606.
2. 406 260.
3. 53 991 402.
4. 3 317 775.
5. 148 123.
6. 0000000.
7. 242 582.
8. 25 512.
9. 2 369 523.
10. 83 746.
11. $50 489 746.
12. $827 371.
13. $8 880 602.
14. 288 397.
15. $7 192 091.
16. 19 969 998.
17. $977.
18. 768 *barrels*, $2 415.
19. 93 665 *city.*
20. 1 183*lbs.*
21. $18 055 117.
22. 700 526 *bushels.*
23. 87 *years*, 164*yrs.*, 77*yrs.*
24. 284*yrs.*, 334*yrs.*, 353*yrs.*
25. 54673 New York and Philadelphia; 155934 Philadelphia and New Orleans; 55 765 New Orleans and Cincinnati; 17 077 Cincinnati and Charleston; 18 341 Charleston and Norfolk; 3 943 Norfolk and Lexington.

MULTIPLICATION.

§ 5.

3. 2 367.
4. 21 185.
5. 43 446.
6. 65 604.
7. 43 072.
8. 83 780.
9. 25 795.
10. 81 468.
11. 217 284.
12. 592 590.
13. 86 436.
14. 4 165 812.
15. 2 181 025.
16. 14 848 788.
17. 29 508 828.
18. 42 770 826.
19. 290 125 636.
20. 698 351 598.
21. 8 642 579 810.
22. 2 087 277 148.
23. 11 653 412 880.
24. 7 430 593 742.
25. 5 360 775 519.
26. 4 938 271 560.
27. 45 406 172 835.
28. 20 367 028 432.
29. 3 775 566 894.
30. 9 646.
31. 15 130.
32. 7 961.
33. 19 481.
34. 12 714.
35. 10 788.
36. 17 577.
37. 25 976.
38. 11 718.

39. 11 070.
40. 1 968.
41. 87 012.
42. 28 880.
43. 128 121.
44. 188 496.
45. 148 149.
46. 78 525.
47. 1 716 820.
48. 26 632 320.
49. 23 747 232.
50. 5 559 570.
51. 13 629 105.
52. 17 673 264.
53. 42 492 405.
54. 31 717 362.
55. 8 015 328.
56. 36 409 842.
57. 29 528 280.
58. 407 381 106.
59. 126 959 008.
60. 15 984.
61. 21 175.
62. 43 659.
63. 327 538.
64. 412 158.
65. 29 949.
66. 661 572.
67. 371 378.
68. 362 967.
69. 404 608.
70. 183 634.
71. 1 741 174.
72. 5 020 645.
73. 19 623 168.
74. 30 813 986.
75. 19 868 928.
76. 26 944 592.
77. 485 550 208.
78. 2 453 033 204 514.
79. 815 508 573 966 072
80. 83 233 769 620 848

CONTRACTIONS.

§ 6. CASE 1.

3. 493 600.
4. 312 799 510.
5. 5 717 500 620.
6. 73 649 446 800.
7. 261 220 258 900.
8. 3 821 291 052 000.
9. 5 400 153 627 000.
10. 37 701 081 600 000.
11. 1 790 045.
12. 2 583 688.
13. 7 461 216.
14. 648 514 496.
15. 7 008 165 804.
16. 6 302 976 336.
17. 36 372 585 250.
18. 280 813 380 159.

§ 7. CASE 2.

2. 14 550.
3. 18 144.
4. 24 768.
5. 28 805.
6. 38 850.
7. 54 630.
8. 1 149 344.
9. 4 419 576.
10. 5 398 208.
11. 3 351 096.
12. 99 999 927.
13. 447 094 752.
14. 878 888 880.
15. 940 910 784.
16. 890 434 656.
17. 160 407 038.
18. 8 625 526 470.
19. 1 133 589 168.

§ 8. CASE 3.

3. 23 463. | 4. 1 759 239. | 5. 68 999 379. | 6. 1 179 457 532

§ 9. CASE 4.

4. 1 969 459 236.
5. 273 656 484.
6. 260 063 163.
7. 2 022 384 600.
8. 2 048 284 728.
9. 64 524 768.

MISCELLANEOUS EXERCISES.

Pages 36, 37 and 38.

1. 82 304 526.
2. 12 345 678 912.
3. 1 543 209 864.
4. 1 234 567 890.
5. 2 057 613 152.
6. 881 834 208
7. 128 600 822.
8. 12 345 678 912.
9. 1 234 158 912.
10. 12 345 678 912.
11. 12 345 678 912.
12. 12 345 678 912.
13. 12 345 678 912.
14. 1 847 107 216.
15. 4 890 096 *inhabitants.*
16. 51 600 *pupils.*
17. 124 914 *bushels.*
18. 7 270 032 *sheep.*
19. 3 447 378*lbs.*
20. 158 000 *miles.*
21. $16 419.
22. $157 333.
23. 4 430 745 *population.*
24. 133 200 *miles.*
25. 5 700 *miles.*
26. $4 871 132.
27. $41 121.
28. $80 922.
29. 43 700 *population.*
30. $32 256.
31. 1 604 175 *days.*
32. $20 025.
33. $247 275.
34. 44 175 *perches.*
35. 806 080 *yards.*
36. 36 542 025.
37. 10 878 945 *cubic inches.*
38. 62 810 *feet.*
39. $2 717.
40. 44 208 000 *acres.*
41. $9 186 765.
42. 3 240 *square feet.*
43. 5 724 *days.*
44. 472 000 *times.*
45. $112 050.
46. 856 980 *miles.*
47. 244 140 625.
48. 384 *miles.*
49. 18 900 *feet.*
50. 782 *pieces,* 21 114 *yards,* $105 570 *value.*
51. $236 624.
52. 2 160*lbs.*
53. 781 200*oz.*
54. 19 768 320 *yards.*
55. 4 165 376 402 969 600*lbs.*

DIVISION.

Page 44. CASE 1.

4. 118 813.
5. 229 218.
6. 158 109. *rem.* 1.
7. 309 182.
8. 145 657.
9. 62 162.
10. 51 734.
11. 138 967.
12. 78 815. *rem.* 4.
13. 123 749.
14. 54 489.
15. 107 822.
16. 13 593.
17. 38 471.
18. 105 420.
19. 874 567.
20. 123 456.
21. 80 868.

22. 125 248.
23. 123 456. *rem.* 6.
24. 49 907.
25. 60 001.
26. 2 035 797.
27. 30 293 289.

Page 45.

CASE 2.

28. 13.
29. 16.
30. 17.
31. 1 901.
32. 2 201.
33. 117 987. *rem.* 9.
34. 25 776. *rem.* 26.
35. 43 683. *rem.* 66.
36. 1 899. *rem.* 42.
37. 36 346. *rem.* 54.
38. 227 806. *rem.* 100.
39. 12 949. *rem.* 709.
40. 17 180. *rem.* 213.
41. 72 056. *rem.* 430.
42. 51 621. *rem.* 127.
43. 42 483. *rem.* 520.
44. 23 359. *rem.* 414.
45. 733 355. *rem.* 160.
46. 179 195. *rem.* 1 622.
47. 1 690. *rem.* 1 918.
48. 17 920. *rem.* 466.
49. 8 426 357.
50. 160 992. *rem.* 23 414.
51. 338 238. *rem.* 205.

CONTRACTIONS.

§ 13.

CASE 1.

3. 236 081. *rem.* 14.
4. 117 823. *rem.* 14.
5. 174 826. *rem.* 32.
6. 560 520. *rem.* 74.
7. 633 862. *rem.* 59.
8. 151 218. *rem.* 187.
9. 491 738. *rem.* 148.
10. 102 079. *rem.* 280.
11. 193 157. *rem.* 2.
12. 1374 616. *rem.* 16.
13. 108 933. *rem.* 278.
14. 4 257. *rem.* 352 776.
15. 8 254. *rem.* 238 012.
16. 7 921. *rem.* 89 628.
17. 22 335. *rem.* 1 177 901.
18. 67 292. *rem.* 2 785 461.
19. 137 296. *rem.* 6 041 614.
20. 62 729. *rem.* 5 776 200.

§ 14.

CASE 2.

3. 1 236 019. *rem.* 18.
4. 1 697 284. *rem.* 23.
5. 453 429. *rem.* 4.
6. 170 035. *rem.* 76.
7. 7 973. *rem.* 25.
8. 712 418. *rem.* 24.
9. 1 524 156. *rem.* 72.
10. 7 234 901. *rem.* 83.
11. 6 546 547. *rem.* 77.
12. 1 770 771. *rem.* 92.
13. 24 715 673. *rem.* 110.
14. 2 570 015. *rem.* 101.
15. 2 583 425. *rem.* 195.
16. 1 398 894. *rem.* 105.
17. 13 607 738. *rem.* 325.
18. 1 820 664. *rem.* 18.
19. 120 309 942. *rem.* 408.
20. 120 107 294. *rem.* 504.
21. 51 044 407. *rem.* 707.
22. 1 376 678. *rem.* 942.
23. 29 582 205. *rem* 74.
24. 1 272 558. *rem.* 2 371.
25. 54 901 889. *rem.* 11 128

§ 15.

2. 413. *rem.* 22.
3. 94. *rem.* 231.
4. 5 349. *rem.* 57.
5. 5 735. *rem.* 35.

CASE 3.

6. 134 478. *rem.* 244.
7. 6 637 102. *rem.* 26.
8. 327 935. *rem.* 923.
9. 34 806. *rem.* 66.

§ 16.

CASE 4.

3. 340. *rem.* 80. | 4. 553. *rem.* 639. | 5. 4 370. *rem.* 6001.

§ 17.

2. 2, 3, 3, 5, 5.
3. 2, 3, 131.
4. 3, 13, 23.
5. 2, 3, 3, 3, 37.
6. 3, 3, 311.

CASE 5.

7. 2, 2, 2, 7, 71.
8. 2, 5, 5, 13, 23.
9. 2, 2, 2, 2, 7, 17, 17.
10. 2, 393 271.
11. 2, 2, 2, 12 345 679.

MISCELLANEOUS EXERCISES.

Pages 50, 51 and 52.

1. 14 814 804.
2. 1 234 567.
3. 22 488 992.
4. 33 580 032.
5. 46 913 280.
6. 142 279 180.
7. 192 591 360.
8. 3 703 680.
9. 123 456.
10. 1 185 120.
11. 99 550 080.
12. 6 715 680.
13. 123 456 789.
14. 123 456 789.
15. 123 456 789.
16. 123 456 789.
17. 123 456 789.
18. 123 456 789.
19. 123 456 789.
20. 123 456 789.
21. 123 456 789.
22. 123 456 789.
23. $14.
24. 424 *men.*
25. 203 *barrels.*
26. 3 *cords.*
27. 207 *persons*, and 122 000 *rem.*
28. $8 432.
29. 48 *trees.*
30. 640 *acres.*
31. 54 *days.*
32. 200 *miles.*
33. 700 *miles.*
34. 14 *days.*
35. 40 *days.*
36. 19 *persons*, and 23 797 *rem.*
37. 126 *lbs.*
38. 19 *men.*
39. $845.
40. 260 329 *lbs.*
41. $1 173.
42. 14.
43. 12 771.
44. $774, and 28 147 *rem.*
45. 65 528 *lbs.*

COMPOUND ADDITION.

§19. FEDERAL MONEY.

	$	cts.	m.
4.	216	68	2.
5.	755	78	5.

	$	cts.	m.
6.	2095	25	9.
7.	1290	40	3.

	$	cts.	m.
8.	2252	99	5.
9.	3207	33	3.

§20. ENGLISH MONEY.

	£	s.	d.
2.	127	16	$8\frac{1}{2}$.
3.	59	10	$2\frac{3}{4}$.
4.	240	10	$8\frac{1}{2}$.
5.	169	6	4.
6.	199	15	4.

	£	s.	d.
7.	260	11	1.
8.	148	1	6.
9.	241	8	7.
10.	226	13	$8\frac{1}{2}$.
11.	122	9	$6\frac{1}{4}$.

	£	s.	d.
12.	1916	3	$9\frac{1}{4}$.
13.	1661	3	$3\frac{3}{4}$.
14.	3647	8	10.
15.	2884	17	8.

§21. TROY WEIGHT.

	lbs.	oz.	dwt.	grs.
3.	31	0	14	8.
4.	11	8	0	23.

	lbs.	oz.	dwt.	grs.
5.	50	3	6	14.
6.	150	3	6	12.

§22. APOTHECARIES WEIGHT.

	℔	℥	ʒ	℈	grs.
3.	40	9	6	1	19.
4.	30	5	0	2	3.

	℔	℥	ʒ	℈	grs.
5.	69	3	6	2	5.
6.	52	4	7	1	0.

§23. AVOIRDUPOIS WEIGHT.

	T.	cwt.	qrs.	lbs.	oz.	drs.
2.	40	18	0	12	1	14.
3.	22	12	3	27.		
4.	49	1	1	2	2	14.

	T.	cwt.	qrs.	lbs.	oz.	drs.
5.	47	5	1	14	13	2.
6.	99	15	0	10	4	13

§24. LONG MEASURE.

2. 206*L.* 1*M.* 0*fur.* 5*p.* 0*yds.* 0*ft.* 0*in.* 2*bar.*
3. 112*M.* 6*fur.* 32*p.* 2*yds.* 2*ft.* 3*in.* 1*bar.*
4. 88*L.* 0*M.* 3*fur.* 6*p.*
5. 1294*M.* 6*fur.* 6*p.*

§25. LAND OR SQUARE MEASURE.

2. 147*A.* 3*r.* 8*p.* 5*ft.*
3. 53*A.* 3*r.* 36*p.* 2*ft.* 14*in.*
4. 196*A.* 0*r.* 23*p.* 2*ft.* 117*in.*
5. 503*A.* 2*r.* 32*p.*
6. 3340*A.* 1*r.* 32*p.*

§26. SOLID, OR CUBIC MEASURE.

3. 102*cds.* 81*ft.* 1248*in.*
4. 299*yds.* 18*ft.* 625*in.*
5. 411 *T.* 1*ft.* 1686*in.*
6. 126*cds.* 88*ft.* 1702*in.*

§27. CLOTH MEASURE.

3. 37*E.Fl.* 2*qrs.* 1*n.*
4. 47 *yds.* 2*qrs.* 3*n.*
5. 69*yds.*
6. 122*EE.* 1*qr.* 2*n.*
7. 46*E.Fl.* 0*qr.* 2*n.*
8. 65*E.Fr.*
9. 181*yds.* 2*qrs.* 1*n.*

§28. WINE MEASURE.

2. 101 *T.* 3*hhd.* 41*gal.* 0*qt.* 1*pt.*
3. 51*P.* 1*hhd.* 9*gals.* 0*qt.* 1*pt.*
4. 105*hhds.* 41*gals.* 2*qts.*
5. 90 *T.* 0*hhds.* 29*gals.* 2*qts.*
6. 19*P.* 1*hhd.* 54*gals.* 1*qt.*
7. 116 *T.* 1*P.* 2*tie.* 4*gals.*

§29. ALE AND BEER MEASURE.

2. 158*hhds.* 16*gals.* 1*qt.*
3. 123 *T.* 0*bt.* 0*hhds.* 22*gals.* 1*qt.*
4. 250*hhds.* 12*gals.* 2*qts.* 1*pt.*
5. 277*hhds.* 10*gals.* 0*qt.* 1*pt.*
6. 187 *T.* 0*bt.* 0*hhd.* 23*gals.* 3*qts.*

§30. DRY MEASURE.

2. 72*ch.* 3*bu.* 3*pks.* 5*qts.*
3. 247*bu.* 3*pks.* 7*qts.* 1*pt.*
4. 93*bu.* 0*pk.* 7*qts.* 1*pt.*
5. 233*bu.* 2*pks.* 3*qts.* 1*pt.*
6. 232*ch.* 16*bu.* 3*pks.* 5*qts*

§31. TIME.

3. 56 *Ys.* 6*mo.* 1*wk.* 6*ds.* 19*h.* 54*min.* 11*sec.*
4. 36 *Ys.* 1*mo.* 2*wks.*
5. 44 *Ys.* 7*mo.* 2*wks.* 2*ds.* 7*h.* 4*min.*
6. 48 *Ys.* 4*mo.* 3*wks.* 2*ds.* 14*h.* 2*min.* 14*sec.*

§32. CIRCULAR MOTION.

	s.	°	*′*	*″*
2.	39	17	55	6
3.	40	29	20	29
4.	2	23	7	31
5.	33	4	9	44
6.	37	10	32	20

MISCELLANEOUS EXERCISES.

1. 1784£ 4*s.*
2. 675*yds.* 3*qrs.* 3*n.*
3. 38*lbs.* 7*oz.* 17*dwt.* 5*grs.*
4. 32*cwt.* 0*qr.* 21*lbs.*
5. 15ℨ 2ʒ 0ʓ 16*grs.*
6. 161*yds.* 2*qrs.* 2*n.*
7. 77*A.* 1*r.* 1*p.*
8. 238*h day.*
9. 634*M.* 3*fur.* 27*p.*
10. 5*T.* 32*gal.* 1*pt.*
11. 117 *years.*
12. 135*bu.*
13. 11*s.* 0° 30′ 40″.
14. 99*cds.* 35*ft.*
15. $9 557.79.
16. $434.53.
17. $1 146.37.
18. $3 421.27.
19. 128*gals.* 1*qt.* 1*pt.*
20. 3*s.* 4° 0′ 21″.
21. 21*lbs.* 5*oz.* 18*dwt.*
22. 13*Ys.* 6*mos.* 4*d.*
23. 2 273£ 14*s.* 10¼*d.*
24. 30*cwt.* 3*qrs.* 3*lbs.*
25. 233*M.* 4*fur.* 25*p.*
26. 200*cds.* 58*ft.*

COMPOUND SUBTRACTION.

§ 35. FEDERAL MONEY.

	$	cts.	m.		$	cts.	m.		$	cts.	m.
4.	88	99	2.	8.	850	10	2.	12.	898	83	9.
5.	229	81	7.	9.	262	20	4.	13.	55	92.	
6.	84	99	2.	10.	215	17	4.	14.	43	94.	
7.	228	98	9.	11.	237	77	4.				

§ 36. ENGLISH MONEY.

	£	s.	d.		£	s.	d.		£	s.	d.
3.	482	18	11¾.	9.	117	18	11¼.	15.	198	19	2½.
4.	10	9	7.	10.	111	6	1½.	16.	135	19	6.
5.	39	18	9½.	11.	55	18	0¾.	17.	210	15	1¼.
6.	43	18	9¾.	12.	1	15	11¾.	18.	0	15	7¾.
7.	189	18	4¾.	13.	4	11	6½.	19.	0	9	11.
8.	204	6	6¼.	14.	2	17	8.	20.	47	6	10½.

§ 37. TROY WEIGHT.

	lb.	oz.	dwt.	gr.		lb.	oz.	dwt.	gr.		lb.	oz.	dwt.	gr.
3.	24	1	7	5.	9.	5	11	3	17.	15.	8	1	18	6.
4.	7	11	19	3.	10.	0	2	11	9.	16.	11	10	1	15.
5.	6	5	16.		11.	0	6	16	23.	17.	20	0	8	3.
6.	0	5	7	22.	12.	5	9	16	20.	18.	15	11	19	21.
7.	2	9	16.		13.			17	9.	19.	5	5	7	21.
8.	5	11	8	8.	14.	4	6	12	14.	20.	12	6	18	23

§38. APOTHECARIES WEIGHT.

	lb	℥	ʒ	℈	grs.
4.	4	10	3	2	5.
5.	4	11	1	1	18.
6.	1	9	7	0	2.
7.	8	5	7	2	10.
8.	2	11	6	2	19.
9.	5	0	7	0	2.
10.	7	0	1	2.	
11.	8	11	7	1.	
12.	5	7	1	1	9.

	lb	℥	ʒ	℈	grs.
13.	4	10	4	1	9.
14.	12	0	5	0	19.
15.	4	6	6	0	18.
16.	2	10	6	2	6.
17.	6	9	5	1	19.
18.	5	4	5	2	19.
19.	8	11	3	1	11.
20.	58	11	1	0	14.

§39. AVOIRDUPOIS WEIGHT.

	cwt.	*qrs.*	*lbs.*	*oz.*	*drs.*
3.	9	0	16.		
4.	17	0	26.		
5.	15	2	11.		
6.	8	1	27.		
7.	2	1	6	3.	
8.	4	1	2	10.	
9.	5	1	1	11.	
10.	2	2	1	1.	
11.	3	3	15	4	11.

	T.	*cwt.*	*qrs.*	*lbs.*	*oz.*	*drs.*
12.		5	2	17	11	7.
13.		9	3	12	15	9.
14.		2	3	10	14	15.
15.		1	3	0	15	5.
16.		16	2	3.		
17.	30	3	2	23.		
18.	12	3	0	8	14	7.
19.	61	15	1	4	5	15.
20.	44	5	1	9	14	8.

§40. LONG MEASURE.

3. 18*deg.* 1*M.* 5*fur.* 5*p.* 1*yd.*
4. 23*M.* 4*fur.* 33*p.* 0*yd.* 2*ft.*
5. 31*deg.* 55*M.* 6*fur.*
6. 12*M.* 4*fur.* 34*p.* 1*yd.*
7. 2*fur.* 1*p.* 0*yd.* 2*ft.* 10*in.* 1*bar.*
8. 18*deg.* 5*M.* 4*fur.* 21*p.* 1*yd.*
9. 34*M.* 6*fur.* 39*p.* 1*yd.*
10. 4*deg.* 10*M.* 6*fur.* 32*p.* 1*yd.* 1*ft.* 11*in.*
11. 8*M.* 0*fur.* 35*p.* 1*yd.*
12. 34*M.* 0*fur.* 39*p.*
13. 32*M.* 3*fur.* 36*p.* 1*yd.*
14. 12*M.* 5*fur.* 36*p.* 1*yd.*
15. 45*M.* 4*fur.* 39*p.*
16. 47*M.* 4*fur.* 38*p.* 1*yd.*
17. 10*M.* 6*fur.* 39*p.* 1*yd.*
18. 111*M.* 6*fur.* 39*p.* 1*yd.*
19. 101*M.* 6*fur.* 39*p.* 1*yd.* 1*ft.*
20. 7*deg.* 6*M.* 6*fur.* 38*p.* 0*yd.* 2*ft.* 11*in.* 1*bar.*

§41. LAND, OR SQUARE MEASURE.

	A.	*r.*	*p.*	*yds.*		*A.*	*r.*	*p.*	*yds.*	*ft.*	*in.*
3.	26	3	0.		12.	7	2	17	2.		
4.	20	3	28.		13.	28	1	37.			
5.	32	2	31.		14.	37	3	5	4.		
6.	166	2	39.		15.	92	0	24	0	8.	
7.	35	3	3.		16.	88	1	30	7	4.	
8.	52	0	34.		17.	101	2	16	2	4.	
9.	6	2	37.		18.	9	2	31	7	7	143.
10.	42	0	36.		19.	41	2	18	0	86	138.
11.	17	0	27	1.	20.	9	2	8	0	17	31.

§42. SOLID, OR CUBIC MEASURE.

	T.	*ft.*	*in.*		*T.*	*ft.*	*in.*		*cd.*	*ft.*	*in.*
3.	64	29	894.	10.	25	0	1720.	17.	3	47	30.
4.	16	37	631.	11.	34	48	373.	18.	7	4	1568.
5.	12	38	815.	12.	91	10	1.	19.	37	98	1679.
6.	21	11	23.	13.	47	48	29.	20.	92	58	1204.
7.	16	39	227.	14.	90	19	108.				
8.	18	11	1588.	15.	103	18	203.				
9.	14	36	1698.	16.	6	46	109.				

§43. CLOTH MEASURE

	yds.	*qr.*	*n.*	*in.*		*yds.*	*qrs.*	*na.*		
3.	54	0	3	1.	9.	149	2	3.	15.	16*E.Fr.* 1*qr.* 3*n.*
4.	59	0	2.		10.	18	3	1.	16.	21*EFl.* 2*qrs.* 3*n.*
5.	25	0	1.		11.	78	2	3.	17.	31*E.H.* 0*qr.* 3*n.*
6.	58	2	3.		12.	87	3	1.	18.	24*yds.* 3*qrs.* 2*n.*
7.	33	2	3.		13.	11*E.E.* 2*qrs.* 3*n.*			19.	172*yds.*
8.	151	3	2.		14.	27*E.E.* 1*qr.* 1*n.*			20.	88*yds.* 2*qrs.* 2*n.* 1*in*

§44. WINE MEASURE.

	T.	*hhds.*	*gals.*	*qts.*	*pt.*		
3.	6	0	46	1.		12.	17*hhds.* 61*gals.* 3*qts.* 1*pt.*
4.	8	1	21	3.		13.	19*hhds.* 61*gals.* 2*qts.* 0*pt.* 3*gi.*
5.		11	62	1	1.	14.	2*T.* 2*hhds.* 30*gals.*
6.		13	61	2	1.	15.	19*T.* 2*hhds.* 51*gals.* 3*qts.*
7.		38	59	2.		16.	7*T.* 1*pun.* 0*tie.* 19*gal.*
8.		9	53	3.		17.	27*T.* 1*pun.* 0*tie.* 5*gals.* 1*qt.*
9.		42	60	1.		18.	5*T.* 1*pun.* 0*hhds.* 12*gals.* 3*qts*
10.		3	61	3.		19.	22*T.* 0*P.* 0*hhd.* 60*gal.* 3*qts.*
11.		83	61	3.		20.	36*hhds.* 61*gals.* 3*qts.* 1*pt.* 1*gi.*

§45. ALE AND BEER MEASURE.

	hhds.	*gal.*	*qt.*	*pt.*
3.	2	11	0	1.
4.	58	22	1	1.
5.	34	52	2.	
6.	18	25	3	1.
7.	26	46	3.	
8.	125	44	2.	
9.	81	52	3.	
10.	37	16	3.	
11.	170	4	1	1.

	T.	*bt.*	*hhds.*	*gals.*	*qts.*
12.	17	1	1	19.	
13.			223	2	3.
14.	14	0	1	53.	
15.	4	1	0	20.	
16.	9	1	0	7	1.
17.			108	53	1.
18.	10	1	1	48.	
19.			23	53	3.
20.	27	1	1	20.	

§46. DRY MEASURE.

	bu.	*pks.*	*qts.*	*pts.*
3.	27	2	6	1.
4.	29	2	6	1.
5.	5	2	7.	
6.	48	2	7.	
7.	40	1	7.	
8.	98	2	7.	
9.	29	2	5.	
10.	221	1	7.	
11.	292	3	7.	

	chal.	*bu.*	*pks.*	*qts.*	*pts.*
12.		372	1	7.	
13.		726	2	7.	
14.		688	3	7	1.
15.		128	1	6	1.
16.		799	0	7.	
17.	4	34	0	7.	
18.	7	35	3	7.	
19.	15	18	3	6	1.
20.	23	35	2	3.	

§47. TIME.

	Y.	*mo.*	*w.*	*d.*	*h.*	*min.*	*sec.*
2.	7	11	2	5	19	51	42.
3.	3	12	0	6.			
4.	4	11	1	4.			
5.	5	12	0	6.			
6	5	10	3	5.			
7.		3	3	0	20.		
8.		4	3	6	17.		
9.		1	2	5	23	3.	
10.		5	0	3	7	34.	
11.		3	2	1	22	58	6.

	Y.	*mo.*	*w.*	*d.*	*h.*	*min.*	*sec.*
12.	2	12	2	5	23	13	58.
13.				3	19	59	9.
14.				6	19	18	56.
15.			2	5	22	58	59.
16.	13	11	3	6	20.		
17.	6	11	3	6	14.		
18.	11	9	2	5	22	59.	
19.		3	3	6	21	1	27.
20.				1	2	59	59.

§48. CIRCULAR MOTION.

	°	′	″
3.	5	16	51.
4.	13	48	58.
5.	4	56	57.
6.	9	15	53.

	°	′	″
7.	9	21	59.
8.	11	58	3.
9.	2	30	12.
10.	6	36	51.

	s.	°	′	″
11.		10	53	49
12.		23	43.	
13.	3	20	47	57.
14.	2	25	56	19.

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	°	′	″		°	′	″		s.	°	′	″
15.	3	58	39.	17.	3	59	49.	19.	3	0	53	3.
16.	14	9	47.	18.	17	58	59.	20.	6	25	55	36.

MISCELLANEOUS EXERCISES.

1. $6 890.57.
2. $3 260.33.
3. $6 823.48.
4. $428.88.
5. $10 900.
6. $14 834.14.
7. 100£ 0*s.* 0¼*d.*
8. 153£ 8*s.* 2¾*d.*
9. 22*yds.* 0*qr.* 2*n.*
10. 2 *T.* 9*cwt.* 1*qr.* 10*lbs.*
11. 1 *Y.* 9*mo.* 1*w.* 2*ds.*
12. 12*M.* 7*fur.* 3*p.*
13. 6 *Y.* 3*mo.* 19*ds.*
14. 9 *Y.* 3*mo.* 22*ds.*
15. 11° 31′ 31″.
16. 3*s.* 21° 51′ 52″.
17. 20*cds.* 20*ft.*
18. 4 282*gals.* 0*qt.* 1*pt.*
19. 3*lbs.* 4*oz.* 1*dwt.* 19*grs.*
20. 4℔ 10℥ 6℥ 1℈ 5*grs.*
21. 7*A.* 3*r.* 32*p.*
22. 33 *Y.* 2*mo.* 15*ds.*, and 51 *Y.* 2*m.* 15*dys.*
23. 33 *Y.* 11*mo.* 17*ds.*
24. 2 *Y.* 4*mo.* 22*ds.*
25. 7 *years.*
26. 32 *Y.* 8*mo.* 5*ds.*
27. 7*M.* 2*fur.* 36*p.*
28. 257 *days.*
29. 14*h.* 0*min.* 38*sec.*
30. 1*M.* 3*fur.* 33*p.* 5*yds.* 0*ft* 6*in.*

MULTIPLICATION OF FEDERAL MONEY.

§49.

3. $259.20.
4. $312.39.
5. $316.80.
6. $87.84.
7. $380.26.8.
8. $74.48.
9. $187.20.
10. $328.50.
11. $57.75.
12. $299.25.
13. $6.72.
14. $38.25.
15. $179.54.
16. $513.
17. $156.25.
18. $424.12.
19. $342.51
20. $142.72.
21. $369.60.
22. $3.15.
23. $1 244.50.
24. $73.50.
25. $517.50.
26. $2 778.75
27. $244.16.
28. $128.25.
29. $45.99.
30. $168.30.
31. $86.10.
32. $36 575.
33. $1 368.
34. $39.44.
35. $263.88.
36. $11 799.
37. $39.68.
38. $9 800.
39. $1 405.
40. $7 891.40.
41. $55.44.
42. $2 200.50.
43. $24.12.
44. $1 251.36.
45. $15.47.
46. $68.64.
47. $430.71.
48. $52.92.
49. $375.
50. $761.25.
51. $123.25.
52. $56.25.
53. $103.32.
54. $164.28.
55. $130.68.
56. $24 791.78.
57. $720.
58. $16 099.20.
59. $30 008.22.

REDUCTION.

§51. REDUCTION DESCENDING.

3. 11 909*qts.*
4. 167 020 *grains.*
5. 3 486*oz.*
6. 4 134 *inches.*
7. 68½ *inches.*
8. 81 801 *sq. feet.*
9. 12 420 864 *solid in.*
10. 1 925*qts.*
11. 1 981 *hours.*
12. 2 359*qts.*
13. 734 842 *perches.*
14. 1 306 160*oz.*
15. 569 232 *grains.*
16. 107 276 *grains.*
17. 1 968 *grains.*
18. 169 440*lbs.*
19. 1 182 *nails.*
20. 19 008 000*bar.*
21. 16 379 *yards.*
22. 58 946 *perches.*
23. 336 801 *farthings.*
24. 20 412 *pence.*
25. 7 440 *grains.*
26. 7 596 *quarters.*
27. 3 072 960 *inches.*
28. 39 360 *perches.*
29. 1 008 *pints.*
30. 89 856 *pints.*
31. 323 568 000 *seconds.*
32. 1 239 502 *seconds.*
33. 1 244 160 *inches.*
34. 41 472 *quarts.*
35. 1 408 *nails.*
36. 4 560 *mills.*
37. 187 800 *mills.*
38. 4 100 *cents.*
39. 120 000 *mills.*
40. $160.

§52. REDUCTION ASCENDING.

2. 104£ 3*s.* 4¾*d.*
3. 27*lbs.* 2*oz.* 10*dwt.*
4. 144*oz.*, or 12*lbs.*
5. 726 *yards.*
6. 100*E.E.*
7. 99 *leagues.*
8. 4 840 *sq. yards.*
9. 730*ds.* 11*h.* 37*min.* 38*sec.*
10. 35 480 *guineas.*
11. 9 313£ 10*s.*
12. 385*lbs.* 12*oz.* 13*dr.*
13. 123 *acres.*
14. 3℔ 9℥ 4ʒ 2℈.
15. 1 266*E.Fl.*
16. 176 *boxes.*
17. 48*M.* 4*fur.*
18. 9*pun.*
19. 432 *butts.*
20. 157*bu.* 3*pks.* 6*qts.*
21. 15 *tons.*
22. 15 *cords.*
23. 6 *signs.*
24. 44 *tons*, 10*ft.*
25. 10 *tuns.*
26. 27*hhds.* 11*gals.*
27. 41 *weeks.*
28. 1 *mile.*
29. 49*chal.* 9*bu.* 2*pks.*
30. 710 *cents.*
31. 82 645 *dimes.*
32. $47.
33. 63 *Eagles.*
34. 79*hhds.*
35. 33*bu.* 3*pks.*
36. 365*ds.* 5*hr.* 48*min.* 49*sec.*
37. 76£ 0*s.* 0½*d.*

MISCELLANEOUS EXERCISES.

Pages 90 and 91.

1. 715 *crowns*, 8 *pence*.
2. 7 *shirts*.
3. 53 004 *hours*.
4. 2 304*drs*.
5. 67 *packages*.+
6. 10 836*E.Fl*.
7. 137 880 468 *sq. in*.
8. 288 *tankards*.
9. 7 080 *hours*.
10. 5 016 000 000 000 *half dolls*.
11. 810 540 *minutes*.

COMPOUND MULTIPLICATION.

§ **54.** CASE 1.

4. 43£ 8*s*. 9*d*.
5. 262£ 14*s*. 6*d*.
6. 22*lbs*. 7*oz*. 4*dwt*.
7. 62*lbs*. 4*oz*. 17*dwt*. 17*grs*.
8. 5℔ 7ℨ 5ℨ 1℈.
9. 13℔ 10ℨ 5ℨ 2℈ 14*grs*.
10. 89*T*. 10*cwt*. 1*qr*.
11. 45*T*. 8*cwt*. 0*qrs*. 21*lbs*. 0*oz*. 10*drs*.
12. 26*L*. 2*M*. 0*fur*. 13*p*. 0*yd*. 0*ft*. 9*in*.
13. 24*L*. 1*M*. 5*fur*. 26*p*. 3*yds*. 0*ft*. 5*in*. 1*bar*.
14. 158*A*. 1*r*. 6*p*. 99*ft*.
15. 224*A*. 3*r*. 28*p*. 183*ft*. 88*in*.
16. 333*T*. 9*ft*. 1488*in*.
17. 155*cds*. 70*ft*. 570*in*.
18. 118*yds*. 3*qrs*.
19. 69*E.E*. 1*qr*. 2*n*.
20. 97*E.Fl*.
21. 85*T*. 3*hhds*. 54*gal*. 1*qt*.
22. 32*P*. 1*hhd*. 41*gals*. 3*qts*. 1*pt*.
23. 79*hhds*. 23*gals*. 2*qts*. 1*pt*.
24. 263*T*. 0*bt*. 1*hhd*. 32*gals*. 3*qts*.
25. 147*bu*. 1*pk*. 3*qts*.
26. 155*chal*. 27*bu*. 2*pks*.
27. 55*Y*. 7*mo*. 1*wk*. 6*d*.
28. 1*Y*. 7*mo*. 3*wks*. 2*d*. 2*h*. 13*min*. 36*sec*.
29. 14*s*. 19° 22′ 45″.
30. 371*lbs*. 11*oz*. 11*dwt*.
31. 7*T*. 2*cwt*. 0*qr*. 1*lb*. 10*oz*.
32. 116*T*. 6*cwt*. 2*qrs*. 19*lbs*. 11*oz*.
33. 9℔ 10ℨ 5ℨ 2℈ 4*grs*.

§ 55. CASE 2.

3. 202*lbs.* 0*oz.* 5*drs.*
4. 179 *T.* 1*cwt.* 3*qrs.* 9*lbs.* 9*oz.* 4*drs.*
5. 321 *L.* 1*M.* 2*fur.* 37*p.* 0*yd.* 2*ft.* 6*in.*
6. 1 624 *A.* 2*r.*
7. 848 *T.* 24*ft.* 828*in.*
8. 850*cds.* 27*ft.* 288*in.*
9. 587*yds.* 1*qr.*
10. 570 *E. Fr.*
11. 1 531 *T.* 3*hhds.* 42*gals.*
12. 630 *P.* 0*hhd.* 48*gals.*
13. 143 *T.* 1*bt.* 1*hhd.* 48*gals.*
14. 2 651*bu.*
15. 610*chal.* 34*bu.* 0*pk.* 1*qt.*
16. 352 *Y.* 4*mo.* 1*wk.* 0*d.* 18*h.* 37*min.* 48*sec.*
17. 1 084 *Y.* 5*mo.* 2*wks.* 2*ds.* 6*h.*
18. 84*s.* 5° 15′ 56″.
19. 3 666£ 1*s.* 10½*d.*
20. 1 458*yds.* 0*qr.* 3*n.*
21. 3 119*A.* 0*r.* 30*p.*
22. 1 491 *T.* 8*cwt.* 2*qrs.* 8*lbs.*

§ 56. CASE 3.

2. 5£ 5*s.* 10¾*d.*
3. 11£ 11*s.* 1*d.*
4. 28£ 11*s.* 10*d.*
5. 130£ 3*s.* 3*d.*
6. 1 261£ 19*s.* 5½.
7. 11 *T.* 1*cwt.* 1*qr.* 14*lbs.*
8. 45£ 13*s.* 4*d.*
9. 47£ 10*s.* 8½*d.*
10. 174£ 2*s.* 8½*d.*
11. 223£ 6*s.*

§ 57. CASE 4.

2. 3 771£ 16*s.* 5½*d.*
3. 396 240£.
4. 4 203£ 8*s.* 3*d.*
5. 4 539£ 8*s.* 4*d.*
6. 166£ 0*s.* 9*d.*
7. 5£ 4*s.* 9¾*d.*

MISCELLANEOUS EXERCISES.

Pages 95, 96 and 97.

1. 9£ 10*s.*
2. 379£ 18*s.*
3. 430£.
4. 3 105£ 3*s.*
5. 5 564£ 11*s.* 2¼*d.*
6. 2 078*yds.* 0*qr.* 2*n.*
7. 45£ 10*s.*
8. 472£ 10*s.*
9. 424*cwt.* 1*qr.* 4*lbs.*
10. 14*lbs.* 8*oz.* 10*dwt.*
11. 382*M.* 6*fur.* 20*p.*
12. 39£.
13. 5 346 916£ 9*s.* 10½*d.*
14. 5£ 7*s.* 4*d.*
15. 86£ 12*s.* 6*d.*
16. 2 342*A.* 3*r.* 10*p.*

Pages 96—100.]

17. 510*ft.* 1 656*in.*
18. 113*cds.* 109*ft.*
19. 143℔ 6ʒ 3ʒ.
20. 286*s.* 15° 12′.
21. 792 480£.
22. 6 939*ds.* 14*h.* 27*min.* 12*sec.*
23. 263£ 5*s.* 9*d.*
24. 2 *T.* 10*cwt.* 1*qr.* 12*lbs.*
25. 2 *T.* 0*cwt.* 1*qr.* 4*lbs.*
26. 6£ 10*s.* 3¾*d.*
27. 42£ 3*s.* 9*d.*
28. 1 024 *guineas.*
29. 91£ 13*s.* 4*d.*
30. $219.
31. 24£ 0*s.* 3*d.*
32. $257.32.
33. $7 630.
34. $537.
35. $69 938.75.
36. $860.
37. $82 962.
38. $7 692.75.
39. 6 847*yds.* 3*qrs.*

DIVISION OF FEDERAL MONEY.

§ 58.

2. $20.51.3.
3. $0.12.5.
4. $7.50.
5. $0.90.5.
6. $2.35.
7. $0.87.5.
8. $1.80.3+
9. 8*cts.*
10. $0.12.5.
11. $950.
12. $6.
13. $2 222.22.2.+
14. $0.45.
15. $4.25.
16. $97.90.
17. $28.87.
18. $12.90.
19. $24.40.6.+
20. $1.89.5.
21. $1.50.
22. $333.33.3.+
23. 30*cts.*
24. $0.46.+
25. $2.44.9.+
26. $22.53.
27. $0.09.5.
28. $13.50.
29. $1.05.
30. 8*cts.*
31. $5.50.
32. 22*cts.*
33. $0.33.3.+
34. $2.75.
35. 31*cts.*
36. $5.21.
37. 29*cts.*
38. 37*cts.*
39. $4.25.
40. $7.21.
41. $0.08.5.
42. $0.11.9.+
43. $0.13.4.+
44. $0.07.5.
45. 16*cts.*
46. $2.24.7.
47. $0.06.1.+
48. $0.10.5.
49. $2.12.5.
50. $1.91.
51. $1.83.
52. $3.62.5
53. $450.
54. $3.90.
55. $0.32.2.+
56. $46.68.
57. 61*cts.*
58. $12.12.5.+
59. 15*cts.*
60. $638.60.

COMPOUND DIVISION.

§ 60. CASE 1.

3. 23£ 4*s.* 3¼*d.*
4. 2*lbs.* 4*oz.* 12*dwt.* 17*grs.*
5. 3℔ 4℥ 5℥ 1℈ 9*grs.*
6. 3 *T.* 7*cwt.* 2*qrs.* 24*lbs.* 15*oz.* 6*drs.*
7. 10*L.* 2*M.* 3*fur.* 32*p.* 4*yds.* 0*ft.* 5*in.* 2*bar.*
8. 10*A.* 0*r.* 13*p.* 15*yds.* 1*ft.* 21*in.*
9. 11 *T.* 4*ft.* 51*in.*
10. 4*cds.* 16*ft.* 207*in.*
11. 10*yds.* 3*qrs.* 1*n.*
12. 7*E.E.* 0*qrs.* 2*n.*
13. 2*hhds.* 11*gals.* 3*qts.*
14. 8 *T.* 0*bt.* 0*hhd.* 57*gals.* 3*qts.*
15. 11*bu.* 0*pk.* 1*qt.*
16. 6*chal.* 3*bu.* 0*pk.* 3*qts.*
17. 7*mo.* 2*wks.* 2*d.* 6*h.* 8*min.* 19*sec.*
18. 1*s.* 2° 25′ 45″.

§ 61. CASE 2.

2. 2£ 14*s.* 11½*d.*+
3. 3£ 5*s.* 4*d.*+
4. 18*cwt.* 3*qrs.* 23*lbs.*
5. 1£ 1*s.* 4*d.*
6. 1 *T.* 5*cwt.* 0*qrs.* 22*lbs.*
7. 1℔ 1℥ 0℥ 2℈ 12½*grs.*
8. 9*cwt.* 1*qr.* 22*lbs.*+
9. 7*cwt.* 1*qr.* 12*lbs.*+

§ 62. CASE 3.

2. 12*yds.* 0*qrs.* 2*n.*+
3. 2*A.* 0*r.* 3*p.*+
4. 9*M.* 4*fur.* 39*p.* 0*yds.* 2*ft.* 8*in.*+
5. 2*mo.* 0*wk.* 4*ds.* 20*h.* 25*min.* 13*sec.*+
6. 4*cwt.* 3*qrs.* 10*lbs.* 4*oz.*+
7. 32*cwt.* 0*qr.* 3*lbs.*+

§ 63. CASE 4.

2. 24. | 3. 128*lbs.* | 4. 2 112. | 5. 75*yds.*

MISCELLANEOUS EXERCISES.

Pages 103 and 104.]

1. 2£ 4*s.* 5½*d.*
2. 19*s.* 3¾*d*
3. £4 10*s.*
4. ¼*d.*
5. 37 *chests.*
6. 3*oz.* 4*dwt.* 11*grs.*
7. 8*s.* 8¾*d.*
8. 73 *revolutions.*
9. 216*gals.* 2*qts.*+

Page 104.]

10. 240 *guineas.*
11. 22.
12. 4 *years.*
13. 6 *pieces*
14. 1¾*d.*+
15. 10 560 *times.*
16. 1£ 16*s.*
17. 945*lbs.*
18. 695£ 6*s.* 10*d.*
19. 2½*d.*
20. 470£ 10*s.*

MISCELLANEOUS EXERCISES.

ORAL.

IN THE FIRST PART OF THE UNITED STATES ARITHMETIC.

Pages 105—108.]

1. $22.
2. 9 *barrels.*
3. $6
4. $140.
5. $21.
6. $18.
7. $7.
8. 56*cts.*
9. $97.
10. $39.
11. $10.
12. $1.30.
13. $12.
14. 26 *miles.*
15. $22.
16. $9.
17. $38.
18. $6.
19. $7.
20. $9.
21. $6.
22. $13.
23. $6.
24. $25.
25. $24.
26. $78.
27. $2.
28. $2.
29. $3.
30. $9.
31. $2.
32. $4.
33. 9 *miles.*
34. 6 *bags.*
35. 8 *yards.*
36. 3 *tons.*
37. 45*gals.*
38. 13 *acres.*
39. $49.
40. $7.
41. $7.
42. $12.
43. $8.
44. $20.
45. 98*cts.*
46. 96*cts.*
47. 72*cts.*
48. $42.
49. $84.
50. $30.
51. 24 *miles.*
52. $3.08.
53. $18.
54. $5.25.
55. $2.16.
56. $143.
57. $300.
58. $150.
59. $4.50.
60. $48.
61. 126 *yards.*
62. $12.
63. $40.
64. $35.
65. $72.
66. $36.
67. $7.50.
68. $9.
69. $21.
70. $6.40.
71. $28.
72. $288.
73. 8 *tons.*
74. 11 *miles.*
75. $3.
76. $2.
77. $3.
78. $6.
79. 12*cts.*
80. 50*cts.*
81. $8.
82. 25*cts.*
83. 4 *miles.*
84. 15 *hours.*
85. 6 *days.*

MISCELLANEOUS EXERCISES.

WRITTEN.

IN THE FIRST PART OF THE UNITED STATES ARITHMETIC.

Pages 109—113.]

1. 216 *diff.*, 93 312 *product*, 2 *quot.*
2. 29 134 889.
3. 64 895.
4. 324.
5. 118 650.
6. 2 106 *miles.*
7. 8 *years.*
8. 274 *T.* 0*cwt.* 3*qrs.* 26*lbs.*
9. 149 *days.*
10. 307 583 892 *drachms.*
11. 16*M.* 2*fur.* 26*p.* 3*yds.* 1*ft.* 8*in.* 2*bar.*
12. $5 *per yd.*, and $235 *for* 47*yds.*
13. $4.50 *per barrel*, and 76 *cost* $342.
14. $193.60.
15. 1 620 *days*, 1 *man*, 81 *days*, 20 *men.*
16. 15 *days.*
17. 15 *days.*
18. $8 *man*, and $4 *boys.*
19. 831 134 850 306 924.
20. 2, 3, 5, 7, 11 and 13.
21. 4 *days*, and 9 *days.*
22. 13 480 *roubles.*
23. 115*gals.*
24. $437 525.
25. 11 *hours.*
26. 394.
27. 163 *years.*
28. $43.42.
29. $17 437.50.
30. 50 *cents.*
31. 2 155.83+
32. 98*dys.*, 8*h.* 52*m.* 48*s.*
33. 6 *cents.*
34. 688 *boxes.*
35. $785 *one*, $965 *the other*
36. 4 *miles per hour*, and 208 *miles in* 52 *hours.*
37. 1*st. wt.* 6*cwt.* 3*qrs.* 26*lbs.* *Value* $46.92.
    2*d. wt.* 8*cwt.* 3*qrs.* 14*lbs.* *Value* $69.58.
38. 30 *days.*
39. $885.83 *loss.*
40. 2 152$\frac{5}{6}$ *feet.*
41. 829 137$\frac{1}{2}$ *days.*
42. $25 187.50.
43. $189 000 000.
44. $311 111$\frac{1}{9}$.
45. 4*Y.* 0*mo.* 10*ds.*
46. 67*Y.* 9*mo.* 23*ds.*
47. 2*Y.* 6*mo.* 20*ds.*
48. $2.
49. $270 *more than in Prussia*, and $273 *more than in Genoa.*
50. 17 130*ft.*
51. 1*st.*, 671 *years*, 2*d.*, 241 *years.*
52. 2 092 *years.*
53. $100 740 000.
54. 246 *to a square mile.*
55. 2 473 122 *inhabitants.*
56. 31 069$\frac{3}{7}$ *miles.*
57. 3 *times.*+
58. 157 *years.*
59. 59 *years.*
60. $84 960.07

Pages 114—119 of Part First.]

61. 149 *years.*
62. 64 *times.*
63. 917.
64. 63 *years.*
65. $1 175 000.
66. 288 *years.*
67. 25° 52′.
68. 140 *years*, 2 *months.*
69. 197 916⅔ *miles.*
70. 58$\frac{14}{17}$ *days.*
71. 2 381 *years.*
72. 34 880½*cwt. of iron*, and 294 608⅖ *cwt. of coals.*
73. $171 878.04.
74. 1 656 000 *miles.*
75. 264 *miles.*
76. 520 250.
77. 30 060 *men.*
78. 793 632 *times.*
79. 7 987 680 *letters.*
80. 10 *years.*
81. $52 500 *each*, $157 500 *all.*
82. 7 590 *times in an hour*, 1 897 *times in a quarter of an hour*, and 2 *rem.*, 126 *times in a minute*, and 7 *remainder.*
83. 200.
84. 852, and 480 *over.*
85. 4 629 *slain.*
86. 864 *days.*
87. 1 160 *pieces of each kind.*
88. 8½*d.*
89. 15*drs. of lead for* 1*gr. of silver*, 21*lbs.* 12*oz.*+ for $1.
90. 144*ft.*
91. 5 280 *ft. on a side*, 27 878 400 *square ft. of surface*, 3 520 *lots.*
92. 157*bu.* 3*pks.* 7*qts.* 1*pt.*+
93. $25 076.26⅔.
94. $18.728.11$\frac{1}{9}$.
95. $3 104.385.+
96. $7 000 000.
97. 191 964 *T.* 5*cwt.* 2*qrs.* 24*lbs.*
98. 63 700 000 *times.*
99. 2*wks.* 5*d.* 10*h.*
100. 395$\frac{4}{6}$ *times.*
101. 65 517 *paupers.*+
102. 20*cts.* 4 *mills.*+
103. $8 108 797.50.
104. $7 255 000.
105. $2 049 799.86.
106. $1.25.5+ *per acre.*
107. 16$\frac{1}{10}$ *popul. to a sq. mile.*
108. $19 500 000.
109. 5 758*d.* 8*h.*
110. 115$\frac{1}{6}$ *times.*
111. $87 502 299.10.7+
112. $50 094 281.25.
113. 5*M.* 0*fur.* 6*p.* 0*yd.* 1*ft.*
114. 1*M.* 1*fur.* 33*p.* 5*yds.* 0*ft.* 6*in.*
115. 1 320 000*ft. Thames*, and 2 217 600*ft. Delaware.*
116. 8*A.* 2*r.* 34*p.*+
117. 3$\frac{17}{81}$ *times.*
118. 5*fur.* 12*p.* 0*yds.* 2*ft.*
119. 192 *T.* 17*cwt.* 0*qrs.* 16*lbs.*
120. 5 100 000 000*lbs.*
121. 18*M.* 0*fur.* 20*p.* 0*yds.* 0*ft.* 5*in.*+
122. 300 004.
123. 37 500£.
124. 20 243 *T.* 1*cwt.* 0*qr.* 17*lbs.* 482 387$\frac{1}{4}$ *chests*, 485. *wks.* 5*ds.* 20*h.*
125. $23 467 500, 1 335 040 000 *men.*
126. 37 409£ 1*s.* 7¾*d.*+

127. 24 *cts.* 2 *mills*+ *per lb.*, and 20 *cts.* 5 *mills*+ *for each individual.*
128. $3.95.9+ *per week*, 26+ *men.*
129. 50 *of each.*
130. 30 *persons.*
131. 780 984+ *fires.*
132. 9 795 337 *T.* 10*cwt.*
133. 120+ *weeks.*
134. 201 914+ *hhds.* 4 407 824+ *years.*
135. 1 982 061*M.* 2*fur.* 36*p.* 2*yds.*
136. $47 756 640.
137. $13 089 740.62½ *value of both.*
138. 10 *cts.* 2 *mills.*+
139. 684 *weeks* 3 *days.*+
140. 247*lbs.* 9*oz.*+ *Penn'a*, 176*lbs.* 15*oz.*+ *G. Brit.*

FRACTIONS.

§ 65. CASE 1. [Pages 108—112.

2. 36. | 4. 2. | 6. 6. | 8. 37. | 10. 2.
3. 54. | 5. 365. | 7. 3. | 9. 8.

§ 66. CASE 2.

2. 120. | 5. 1092. | 8. 1800.
3. 24. | 6. 280. | 9. 180.
4. 420. | 7. 630. | 10. 2520.

REDUCTION OF FRACTIONS.

§ 67. CASE 1.

2. $\frac{3}{17}$. | 6. $\frac{7}{8}$. | 10. $\frac{14}{17}$. | 14. $\frac{117}{196}$.
3. $\frac{1}{5}$. | 7. $\frac{144}{167}$. | 11. $\frac{5}{13}$. | 15. $\frac{2}{13}$.
4. $\frac{55}{64}$. | 8. $\frac{1}{8}$. | 12. $\frac{89}{250}$. | 16. $\frac{3}{7}$.
5. $\frac{9}{13}$. | 9. $\frac{1}{3}$. | 13. $\frac{3}{4}$. | 17. $\frac{431105}{2293756}$.

§ 68. CASE 2.

2. $\frac{1017}{1275}$. | 5. $\frac{1815}{18}$. | 8. $\frac{8229}{16}$. | 11. $\frac{12410}{19}$.
3. $\frac{1259}{13}$. | 6. $\frac{4162}{120}$. | 9. $\frac{5919}{216}$. | 12. $\frac{3979}{8460}$.
4. $\frac{738}{27}$. | 7. $\frac{3848}{21}$. | 10. $\frac{2168}{17}$.

§ 69. CASE 3.

2. 8. | 5. $1064\frac{8}{11}$. | 8. $100\frac{5}{6}$. | 11. $653\frac{5}{19}$.
3. $56\frac{13}{22}$. | 6. $84\frac{3}{4}$. | 9. $346\frac{7}{120}$. | 12. $1209\frac{187}{814}$.
4. 183. | 7. $96\frac{11}{13}$. | 10. $127\frac{4}{17}$.

§ 70. CASE 4.

2. $\frac{63}{9}$. | 3. $\frac{156}{13}$. | 4. $\frac{266}{14}$. | 5. $\frac{9000}{90}$.

§ 71. CASE 5.

3. $\frac{9}{16}$.
4. $\frac{37}{333}$.
5. $\frac{1}{6}$.
6. $\frac{1}{11}$.
7. $\frac{65}{338}$.
8. $\frac{1540}{741}$.
9. $\frac{104}{91}$.
10. $\frac{74}{64}$.
11. 275.
12. 114.
13. $\frac{10584}{28}$.

§ 72. CASE 6.

RULE 1.

2. $\frac{144}{288}$, $\frac{192}{288}$, $\frac{240}{288}$, and $\frac{252}{288}$.
3. $\frac{400}{1200}$, $\frac{720}{1200}$, $\frac{6600}{1200}$, and $\frac{1080}{1200}$.
4. $\frac{36}{60}$, $\frac{15}{60}$, $\frac{40}{60}$, and $\frac{240}{60}$.
5. $\frac{126}{210}$, $\frac{70}{210}$, $\frac{160}{210}$, and $\frac{1050}{210}$.
6. $\frac{80}{360}$, $\frac{72}{360}$, $\frac{2520}{360}$, $\frac{270}{360}$, and $\frac{1080}{360}$.
7. $\frac{768}{1536}$, $\frac{1024}{1536}$, $\frac{1152}{1536}$, $\frac{960}{1536}$, and $\frac{1344}{1536}$.
8. $\frac{1248}{2496}$, $\frac{1040}{2496}$, $\frac{19344}{2496}$, and $\frac{576}{2496}$.
9. $\frac{13662}{18480}$, $\frac{17325}{18480}$, $\frac{16170}{18480}$, and $\frac{13200}{18480}$.
10. $\frac{8448}{9984}$, $\frac{11232}{9984}$, $\frac{8324}{9984}$, and $\frac{6240}{9984}$.
11. $\frac{2507733}{3940005}$, $\frac{3013301}{3940005}$, and $\frac{3756683}{3940005}$.

§ 73. RULE 2.

2. $\frac{21}{36}$ and $\frac{22}{36}$.
3. $\frac{36}{90}$, $\frac{60}{90}$, $\frac{50}{90}$, and $\frac{42}{90}$.
4. $\frac{8}{24}$, $\frac{18}{24}$, and $\frac{21}{24}$.
5. $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, and $\frac{48}{60}$.
6. $\frac{16}{48}$, $\frac{36}{48}$, $\frac{40}{48}$, $\frac{42}{48}$, $\frac{33}{48}$, and $\frac{24}{48}$.

§ 74. CASE 7.

3. $\frac{1}{128}$ *of a cwt.*
4. $\frac{1}{8640}$ *of a lb.*
5. $\frac{1}{1008}$ *of a hhd.*
6. 12 *nails.*
7. $\frac{1}{8064000}$ *of a week.*
8. $2\frac{220}{3}$ *of a penny.*
9. $\frac{56}{13}$ *of a day.*
10. $\frac{40}{147}$ *of a guinea.*
11. $\frac{9}{13}$ *of a pint.*
12. $\frac{4}{5}$ *of a nail.*
13. $\frac{9}{16}$ *of a mile.*
14. $\frac{7}{8}$ *of a dwt.*
15. $\frac{6}{7}$ *of a lb.*
16. $\frac{53}{6370}$ *of a gal.*
17. $\frac{4}{7}$ *of a gill.*
18. $\frac{1}{2}$ *of a yard.*
19. $\frac{11}{17}$ *of a yard.*

§ 75. CASE 8.

2. 12*h.* 55*min.* $23\frac{1}{13}$*s.*
3. 9*oz.* $2\frac{3}{7}$*dr.*
4. 3*hhds.* 31*gal.* 2*qts.*
5. 6*s.* $8\frac{1}{2}$*d.*
6. 12*s.* 3*d.*
7. 12*cwt.* 2*qrs.*
8. 6*oz.* 15*dwt.*
9. 3*qrs.*
10. 1*pk.* $1\frac{5}{9}$*qt.*
11. 15 *bushels.*
12. 2*r.* 20*p.*
13. 2*qrs.* $3\frac{1}{9}$*n.*
14. 1*fur.* $26\frac{3}{8}$*p.*
15. 35 *perches.*

§ 76. CASE 9.

2. $\frac{61}{96}$.
3. $\frac{9}{16}$.
4. $\frac{5}{6}$.
5. $\frac{11}{24}$.
6. $\frac{67}{90}$.
7. $\frac{269}{272}$.
8. $\frac{23}{32}$.
9. $\frac{1083}{1309}\frac{19}{56}$.
10. $\frac{337}{640}$.
11. $\frac{4}{5}$.
12. $\frac{23}{34}$.
13. $\frac{3}{10}$.
14. $\frac{7}{20}$.

§ 77. CASE 10.

4. $\frac{20}{31}$.
5. $\frac{19}{27}$.
6. $\frac{99}{182}$.
7. $\frac{104}{171}$.
8. $\frac{39}{40}$.
9. 15.
10. $\frac{2}{4}$.
11. $\frac{64}{255}$.

ADDITION OF FRACTIONS.

§ 78. CASE 1.

2. $1\frac{5}{19}$.
3. $2\frac{3}{29}$.
4. $1\frac{29}{99}$.
5. $2\frac{191}{777}$.
6. $2\frac{2746}{9251}$.
7. $2\frac{43}{192}$.
8. $1\frac{3992}{6995}$.
9. $1\frac{601}{714}$.

§ 79. CASE 2.

4. $4\frac{11}{252}$.
5. $5\frac{8}{35}$.
6. $6\frac{12273}{50400}$.
7. $8\frac{3}{6}$.
8. $4\frac{5}{16}$.
9. $13\frac{109}{112}$.
10. $1\frac{17}{240}$.
11. $10\frac{1}{60}$.
12. $283\frac{4}{15}$.
13. $24\frac{1}{63}\frac{144}{612}\frac{3}{}$.

§ 80. CASE 3.

3. 6*cwt.* 1*qr.* 21*lbs.* 12*oz.* $7\frac{1}{5}$*drs.*
4. 14*bu.* 0*pks.* 6*qts.* $1\frac{9}{11}$*pt.*
5. 8*oz.* 8*dwt.* 3*grs.*
6. 440*yds.* 1*ft.* 9*in.*
7. 1£ 9*s.* $4\frac{1}{5}$*d.*
8. 4*M.* 0*fur.* $35\frac{5}{9}$*p.*
9. 37*A.* 0*r.* $18\frac{14}{17}$*p.*
10. 19*cwt.* 0*qr.* $11\frac{17}{29}$*lbs.*
11. 5*fur.* $17\frac{11}{21}$*p.*
12. 1£ 0*s.* $4\frac{146}{385}$*d.*
13. 25*M.* 7*fur.* $12\frac{44}{63}$*p.*
14. 2*A.* 2*r.* $28\frac{80}{693}$*p.*
15. 14*cwt.* 2*qrs.* $3\frac{28}{31}\frac{47}{5}$*lbs.*
16. 1*fur.* 28*p.* $1\frac{19}{180}$*yds.*
17. 6*yds.* 6*ft.* $36\frac{344}{450}\frac{}{45}$*in.*

SUBTRACTION OF FRACTIONS.

§ 81. CASE 1.

2. $5\frac{1}{4}$.
3. 24.
4. $\frac{5}{36}$.
5. $\frac{33}{86}$.
6. $\frac{253}{2568}$.
7. $\frac{467}{4873}$.

§ 82. CASE 2.

3. $\frac{309}{700}$.
4. $\frac{281}{917}$.
5. $\frac{119}{3540}$
6. $\frac{4}{7}$.
7. $2\frac{3}{35}$.
8. $1\frac{7}{12}$.
9. $\frac{280}{891}$.
10. $17\frac{1}{20}$.
11. $\frac{49}{185}$.

§ 83. CASE 3.

2. $9s.\ 11\frac{13}{21}d.$
3. $13cwt.\ 3qrs.\ 18lbs.$
4. $1M.\ 4fur.\ 16p.$
5. $11dwt.\ 3grs.$
6. $9s.\ 10\frac{1}{2}d.$
7. $23bus.\ 1pk.\ 4qts.$
8. $9A.\ 0r.\ 32p.$
9. $6M.\ 5fur.\ 17\frac{3}{5}p.$
10. $6cwt.\ 1qr.\ 24\frac{4}{15}lbs.$
11. $6fur.\ 21\frac{37}{63}p.$
12. $5fur.\ 13p.\ 5\frac{3}{26}yds.$
13. $29\frac{1}{8}\ perches.$
14. $4s.\ 6\frac{84}{91}\frac{9}{0}d.$
15. $1fur.\ 18\frac{50}{63}p.$
16. $5yds.\ 3\frac{36}{91}\frac{37}{04}\frac{3}{5}ft.$

MULTIPLICATION OF FRACTIONS.

§ 84. CASE 1.

3. $\frac{1}{18}$.
4. $\frac{115}{336}$.
5. $9\frac{9}{140}$.
6. $47\frac{7}{8}$.
7. $\frac{6}{175}$.
8. $140\frac{28}{30}$.
9. $92\frac{4}{5}$.
10. $\frac{1}{65}$.
11. $70\frac{1}{3}$.
12. $\frac{9}{3}\frac{8}{0}\frac{8}{9}\frac{0}{6}\frac{0}{9}$.

§ 85. CASE 2.

2. 24.
3. 8.
4. $280\frac{8}{9}$.
5. $4573\frac{9}{11}$.
6. $14016\frac{1}{7}$.
7. 76.

Page 125. MISCELLANEOUS EXERCISES.

1. $1r.\ 32p.\ 30\frac{24}{217}yds.$
2. $1cwt.\ 0qr.\ 17lbs.\ 3\frac{9}{13}oz.$
3. $15M.\ 6fur.\ 37\frac{1}{3}p.$
4. $8s.\ 7\frac{43}{95}d.$
5. $5yds.\ 8ft.\ 68\frac{11}{14}in.$
6. $2ft.\ 134\frac{14}{23}in.$
7. $8yds.\ 10ft.\ 1382\frac{2}{5}in.$
8. $3£\ 5s.\ 9\frac{23}{39}d.$
9. $5\ T.\ 8cwt.\ 1qr.\ 9\frac{1}{3}lbs.$
10. $7£\ 1s.\ 4\frac{6}{90}\frac{1}{0}d.$
11. $2\ poles.$
12. $1M.\ 2fur.\ 12\frac{4}{33}p.$
13. $456A.\ 0r.\ 38\frac{2}{21}p.$
14. $68yds.\ 7ft.\ 8\frac{56}{62}in.$
15. $1fur.\ 38p.\ 2\frac{1}{5}ft.$
16. $\$1.43\frac{1}{4}$.
17. $\$1.88\frac{1}{4}$.
18. $\$2.78\frac{2}{3}$.
19. $\$59.25$.
20. $\$150.22$.

DIVISION OF FRACTIONS.

§ 86. CASE 1.

3. $4\frac{3}{10}$. | 5. $2\frac{1}{2}$. | 7. $2\frac{1}{20}$. | 9. $8\frac{10}{11}$. | 11. $\frac{3}{16}$.
4. $2\frac{16}{49}$. | 6. $1\frac{7}{8}$. | 8. $\frac{20}{41}$. | 10. $\frac{4}{7}$. | 12. $\frac{49}{512}$.

§ 87. CASE 2.

2. 10. | 5. $200\frac{2}{7}$. | 8. $117\frac{1}{2}$.
3. 108. | 6. $306\frac{6}{11}$. | 9. $100\frac{4}{5}$.
4. $29\frac{6}{7}$. | 7. $161\frac{1}{2}$. | 10. $36\frac{3}{4}$.

§ 88. CASE 3.

2. $\frac{3}{176}$. | 4. $\frac{5}{144}$. | 6. $\frac{1}{112}$.
3. $\frac{11}{576}$. | 5. $\frac{7}{88}$. | 7. $\frac{1}{2500}$.

§ 89. CASE 4.

2. $2\frac{11}{64}$. | 4. $17\frac{19}{33}$. | 6. $4\frac{89}{117}$.
3. $4\frac{1}{6}$. | 5. $4\frac{241}{288}$. | 7. $3\frac{19}{22}$.

MISCELLANEOUS EXERCISES.

Pages 127 and 128.

1. $8\frac{1}{8}$.
2. $60\frac{3}{4}$.
3. $58\frac{1}{2}$.
4. $22\frac{29}{56}$.
5. $23\frac{125}{154}$.
6. $2\frac{2}{27}$.
7. $2\frac{1}{56}$.
8. $\frac{100}{627}$.
9. $8\frac{5}{42}$.
10. $1ft.\ 91\frac{7}{11}in.$
11. $29\frac{2}{57}$ *poles.*
12. $8\frac{17}{420}$ *miles.*
13. 2 *roods,* $1\frac{1}{430}$ *perches.*
14. $3cwt.\ 0qr.\ 14\frac{788}{1912}lbs$
15. $11yds.\ 0ft.\ 1393\frac{432}{512}in.$
16. $\$9.47\frac{7}{19}$.
17. $\$3.20\frac{40}{59}$.
18. $\$12.$
19. $\$4.18\frac{1432}{2821}$.
20. $\$2.82\frac{18}{61}$.

DECIMALS.

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§ 91. ADDITION OF DECIMALS.

2. 36.3179. | 4. 386.63531. | 6. 84691.9698.
3. 391.9695. | 5. 282.6051. | 7. 644.5971.

§ 92. SUBTRACTION OF DECIMALS.

2. 226.28347.
3. 4.890349.
4. .04069.
5. 4899.9993.
6. 7.0072.
7. .4114.
8. 40.408.
9. 364.635

§ 93. MULTIPLICATION OF DECIMALS.

2. .15417.
3. .00020049.
4. .0499536.
5. .068191.
6. 6.213252.
7. .0074046.
8. 3.410556.
9. 7296.3072
10. .0070001.
11. 101.6064.

§ 94. DIVISION OF DECIMALS.

2. 1.3318+
3. 10098.3+
4. 303.0303+
5. .0018471+
6. 17.8253+
7. .005387399+
8. .000166+
9. 10.
10. 100 000.
11. 1.10086+
12. .0001.
13. 98125.

REDUCTION OF DECIMALS.

§ 95. CASE 1.

3. .95 and .8823+
4. .555+, .388+ and .666+
5. .36111+ and .69565+
6. .514388+
7. .44402+
8. .604236+
9. .7040448+
10. .44419+
11. .1293+
12. .7734+
13. .171875.
14. 158.066298+

§ 96. CASE 2.

3. $\frac{1369}{2000}$.
4. $\frac{1373}{2100}$.
5. $\frac{9}{16}$.
6. $\frac{1643}{2000}$.
7. $\frac{37118}{50000}$.
8. $\frac{18087}{250000}$.
9. $\frac{36423}{50000}$.
10. $\frac{83497}{100000}$.
11. $\frac{196619}{250000}$.

§ 97. CASE 3.

2. 17*cwt.* 1*qr.*
3. 1*pk.* 1*qt.* 1.52*pt.*
4. 1194*yds.* 2*ft.* 7.104*in.*
5. 211*ds.* 16*h.* 48*min.*
6. 16*s.* 7$\frac{3}{4}$*d.*
7. 2*qrs.* 19*lbs.* 5.12*oz.*
8. 2*dwt.* 2.112*grs.*
9. 1*qr.* 3.5392*n.*
10. 1*s.* 11$\frac{1}{4}$*d.*
11. 1*rood*, 21.92*perches.*
12. 124.7455$\frac{19}{33}$ *sq. in.*
13. 23° 33′ 43.2″.

§ 98. CASE 4.

3. .279464+ *ton.*
4. .05173+ *lb.*
5. .02957+ *cwt.*
6. .1232+ *Y.*
7. .3875*M.*
8. .7777+ *yd.*
9. .625*ft.*
10. .3375*A.*
11. .00376*cwt.*
12. .09239+ *gui.*

MISCELLANEOUS EXERCISES.

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1. 4*M.* 7*fur.* 34.056*p.*
2. 15*cwt.* 3*qrs.* 18.8519$\frac{3104}{7777}$*lbs.*
3. 4*yds.* 7*ft.* 70.26+ *in.*
4. 4*A.* 3*r.* 25.732$\frac{1160}{3025}$*p.*
5. 6*s.* 7.38+ *d.*
6. 18*A.* 1*r.* 31.4642*p.*
7. 37£ 0*s.* 8.4*d.*
8. 10*cwt.* 3*qrs.* 24.14$\frac{40}{99}$*lbs.*
9. 73*ft.* 84.284*in.*
10. 12*M.* 7*fur.* 3.530$\frac{10}{11}$*p.*
11. 2*lbs.* 10.21952*oz.*
12. 3*wks.* 5*ds.* 21.22752*h.*
13. 5° 5′ 50.244″.
14. 109*A.* 1*r.* 6.55216*p.*
15. 148*cwt.* 1*qr.* 10.9984*lbs.*
16. 2 *rood*, 6.80+ *perches.*
17. 6.92$\frac{94}{1073}$.
18. 214.6$\frac{446}{1748}$.
19. 1.204224 *drams.*
20. 4.356$\frac{308}{787}$*yds.*

DUODECIMALS, OR CROSS MULTIPLICATION.

§ 99.

	ft.	*in.*	*′*	*″*
3.	68	10	8.	
4.	110	2	6.	
5.	227	4	0.	
6.	76	2	8.	
7.	116	3	1	3.

	ft.	*in.*	*′*	*″*	*‴*
8.	27	6	8	8	6.
9.	59	0	5	9	8.
10.	70	0	9	11	10.
11.	90	8	10	2	4.
12.	84	7	10	0	9.

REDUCTION OF CURRENCIES.

§ 101. CASE 1.

2. $302.83+
3. $344.
4. $431.61.
5. $351.18¾.
6. $486.
7. $112.70.
8. $88.88+
9. $192.
10. $432.88+

§ 102. CASE 2.

	£	*s.*	*d.*
2.	150.		
3.	20	6.	
4.	91	4.	
5.	95	3.	
6.	83	11	4¾.
7.	273	12.	
8.	368.		
9.	42	19	6.
10.	814	10.	

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§ 103. THE SINGLE RULE OF THREE.

5. 73 *bushels.*
6. 96 *yards.*
7. $16\frac{8}{57}$ *cents.*
8. $201.60.
9. $274.54.
10. $203.50.
11. $876.
12. $104.40.
13. 5 *cents.*
14. $225.
15. $432.
16. 200 *gallons.*
17. 588 *miles.*
18. 14 *days.*
19. $684.
20. $2664.
21. $341.66\frac{2}{3}$.
22. $58.80.
23. 540 *acres.*
24. 30 *weeks.*
25. 17 *weeks.*
26. 50 *yards.*
27. $45.32.
28. $489.88.
29. 100 *barrels.*
30. $40.56\frac{1}{4}$.
31. 1 *quart.*
32. $7.28+
33. $711.75.
34. $19.23+
35. 75 *miles.*
36. 8*oz.*
37. $1.55.
38. $14.98.
39. $52545.
40. $16.87\frac{1}{2}$.
41. $1766.60.
42. 280 *yards.*
43. 378 *feet.*
44. 600 *men.*
45. $358.
46. $5461\frac{1}{3}$ *inches.*
47. $9\frac{1}{3}$ *days.*
48. 145*lbs.* 10*oz.*
49. 13*hs.* 20*min.*
50. 210 *feet.*
51. $41\frac{1}{2}$ *days.*
52. $17\frac{4}{13}$ *days.*
53. 83*ft.* 3*in.*
54. $89.40.
55. 75 *men.*
56. 72 *men.*
57. 50 *men.*
58. $1.35.
59. $37.50.
60. 84 *yards.*
61. 50 *feet.*
62. 18 *inches,*
63. 80 *yards.*
64. $134\frac{4}{9}$ *yards.*
65. 8 *miles.*
66. $8333\frac{1}{3}$ *yards.*
67. $37\frac{1}{2}$ *miles.*
68. $4433\frac{1}{3}$*lbs.*
69. 1£ 5*s.* 8*d.*
70. $142.18\frac{3}{4}$.
71. $140.
72. $38.32\frac{1}{2}$.
73. $760.
74. $\frac{3}{7}$.
75. $18.
76. 4*yds.* 3*qrs.* 2*n.*
77. $34\frac{1}{6}$ *yards.*
78. 8*oz.*
79. 24 *cents.*
80. $14\frac{12}{20}\frac{2}{7}$.
81. 5 *cents.*
82. $70.20.
83. $2315.86\frac{2}{23}$.
84. 147 *men.*
85. 69.6203125.
86. 108 *cows.*
87. 14.137 *yards.*
88. 75 *feet.*
89. $4\frac{2}{2}\frac{3}{4}\frac{6}{1}$ *hours.*
90. $5.299+
91. 4*l.*
92. 78 *days.*
93. 12 *weeks.*
94. 4 *months.*
95. 12 *days.*
96. $4\frac{1}{2}$ *days.*
97. 60 *perches.*
98. 37*ft.* 6*in.*
99. 90 *yards.*
100. 18 *yards.*
101. 15*yds.* 1*ft.* 8*in.*
102. $12\frac{1}{2}$ *miles.*
103. 301 *yards.*
104. $105142.81+

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§ 104. THE DOUBLE RULE OF THREE.

5. 4 *cows.*
6. 1 000 *acres.*
7. 15 *extra.*
8. 40 *squares.*
9. $74\frac{3}{8}$ *acres.*
10. 10 *days.*
11. $\$166\frac{2}{3}$.
12. 14 *days.*
13. $150\frac{25}{54}$ *stones.*
14. 128 *acres.*
15. 120 *acres.*
16. 60 *bushels.*
17. 192 *rods.*
18. 80 *cents.*
19. $60.60.
20. $\$104\frac{1}{6}$.
21. 640£.
22. 12 *men.*
23. 15 *days.*
24. $247\frac{2}{3}$ *rods.*
25. $280\frac{4}{5}$ *gallons.*
26. $3\frac{56}{1173}$ *weeks.*
27. $4\frac{1}{2}$ *lbs.*
28. 6 *per cent.*
29. 162.5 *acres.*
30. $\$305.03$.
31. 20 *men.*
32. $\$22.55+$
33. 18 *days.*
34. 126+ *men.*
35. $3\frac{3}{4}$ *months.*
36. 2250 *men.*
37. 15 *men.*
38. 75 *days.*
39. $37\frac{1}{8}$ *horses.*
40. $533\frac{1}{3}$ *cubic yds.*
41. 23 *days.*
42. 57*ds.* $7\frac{11}{23}$*h.*
43. 28 *horses.*
44. 17 280 *bushels.*
45. $5\,483\frac{23}{195}$ *soldiers*
46. $288\frac{59}{207}$ *days.*
47. 12 *men.*
48. 14*wks.* 2*ds.*
49. $\$619.20$.
50. 72 *inches.*

§ 105. ANALYSIS.

7. $\$307.20$.
8. 96 *yards.*
9. $\$590.217$.
10. $\$177.32$.
11. $\$13$.
12. 1 217*lbs.*
13. $\$9.85125$.
14. $410\frac{70}{73}$ *barrels.*
15. $\$14.04$.
16. $\$2\,871$.
17. $\$7\,974.857$.
18. $\$180$.
19. $10\frac{2}{3}$ *days.*
20. 20 *perches.*
21. 32 *inches.*
22. 72 *yards.*
23. 50 *yards.*
24. 64 *yards.*
25. $\frac{5}{24}$.
26. $\frac{245}{2005}$.
27. $1\frac{7}{20}$ *hours.*
28. $\$0.78\frac{6}{11}$.
29. $\$3.36$.
30. $3\frac{31}{245}$ *cents.*
31. $\$3\,060.90$.
32. 88.48*lbs.*
33. $\$83.72$.
34. $9\frac{3}{5}$ *mills.*
35. $\$138.69$.
36. 72 *acres.*
37. $\$375$.
38. 11*s.* 3*d.*
39. $\$480$.
40. 7 *days.*
41. 4 *men.*
42. 5 *days.*
43. 9 *persons.*
44. $217\frac{7}{11}$ *gallons.*
45. $\$220.50$.
46. 20 *horses.*
47. 7*mo.* 6*ds.*
48. 4 *horses.*
49. 18 *bushels.*
50. 3 528 *bricks.*
51. $75\frac{12}{15}$ *days.*
52. $\$76.50$.
53. 20 *men.*

PRACTICE.

§107. CASE 1.

£ s. d.	£ s. d.	£ s. d.
2. 1 9 1¼.	5. 83 1 5.	8. 208 1 6.
3. 9 14 0½.	6. 125 18 11½.	9. 2 077 9 6¾.
4. 19 2 6.	7. 86 19 11¼.	

§108. CASE 2.

$. cts.	$ cts.	$ cts.	$ cts.
2. 388 80.	4. 2 253 12½.	6. 2 098.	8. 2 484 30.
3. 824 46.	5. 4 663 50.	7. 2 660 67.	9. 7 921 31¼.

§109. CASE 3.

£ s. d.	£ s. d.	£ s. d.
2. 26 17 4¾.	11. 165 13 4.	20. 310 3 3.
3. 16 15 6.	12. 138 8 0¾.	21. 185 8 3.
4. 58 10.	13. 74 11 5.	22. 374 6 9¼.
5. 82 1 2½.	14. 119 0 6.	23. 278 5 9¾.
6. 68 13 9.	15. 80 9 6.	24. 274 7 2.
7. 84 9 8.	16. 69 8 3.	25. 343 10.
8. 129 6 7.	17. 166 5 7½.	26. 217 7 11½.
9. 150 1 10½.	18. 194 7 3¼.	27. 478 1 6¾.
10. 54 12 1¼.	19. 179 9 10.	

§110. CASE 4.

£ s.	£ s.	£ s.	£ s.
5. 1 962 18.	8. 2 578 19.	11. 6 697 12.	14. 11 111 8.
6. 1 468 16.	9. 2 344 16.	12. 2 769 6.	15. 79 003 4.
7. 2 853 12.	10. 3 193 10.	13. 1 015 11.	16. 92 009 8.

§111. CASE 5.

3. 2 078£.	7. 1 500£ 16s.	11. $1 056.66⅔.
4. 1 244£ 0s. 3d.	8. $3 375.	12. $2 097.
5. 1 315£.	9. $3 879.75.	13. $19 289.06¼.
6. 3 312£ 13s.	10. $243.75.	14. $103 011.66¼.

§112. CASE 6.

3. 3 625£ 7s. 6d.
4. 30 296£ 6s. 8d.
5. 25 867£ 12s. 6d.
6. 5 772£ 12s. 4d.
7. 7 573£ 8s. 1d.
8. $11 276.47½.
9. $15 073.33⅓.
10. $18 958.75.
11. $12 529.16⅔.
12. $69 710.66⅔.

§ 113. CASE 7.

2. $770.35.
3. 71£ 5*s.* 3*d.*
4. 142£ 17*s.* 6¼*d.*
5. 320£ 17*s.* 5½*d.*
6. 216£ 4*s.* 6*d.*
7. 23£ 4*s.* 1½*d.*
8. 89£ 3*s.* 2¼*d.*
9. 883£ 8*s.* 9½*d.*
10. 1 619£ 11*s.* 1¾*d.*
11. 53£ 16*s.* 4*d.*
12. $3 456.38.7.
13. $224.40½.
14. $6 768.56¼.
15. $278.14.5.
16. $213.86.7.
17. $207.49.

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1. 7£ 6*s.* 9*d.*
2. 11*s.* 10.
3. $80.21.
4. $50.62½.
5. 8£ 14*s.* 6*d.*
6. 14£ 17*s.*
7. 364£ 16*s.*
8. 58£ 2*s.*
9. $247.92½.
10. 1 148£.
11. $585.
12. $4 313.02.9.
13. $744.03½.
14. 399£ 19*s.* 5¾*d.*
15. 2£ 12*s.* 6*d.*
16. 2£ 4*s.* 5¾*d.*
17. 49£ 8*s.* 3*d.*
18. 1 700£ 1*s* 6¾*d.*
19. 1 099£ 7*s.* 4*d.*
20. 331£ 14*s.* 8¼*d.*

TARE AND TRET.

§ 115. CASE 1.

2. 27*cwt.* 3*qrs.* 22*lbs. neat weight.*, $324.17.8 *amount.*
3. 12*cwt.* 2*qrs.* 18*lbs. neat weight*, $124.45.4 *amount.*
4. 2 *T.* 16*cwt.* 2*qrs.* 4*lbs. neat weight*, $25.21.5 *amount.*
5. 22*cwt.* 3*qrs.* 7*lbs. neat weight*, $217.32.8 *amount.*

§ 116. CASE 2.

2. 9*cwt.* 0*qr.* 6*lbs. neat weight*, $113.16.9 *value.*
3. 130*cwt.* 0*qr.* 15*lbs. neat weight*, $1 184.21.8 *value.*
4. 52*cwt.* 0*qr.* 14*lbs. neat weight*, $439.93⅓ *value.*
5. 31*cwt.* 0*qr.* 21*lbs. neat weight*, $1 816.36 *value.*

§ 117. CASE 3.

2. 89*cwt.* 2*qrs.* 7*lbs.* 3*oz. neat weight*, $648.44.4 *amount.*
3. 54*cwt.* 0*qr.* 3*lbs.* 6*oz. neat weight*, $440.88.6 *amount.*
4. 2 475*lbs. neat weight*, $1 460.25 *value.*
5. 39*cwt.* 3*qrs.* 23*lbs.* 4*oz. neat weight*, $214.17.2 *value.*

§ 118. CASE 4.

2. 8*cwt.* 1*qr.* 1*lb. neat weight*, $91.92 *amount.*
3. 37*cwt.* 2*qrs. neat weight*, $630 *amount.*
4. 1 225*lbs. neat weight*, $704.37½ *value.*
5. 63*cwt.* 0*qr.* 11*lbs.* 5*oz. neat weight*, $311.71.8 *value.*

§ 119. CUSTOM HOUSE BUSINESS.

2. $6.37½.
3. $1.53½.
4. $7.42½.
5. $56.73.
6. $7.46½.
7. $1366.49½

§ 121. SIMPLE INTEREST.

3. 14.46.
4. 14.04.
5. 162.
6. 160.423.
7. 83. 82.
8. 354.31.
9. 679.316.
10. 378.508.
11. 47.515.
12. 54.648.
13. 81.788.
14. 75.095.
15. 54.229.
16. 224.668.
17. 531.362.
18. 140.345.
19. 14.6925.
20. 72.75.
21. 107.231.
22. 194.371.
23. 620.0175
24. 478.533.
25. 324.633.
26. 1025.613.
27. 6 878.85.
28. 576.882.
29. 1 344.998.
30. 13 960.605.
31. 108.531.
32. 390.164.
33. 429.718.
34. 243.609.
35. 1369.444.
36. 330.775.
37. 302£ 5*s.* 8¾*d*

§ 122. CASE 1.

3. 4.82.
4. 8.736.
5. 32.978.
6. 69.525.
7. 60.289.
8. 74.149.
9. 97.266.
10. 1.216.
11. 8.493.
12. 1 649.954.
13. 1 126.327.
14. 78.326.
15. 371.541.
16. 2 285.521.
17. 850.813.
18. 2 591.589.
19. 1 384.947.
20. 1 794.04.

§ 123. CASE 2.

2. 3.556.
3. 16.748.
4. 40.894.
5. 52.687.
6. 38.221.
7. 0.732.
8. 2.005.
9. 96.634.
10. 1419.516.
11. 818.298.
12. 703.01¼.
13. 1 022.16⅔.
14. 3 120.50.
15. 988.406.
16. 707.854.
17. 1 099.822.

§ 124. CASE 3.

$	$	$
2. 45.43.	8. 69.602.	13. 18.642.
3. 2.072.	9. 26.872.	14. 220.98.
4. 47.262.	10. 24.368.	15. 793.848.
5. 55.071.	11. 0.828.	16. 4 601.722.
6. 1.361.	12. 27.125.	17. 8 341.95.
7. 61.		

§ 125. CASE 4.

$	$	$
2. 39.862.	9. 42.601.	15. 424.45.
3. 52.117.	10. 640.811.	16. 253.798.
4. 86.88.	11. 3.957.	17. 1 532.031.
5. 333.438.	12. 100.93.	18. 2 499.885.
6. 63.768.	13. 94.30.	19. 766.143.
7. 8.866.	14. 20.067	20. 3 469.37.
8. 1 670.658.		

§ 126. CASE 5.

Yrs.	*Yrs.*	*Yrs.*	*Yrs.*	*Yrs.*
2. 4.	5. 5.	7. $4\frac{1}{4}$.	9. 7.	11. $16\frac{2}{3}$
3. 5.	6. 3.	8. $2\frac{1}{2}$.	10. 6.	12. 20.
4 4.				

§ 127. CASE 6.

$	$	$
2. 550.75.	5. 543.	8. 701.
3. 450.	6. 684.	9. 275.
4. 500.	7. 932.	10. 810.50.

§ 128. CASE 7.

2. 6 *per cent.*	5. 6 *per cent.*	8. 6 *per cent.*
3. 4 "	6. 6 "	9. $3\frac{1}{2}$ "
4. 6 "	7. 5 "	10. 4 "

---

§ 129. PARTIAL PAYMENTS.

$	$	$
3. 1 714.997.	5. 862.663.	7. 241.54.
4. 475.524.	6. 388.87.	

§ 130. COMPOUND INTEREST.

2. $47.754.	6. $420.993 *amount.*	8. $21.567.
3. 286.524.	66.193 *interest.*	9. 106.159.
4. 1 431.022.	7. 157.486.	10. 54.547.
5. 945.783.		

§ 131. COMMISSION.

2. $6.18¾.	4. $27.206.	6. $1 900.
3. 98.70.	5. 17.075.	7. 10.434.

§ 132. BROKERAGE.

2. $4.869.	4. $5.788.	6. $18.37½.
3. 42.967.	5. 12.483.	7. 4.401.

§ 133. INSURANCE.

2. $63.75.	4. $4 494.975.	6. $1 680.
3. 28.999.	5. 825.	

§ 134. STOCKS.

3. $953.33⅓.	5. $3 402.	7. $10 587.50.
4. 788.40.	6. 1 230.62½	

DISCOUNT.

§ 136. CASE 1.

2. $597.706.	7. $833.906.	12. $900.
3. 282.18¾.	8. 1 600.	13. 4 950.
4. 600.	9. 3 200.	14. 11 569.438.
5. 1 310.679.	10. 1 800.	15. 270.
6. 148.44.	11. 234.166.	

§ 137. CASE 2.

2. 28.234.	7. 3.60.	12. 13.753.
3. 26.855.	8. 65.463.	13. 17.06.
4. 38.135.	9. 41.914.	14. 39.43.
5. 26.25.	10. 24.842.	15. 56.662.
6. 13.106.	11. 140.185.	

BANKING.*

§ 139. CASE 1.

2. 9.40 *discount.* 590.60 *present worth.*	5. 23.50.	9. 5 102.784.
3. 8 926.50.	6. 196.	10. 3 809.48.
4. 15.02.	7. 3 525.60.	11. 3.829.
	8. 1 480.99.	12. 3 957.334.

§ 140. CASE 2.

2. 7 501.26.	4. 151.88.	6. 1 520.26.
3. 998.11.	5. 6 264.50.	

* In a few copies of the United States Arithmetic, being those first published, it is stated that banks calculate the discount (or, more correctly, the interest) on a note discounted, for the number of days intervening between the day when discounted and the last day of grace, when the note is payable. The answers to the examples, resulting from this rule, are given below. But banks in this country universally take interest for one day more than the time stated above, because they cannot reloan a sum of money until the day after it is paid into bank, as the payment may not be made until a moment before the hour for ceasing business. The answers given above are adapted to the actual usage of banks.

CASE 1.

2. $9.30 *discount.* $590.70 *present worth.*	6. $195.30.	11. $3.716.
3. $8 928.	7. $3 526.20.	12. $3 958.
4. $14.87½.	8. $1 481.24.	13. $35.75.
5. $23.25.	9. $5 103.648.	14. $2.555.
	10. $3 810.13½.	15. $47.356.

CASE 2.

2. $7 500.	4. $151.86.	6. $1 520.
3. $997.93.	5. $6 263.59.	

§ 141. EQUATION OF PAYMENTS.

2. 6 *months.*
3. 4*mos.* 13*ds.*
4. 6 *months.*
5. 4*mos.* 5*ds.*
6. 12*mos.* 15*ds.*
7. 6*mos.* 23*ds.*
8. 5 *months.*
9. 6 *months.*
10. 11 *months.*
11. 7 *months.*
12. $2\frac{1}{2}$ *months.*
13. 6 *months.*
14. 6*mos.* 29*ds.*
15. 77 *days.*

BARTER.

§ 143. CASE 1.

3. 182 *yards.*
4. 47*cwt.*
5. 21*cwt.* 1*qr.* 14*lbs.*
6. 75 *dozens.*

§ 144. CASE 2.

2. 242*yds.* | 3. 1 272*lbs.* | 4. 1 705$\frac{5}{19}$*lbs.* | 5. 802$\frac{1}{2}$*lbs.*

§ 145. CASE 3.

2. \$2.45$\frac{5}{9}$. | 3. .075*cts.*+ | 4. \$2.

§ 146. CASE 4.

2. 10$\frac{3}{5}$*cts.* | 3. 74$\frac{1}{4}$*cts.* | 4. \$7.20.

Page 191.] MISCELLANEOUS EXERCISES.

1. \$2.50.
2. 1 120*lbs.*
3. 926$\frac{3}{4}\frac{4}{1}$ *yards.*
4. 210$\frac{1}{7}$ *bushels.*
5. 26$\frac{1}{4}$*lbs.*
6. 19$\frac{1}{2}$*cts. per lb.*, and 800*lbs. for* 100 *doz.*
7. 9*cts. per lb.*, and 1867$\frac{1}{2}$ *ells.*
8. 4$\frac{4}{5}$*cts.*
9. \$55.648.
10. 218$\frac{3}{11}$ *bushels, at* \$1.04$\frac{1}{2}$.

LOSS AND GAIN.

§ 148. CASE 1.

2. \$4.40. | 3. \$448 *gain.* | 4. \$1 300.

§ 149. CASE 2.

2. 6$\frac{7}{8}$*cts.* | 3. 21$\frac{3}{5}$*cts.* | 4. \$8.792.

§ 150. CASE 3.

2. 10*cts.* | 3. \$18.18. | 4. \$0.612.

§ 151. CASE 4.

4. 16$\frac{2}{3}$ *per cent.*
5. 21$\frac{17}{23}$ *per cent.*
6. \$6,30.
7. 15 *per cent.*
8. 40 *per cent.*

§ 152. CASE 5.

2. $6. | 3. $100.80. | 4. 63+ *cents.*

MISCELLANEOUS EXERCISES.

Pages 194 and 195.]

1. $43.20 *loss.*
2. $2.
3. $4.80.
4. 9*cts.*
5. 75*cts.*
6. $1.80.
7. $380.80 *loss.*
8. $1.5456.
9. $5.568.
10. $1.1728.
11. $23.244.
12. $12\frac{1}{2}$*cts.*
13. 80*lbs.*

§ 154. SINGLE FELLOWSHIP.

3. *A.* $100, *B.* $200.
4. *D.* $250, *E.* $500, *F.* $750, *G.* $1 000.
5. $190, $380, $570, $760.
6. *S.* $256.40, *T.* $524.50, *U.* $567.10.
7. *W.* $1 500, *X.* $1 500, *Y.* $3 000, *Z.* $6 000.
8. *A.* $144, *B.* $576, *C.* $240.
9. *M.* $225.439, *P.* $1 288.227, *L.* $452.892.
10. *C.* $1 052.203, *D.* $909.636, *E.* $1 399.898.
11. *O.* $120, *P.* $160, *Q.* 200.
12. *A.* 175*A* 2*r.* 33.6*p.*, *B.* 190*A.* 1*r.* 16.4*p.*, *C.* 219*A.* 2*r.* 22*p.*
13. *L.* 19 *tuns*, *M.* 20 *tuns*, *N.* 22 *tuns*, *O* 11 *tuns.*
14. *A.* 166.66\frac{2}{3}$, *B.* $125, *C.* $100, *D.* 108.33\frac{1}{3}$.
15. $253.846.
16. *A.* $1 052.63$\frac{3}{19}$, *B.* 789.47\frac{7}{19}$, *C.* 631.57\frac{17}{19}$, *D* 526.31\frac{11}{19}$.

§ 155. DOUBLE FELLOWSHIP.

3. *A.* $75, *B.* $175.
4. *Q.* $150, *R.* $225, *S.* $125.
5. *L.* 32.30\frac{10}{13}$, *M.* 26.92\frac{4}{13}$, *N.* 30.76$\frac{12}{13}$.
6. *A.* $1 050, *B.* $1 050.
7. *A.* $511.8483, *B.* $483.4123, *C.* $204.7393.
8. *T.* 434.89\frac{139}{149}$, *U.* 591.94\frac{94}{149}$, *V.* 773.15\frac{65}{149}$.
9. *A.* $97.688, *B.* $155.10, *C.* $66.41.
10. *A.* 319.81\frac{151}{429}$, *B.* 371.21\frac{91}{429}$, *C.* 358.97\frac{187}{429}$.

§ 156. TAXES.

3. 59.34\frac{3}{8}$.

§ 157. INVOLUTION.

2. 1444.
3. 24389.
4. 184528125.
5. 148035889.
6. 2187.
7. 133.6336.
8. .000032768.
9. $\frac{625}{1681}$.
10. $\frac{2197}{2744}$.
11. $\frac{625}{6561}$.
12. 1048576.
13. 134217728.
14. 65536.
15. 32768.
16. 134217728.

§ 159. THE SQUARE ROOT.

4. 42.
5. 49.
6. 87.
7. 101.
8. 143.
9. 193.
10. 204.
11. 244.
12. 278.
13. 311.
14. 349.
15. 388.
16. 401.
17. 436.
18. 449.
19. 518.
20. 547.
21. 618.
22. 648.
23. 717.
24. 832.
25. 889.
26. 939.
27. 998.
28. 1049.
29. 1617.
30. 1949.
31. 2546.
32. 3228.
33. 3738.
34. 4048.
35. 4519.
36. 5836.
37. 6044.
38. 6533.
39. 7752.
40. 8528.
41. 86.46.
42. 9.006.
43. .9433.
44. 964.4.
45. 98.42.
46. 9.999.
47. 60.9672+
48. 79.492+
49. 84.9764+
50. .632455+
51. .23664+
52. .08882+
53. .094868+
54. 607.3409+
55. 1.78106+
56. .01809+
57. $\frac{3}{4}$.
58. $\frac{7}{8}$.
59. .816+
60. 1.32287+
61. 1.2472+
62. 3.41869+
63. 2.71313+
64. .6454+
65. .2425+
66. 3.2093+
67. 2.5298+
68. 92.9462+
69. 15367.
70. 40093.
71. 27943.
72. 534762.
73. 6950078.

§ 160. THE CUBE ROOT.

2. 51.
3. 87.
4. 99.
5. 135.
6. 149.
7. 175.
8. 289.
9. 399.
10. 498.
11. 597.
12. 7.67.
13. .888.
14. 96.2.
15. .9226+
16. 275.34.
17. 115.7625.
18. 1.92686+
19. $\frac{378}{480}=\frac{63}{80}$.
20. $7\frac{3}{5}$.
21. 2.3908+

§ 161.

2. 1.346079+
3. 1.331949+
4. 1.801028+
5. .8526+
6. .9189+
7. 2.154434+
8. 19.239+
9. 39.41+

§ 162. ARITHMETICAL SYMBOLS.

1. 39.
2. 11.
3. 217.
4. 8.
5. 10.
6. 36.
7. 17.
8. 117.
9. 23.
10. 16.
11. 5.
12. 2.
13. 6.
14. 96.
15. 84.
16. 90.
17. 5.
18. 280.
19. 5.
20. 6.
21. 8.
22. 2.
23. 8.
24. 44892.
25. $\frac{113}{2002}$.
26. $\frac{52}{75}$.
27. $\frac{24}{6}$.
28. $\frac{7}{15}$.
29. 2.00380+
30. 2.9544375.
31. .4187+
32. 7.275+
33. 4.179447+
34. 1.1644+
35. .66527+
36. 13.5.
37. 2.088+
38. .0478+
39. $\frac{1612}{2875}$.
40. $\frac{1}{2}$.
41. 2.9278+

ARITHMETICAL PROGRESSION.

§ 164.

CASE 1.

2. 121 *sum of series.*
3. 300 *strokes.*
4. 17 *miles*, 380 *yards.*
5. 948 *sum of series.*
6. $140.40 *debt.*
7. $48 *cost.*

§ 165.

CASE 2.

2. 1, *common difference.*
3. 6, " "
4. 3, " "
5. 2, " "
6. 10 *cents.*
7. 2, *com. diff.*, 99 *sum of series.*
8. 5 *daily increase*, 366 *miles distance.*

§ 166.

CASE 3.

2. 24 *number of terms.*
3. 52 " "
4. 30 *yards.*
5. 14 *days.*

§ 167.

CASE 4.

2. 5 *years.*
3. 5 *first term.*
4. 3 *first term.*
5. 15 *cents.*

§ 168.

CASE 5.

2. 5 *years.*
3. 3 *first term.*

§ 169.

CASE 6.

2. 74 *last term.*
3. 61 *last term.*

Pages 218—223.]

GEOMETRICAL PROGRESSION.

§ 171. CASE 1.

2. 87 381 *sum of series.*
3. 175 099 "
4. 19 682 "
5. $13 981.01.
6. 19 980 *sum of series.*
7. 33 554 430 *grains.*

§ 172. CASE 2.

RULE 1.

2. 4 096 *last term.*
3. $104 603 532.03.

RULE 2.

2. $62 914.56.
3. $7 680.
4. $204 800.

§ 173. CASE 3.

2. $42 949 672.95.
3. $4 095.
4. $2 152.33.
5. $55 924.05.

§ 174. CASE 4.

2. 4 *ratio.*
3. 3 *ratio.*
4. 2 *ratio.*

---

§ 175. PERMUTATION.

3. 3 628 800 *changes.*
4. 479 001 600 "
5. 403 291 461 126 605 635 584 000 000.
6. 479 001 600 *changes,*
   35 *Y.* 10*ds.* 2*h.* 18*min.* 27*sec. the time.*

---

§ 176. COMBINATION.

2. $10 378 368.
3. 27 113 264 640.

ALLIGATION.

§ 178. CASE 1.

2. 52*cts.*
3. 66+*cts.*
4. $20\frac{8}{19}$ *carets.*
5. 6*oz.* 13*dwt.* 8*grs.*
6. $2.16.
7. $1.57\frac{1}{3}$.

§ 179. CASE 2.

2. 1st ans.
   - 12*lbs.* at 42*cts.*
   - 6 " " 36 "
   - 6 " " 24 "
   - 12 " " 18 "

   2d ans.
   - 6*lbs.* at 42*cts.*
   - 12 " " 36 "
   - 12 " " 24 "
   - 6 " " 18 "

3. 1st ans.
   - 12*bu.* at 48*cts.*
   - 6 " " 60 "
   - 6 " " 72 "
   - 18 " " 78 "

   2d ans.
   - 6*bu.* at 48*cts.*
   - 12 " " 60 "
   - 18 " " 72 "
   - 6 " " 78 "

5. 6*bu.* at 60*cts.*
   - 6 " " 48 "
   - 24 " " 36 "

6. 2*gals.* at 16*cts.*
   - 2 " " 12 "
   - 8 " " 8 "

8. 9*oz.* of 22 *carats*, 2*oz.* of 18 *carats*, 2*oz.* of 17 *carats*, and 2*oz.* of 16 *carats*.
9. 6*lbs.* (or any equal quantity, more or less) *of each*.

§ 180. CASE 3.

2.
- 30*gals.* at $2.88.
- 20 " " 1.44.
- 20 " " 0.96.

3.
- 24*lbs.* at 90*cts.*
- $14\frac{2}{7}$ " " 65 "
- $14\frac{2}{7}$ " " 60 "

5.
- 28*lbs.* at 45*cts.*
- 28 " " 51 "
- 112 " " 54 "

6. 30*bu.* at $1.08, and 30*bu.* at 96*cts.*

§ 181. CASE 4.

2.
- 48 *lbs.* at 96*cts.*
- 32 " " 60 "
- 32 " " 48 "

3.
- 6*oz.* of 24 *carets.*
- 6 " " 22 "
- 18 " " 18 "

5.
- 42*gals.* of *Canary.*
- 21 " " *Malaga.*
- 21 " " *Rhenish.*
- 42 " " *Oporto.*

6.
- 24*lbs.* at 48*cts.*
- 8 " " 60 "
- 8 " " 84 "
- 16 " " $1.08.

---

§ 183. SINGLE POSITION.

2. $144.
3. $320.
4. 60.
5. $1 000.
6. *A.* $300, *B.* $600, and *C.* $2 400.
7. $625.
8. $600.
9. 32*min.* $43\frac{7}{11}$*sec.*
10. $268.80.
11. 60 *years.*
12. $360.
13. $80 *harness*, $160 *horse*, and $480 *carriage.*

§ 184. DOUBLE POSITION.

3. $125 *income.* *A. spends* $100, and *B.*, $150.
4. $300.
5. *A* $310, *B.* $190.
6. *A.* $600, *B.* $1 000, and *C.* $1 400.
7. *First*, $1 422$\frac{2}{9}$, *second*, $2 033$\frac{1}{3}$, and the *third*, $2 544$\frac{4}{9}$
8. *Worked* 30 *days*, and *idle* 10 *days.*
9. 6 *feet.*
10. 24 *minutes past* 10 A.M.

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§ 186. ANNUITIES.

CASE 1.

2. $2818.546. | 3. $394.914. | 4. $2546.88.

§ 187.

CASE 2.

2. $1731.79. | 3. $4158.126.

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§ 188. EXCHANGE.

1. $2520.22+
2. 5211.97\frac{3}{4}$+
3. 941£ 15*s.* 5*d. sterling.*
4. 2003.90$\frac{1}{2}$—.
5. 545.50\frac{1}{2}$—.
6. 4560 *francs*, 65 *centimes.*
7. 876.47\frac{1}{4}$.
8. 1271.07\frac{3}{4}$.
9. 1823 *florins*, 50+*kreutzers.*
10. 913.55\frac{3}{4}$+
11. 1299.87\frac{1}{2}$—.
12. 10 473 *rials of old plate*, 3 *quintos.*
13. 3150.56\frac{1}{4}$.
14. 339.55\frac{1}{4}$+
15. 8258 *milrees*, 160 *rees.*
16. $656 87$\frac{1}{2}$.

17. *Company's rupees* 4800, *cost* 2444.44\frac{4}{9}$.
    1 *Company's rupee cost* 50 *cents* 9$\frac{1}{4}$ *mills.*
18. $3428.57.+ *Canton money of account cost* 3875.55\frac{5}{9}$
    Federal money.
    $1 *Canton money of acct. cost* $1.13.0$\frac{1}{8}$ *Federal money.*
19. 7200 *florins cost* 2893.33\frac{1}{3}$.
    1 *florin cost* 40 *cents* 1$\frac{1}{3}$ *mills.*

§ 196. MENSURATION.

PROBLEM 1.

2. 15*A.* 2*r.* 20*p.* | 3. 17*ft.* 4*in.* 4*ll.*

PROBLEM 2.

2. 45 *feet.* | 3. 31.62 *perches.*

PROBLEM 3.

2. 128 *feet.* | 3. 72 *feet.*

PROBLEM 4.

2. 34 *feet.* | 3. 16 *yards.*

PROBLEM 5.

2. 25.45 *feet.* | 3. 33.94 *chains.*

PROBLEM 6.

2. 9 *feet.* | 3. $15\frac{5}{21}$ *rods.*

PROBLEM 7.

2. 24113.66 *yards.* | 3. 30*A.* 1*r.* 32*rods.*

PROBLEM 8.

2. 36 *perches.* | 3. $20\frac{26}{41}$ *chains.*

PROBLEM 9.

2. 174*ft.* 4*in.* 8*ll.* | 3. 68*A.* 1*r.* 24*p.*

PROBLEM 10.

2. 5*A.* 0*r.* 33*p.* | 3. 2*A.* 2*r.* 32*p.*

PROBLEM 11.

2. 61.498 *acres.* | 3. 6 *yards.*

PROBLEM 12.

2. 18.973, 25.298, and 31.622 *chains.*
3. 9.64, 21.208, and 25.064 *chains.*

PROBLEM 13.

2. 30 *feet.* | 5. 15 *chains.* | 8. 12.369 *yards.*
3. 30.397 *chains.* | 6. 26.999 *yards.* | 9. 9.98 *feet.*
4. 20 *chains.* | 7. 97.754 *miles.* | 10. 42 *feet.*

PROBLEM 14.

2. 2*A.* 1*r.* 36*p.* | 3. 226.741$\frac{2}{3}$ *yards.*

PROBLEM 15.

2. 17*A.* 2*r.* 16$\frac{1}{2}$*p.* | 3. 921.875 *feet.*

PROBLEM 16.

2. 709.5 *feet.* | 3. 312.96 *yards.*

PROBLEM 17.

2. 16.2316 *feet.* | 3. 24928.596 *miles.*

2. 5.2521 *feet.*

PROBLEM 18.

| 3. 40.743 *feet.*

2. 4*A.* 2*r.* 13.4*p.*

PROBLEM 19.

| 3. 2*A.* 0*r.* 18*p.*

2. 34.7 *perches.*

PROBLEM 20.

| 3. 63.4 *perches.*

2. 3*A.* 3*r.* 3*p.*

PROBLEM 21.

| 3. 339.2928 *feet.*

2. 80.095 *feet.*

PROBLEM 22.

| 3. 222.144 *chains.*

2. 99.378 *perches.*

PROBLEM 23.

| 3. 69.57 *yards.*

2. 2646 *inches.*

PROBLEM 24.

| 3. 216 *feet.*

2. 6 *feet.*

PROBLEM 25.

| 3. 4 *feet.*

2. $7\frac{71}{1728}$ *feet.*

PROBLEM 26.

| 3. 438976 *inches.*

2. 50.53 *inches.*

PROBLEM 27.

| 3. 32.53 *inches.*

2. 78.72 *bushels.*

PROBLEM 28.

| 3. $157\frac{1}{2}$ *feet.*

2. $53.208.

PROBLEM 29.

| 3. 1176 *feet.*

3. 18.15 *yards.*

PROBLEM 30.

2. 21.65 *gallons.*

PROBLEM 31.

| 3. 217.89 *feet.*

2. 141.372 *feet.*

PROBLEM 32.

| 3. 71.45952 *feet.*

2. 61.11744 *feet.*

PROBLEM 33.

| 3. 75.3984 *feet.*

2. 1172.6 *yards.*

PROBLEM 34.

| 3. 43647.66 *feet.*

2. 490.875 *feet.*

PROBLEM 35.

| 3. 197808409.26 *miles.*

2. 22.44935 *feet.*

PROBLEM 36.

| 3. 261601621246.35 *miles.*

2. 2.84 *feet.*

PROBLEM 37.

| 3. 7.08 *feet.*

ARTIFICERS' WORK.

§ 198. MASONS' WORK.

2. 33.25 *feet.* $26.60 *value.*

§ 199. BRICKLAYERS' WORK.

4. 16.01 *rods.*
5. 79 105 *bricks.* $514.18¼ *value.*

§ 200. CARPENTERS' AND JOINERS WORK.

2. 17¼ *squares.*
3. 24.3425 *squares.* $35.396 *expense.*
4. 14.25 *squares.*
5. 19.0575 *squares.* $26.204 *expense.*

§ 201. PLASTERERS' WORK.

2. 85.3 *yards of rendering* and 45⅓ *yards of ceiling.* $32.659 *cost.*
3. 128.62 *yards of rendering,* 79⅓ *yards of ceiling,* 78 *ft.* 1*in.* 10″ *of cornice.*
4. 308.4375 *yards.*

§ 202. PAINTERS' AND GLAZIERS' WORK.

2. 84⅝ *yards of walls and ceiling.* $35.65⅛ *cost.*
3. 141½ *yards in the three rooms.* $49.38⅝ *cost.*
4. 516 *lights.* $30.96 *cost.*

§ 203. PAVERS' WORK.

2. 542⅔ *yards*
3. 416 *yards area of the whole yard,* and 52 *yards area of foot path.* $112.84 *expense.*

THE END.